

SCIENTIFIC SECTION

Mathematics

Applications

By a group of supervisors

Interactive E-learning
Application



The Main Book



FIRST TERM
2
SEC.
2024

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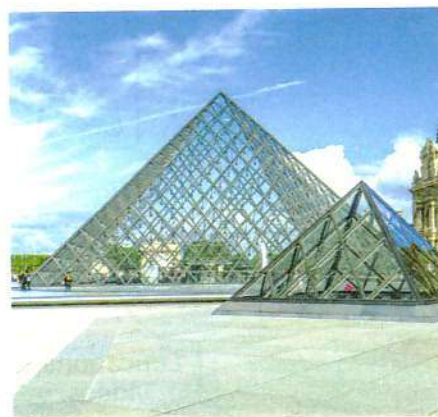
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Revision on vectors



The quantities which we deal with in our life are divided into two kinds of quantities :

(1) Scalar quantities : They are the quantities which are completely determined if we knew their magnitude only as a real number

As : length , mass , time , temperature degree , volume , distance.

(2) Vector quantity : It is a quantity determined by a real number (the magnitude of this quantity) besides the direction.

i.e. The vector quantity is determined completely if we knew its magnitude and its direction.

• **Directed line segment :**

It is a straight line segment having an initial point and an ending point , with direction defined from the initial point to the ending point.

- The norm of the directed line segment (\overrightarrow{AB}) :
the norm of \overrightarrow{AB} is the length of \overrightarrow{AB} and it is denoted by $\|\overrightarrow{AB}\|$
- The two directed line segments are equivalent if they have :
 - The same length (the norm) and the same direction.
- $\overrightarrow{AB} \neq \overrightarrow{BA}$ (they have opposite directions)
- $\overrightarrow{AB} = -\overrightarrow{BA}$
- $\|\overrightarrow{AB}\| = \|\overrightarrow{BA}\|$

- The position vector of a given point (A) with respect to the origin point (O) it is the directed line segment \overrightarrow{OA} , it is denoted by \vec{A}

For example :

In the opposite figure :

If \overrightarrow{OA} is the position vector of the point (A) = (X, y) then :

* $\|\vec{A}\|$ = the length of $\overrightarrow{OA} = \sqrt{x^2 + y^2}$

* If $\|\vec{A}\| = 1$ length unit (unity)
 , then \vec{A} is called the unit vector.

* $\vec{i} = (1, 0)$

, $\vec{j} = (0, 1)$ are two unit vectors

(called two basic unit vectors) in the two directions of the two coordinate axes.

* $\vec{O} = (0, 0)$ it is zero vector which has no direction and it is denoted by \vec{O}

* $\vec{A} = (x, y)$ is called the cartesian form of the vector \vec{A}

* $\vec{A} = x\vec{i} + y\vec{j}$ expresses the vector \vec{A} in terms of the two basic unit vectors.

* $\vec{A} = (\|\vec{A}\|, \theta)$ is called the polar form of the vector \vec{A}

* θ is the measure of the angle made by the vector \overrightarrow{OA} with the positive direction of X-axis, it is called the polar angle.

* $x = \|\vec{A}\| \cos \theta$, then $\cos \theta = \frac{x}{\|\vec{A}\|}$

* $y = \|\vec{A}\| \sin \theta$, then $\sin \theta = \frac{y}{\|\vec{A}\|}$

• If $\vec{A} = (x_1, y_1)$, $\vec{B} = (x_2, y_2)$, then :

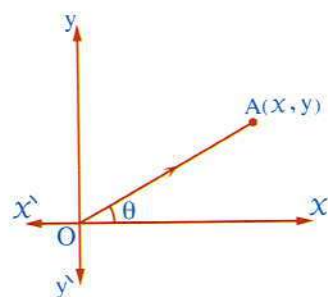
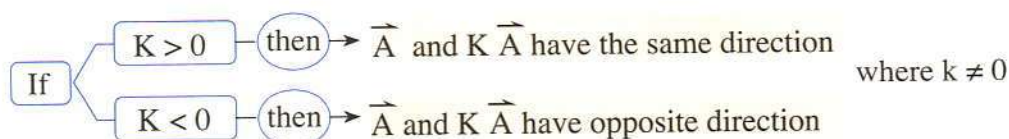
* $\vec{A} = \vec{B}$ if and only if $x_1 = x_2, y_1 = y_2$

$\vec{A} \pm \vec{B} = (x_1 \pm x_2, y_1 \pm y_2)$

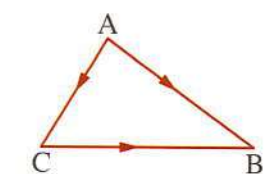
* $\overrightarrow{AB} = \vec{B} - \vec{A} = (x_2 - x_1, y_2 - y_1)$

* $K\vec{A} = K(x, y) = (Kx, Ky)$

* $\vec{A} // K\vec{A}$ with regarding that :

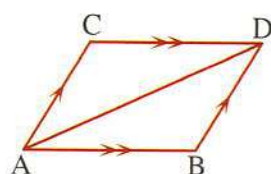


• Adding and subtracting vectors geometrically :



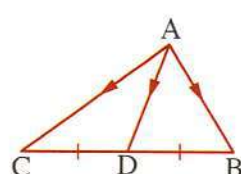
$$\vec{AB} + \vec{BC} = \vec{AC}$$

$$\vec{AB} + \vec{BC} + \vec{CA} = \vec{0}$$



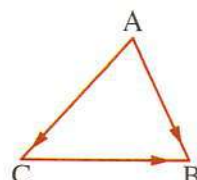
$$\vec{AB} + \vec{BD} = \vec{AD}$$

$$\vec{AB} + \vec{AC} = \vec{AD}$$



$$\vec{AB} + \vec{AC} = 2 \vec{AD}$$

$$\vec{BD} + \vec{CD} = \vec{0}$$



$$\vec{AB} - \vec{AC} = \vec{CB}$$

• Physical applications :

The resultant force \vec{R}

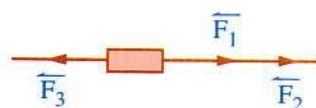
- The resultant of a set of forces acting on a body is operated as the operation of adding vectors

i.e. The resultant force

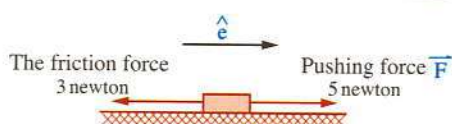
$$\vec{R} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \dots$$

For example :

If we defined a unit vector \hat{e} in the direction of the motion of the body then :



In the case of motion of the body on a rough plane



The resultant force

$$\vec{R} = 5 \hat{e} + (-3 \hat{e}) = 2 \hat{e}$$

i.e. The magnitude of the resultant = 2 newton

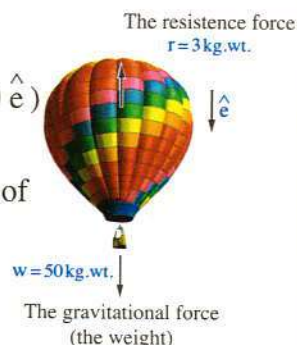
- The direction of the resultant is in the direction of the motion of the body.

In the case of vertical motion

The resultant

$$\text{force} = 50 \hat{e} + (-30 \hat{e}) = 20 \hat{e}$$

i.e. The magnitude of the resultant = 20 Kg.wt.



- The direction of resultant is in the direction of the weight

- If the two forces have the same magnitude and the same line of action but in two opposite directions then the resultant $\vec{R} = \vec{0}$
- If the resultant of a set of concurrent forces = $\vec{0}$ this means the set of forces are in equilibrium.

Example 1

- (1) Write the vector $\vec{A} = (3, -\sqrt{3})$ in the polar form.
- (2) Write in terms of the two basic unit vectors the vector \vec{A} whose norm = 10 length unit and act in the direction of Western North.

Solution

(1) $\therefore \|\vec{A}\| = \sqrt{9 + 3} = 2\sqrt{3}$

$$\therefore \cos \theta = \frac{x}{\|\vec{A}\|} = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2} > 0$$

$\therefore \theta$ lies in the 4th quadrant

$$\therefore \vec{A} = (2\sqrt{3}, 330^\circ)$$

(2) $\therefore \|\vec{A}\| = 10, \theta = 135^\circ$

$$y = \|\vec{A}\| \sin \theta = 10 \sin 135^\circ = 5\sqrt{2}$$

$$\therefore \vec{A} = (-5\sqrt{2}, 5\sqrt{2})$$

$$\sin \theta = \frac{y}{\|\vec{A}\|} = \frac{-\sqrt{3}}{2\sqrt{3}} = -\frac{1}{2} < 0$$

$$\therefore \theta = 360 - 30 = 330^\circ$$

$$\therefore x = \|\vec{A}\| \cos \theta = 10 \cos 135^\circ = -5\sqrt{2}$$

$$\therefore \vec{A} = -5\sqrt{2} \hat{i} + 5\sqrt{2} \hat{j}$$

Example 2

If the forces $\vec{F}_1 = 2\hat{i} + 3\hat{j}$, $\vec{F}_2 = a\hat{i} + \hat{j}$, $\vec{F}_3 = 5\hat{i} + b\hat{j}$ act on a particle

Find the values of a and b if these forces :

(1) Their resultant = $5\hat{i} - 2\hat{j}$

(2) Are in equilibrium.

Solution

$$\begin{aligned} \text{The resultant} &= \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = (2\hat{i} + 3\hat{j}) + (a\hat{i} + \hat{j}) + (5\hat{i} + b\hat{j}) \\ &= (2 + a + 5)\hat{i} + (3 + 1 + b)\hat{j} \end{aligned}$$

(1) \therefore The resultant = $5\hat{i} - 2\hat{j}$

$$\therefore (7 + a)\hat{i} + (4 + b)\hat{j} = 5\hat{i} - 2\hat{j}$$

$$\therefore 7 + a = 5$$

$$\therefore a = -2$$

$$\therefore 4 + b = -2$$

$$\therefore b = -6$$

(2) \therefore The forces are in equilibrium

$$\therefore \vec{R} = \vec{O}$$

$$\therefore (7 + a)\hat{i} + (4 + b)\hat{j} = \vec{O}$$

$$\therefore a = -7, \quad b = -4$$

Unit One

Statics



Lesson

1

Forces - Resultant of two forces meeting at a point.

Lesson

2

Forces resolution into two components.

Lesson

3

The resultant of coplanar forces meeting at a point.

Lesson

4

Equilibrium of a rigid body under the effect of two forces / three forces meeting at a point.

(The triangle of forces rule - Lami's rule).

Lesson

5

Follow : The equilibrium

(Meeting lines of action of three equilibrium forces).

Lesson

1

Forces - Resultant of two forces meeting at a point



The force

"The force is defined as the effect of a natural body upon another one" by pushing, attraction, pressure or repulsion".

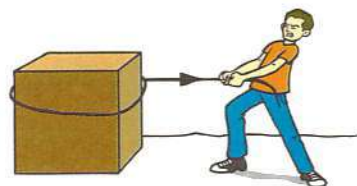
The natural body is a body consisting of material (mass) and volume not equal to zero.

The natural bodies can be classified into two kinds :

- **Rigid bodies (solid bodies)** : They are the bodies whose shapes do not change whatever the forces which are acting on them as solid metals, rocks,



- **Deformable bodies** : They are the bodies whose shapes can be disfigured as strings, liquids, gases, rubber and clay and our study in this unit will be continued to rigid bodies only.

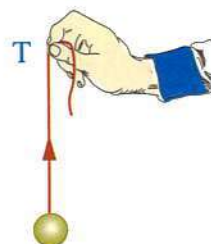


Kinds of forces

There are different kinds of forces , as :

1 Tension force (T) :

As the force in the string (or the rope) when carrying a body at it.



(Tension in the string)

2 Pressure force (P) :

As the force that appears when a body stabilized on a surface.

3 Reaction force (r) :

As the reaction of a smooth surface on a body stabilized on it.

4 Attraction forces and repulsion forces :

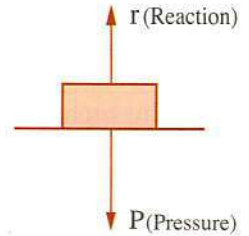
As the forces which formed between magnetic poles , electric charges and astronomical objects.

5 Gravitational forces (weights) :

If we let a body in the air , then it will drop down towards the Earth because the attraction force of the Earth attracts any body towards it.

This force is called gravitational force or weights.

- Notice that : the weight (W) = the body mass \times acceleration of gravity = $m \times g$

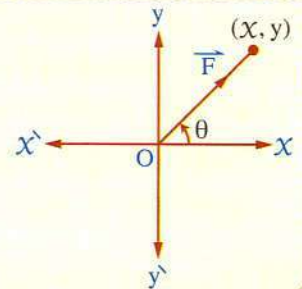


Expressing force

The force is a vector quantity so it can be represented by the same way as the vectors.

i.e. The force can be expressed as follow :

- (1) $\vec{F} = (x, y) \Rightarrow$ the cartesian form.
- (2) $\vec{F} = x\vec{i} + y\vec{j} \Rightarrow$ in terms of the fundamental unit vectors.
- (3) $\vec{F} = (\|\vec{F}\|, \theta) \Rightarrow$ the polar form.



Determination of the force

The force is a vector which passes through a fixed point.

i.e. It acts in a given straight line.

i.e. The force is determined by :

- (1) The magnitude of the force.
- (2) The direction of the force.
- (3) The point of action of the force.

For example :

The football player kicks the ball by a determined force (magnitude of the force) in a determined direction (direction of the force) in a certain point on the surface of the ball (Point of action of the force)



The magnitude of the force

1 The measurement units :

- * The magnitude of the force (The numerical value of the force) is measured by units which are called weight units.

As : gram weight (gm.wt.) , kilogram weight (kg.wt.)

, where $1 \text{ kg.wt.} = 1000 \text{ gm.wt.} = 10^3 \text{ gm.wt.}$

- * There are other units to measure the magnitude of force (they are called absolute units)

As : The dyne , the newton :

, where $1 \text{ newton} = 100\,000 \text{ dyne} = 10^5 \text{ dyne}$

- * The weight units connect with the absolute units by the relation :

$1 \text{ kg.wt.} = 9.8 \text{ newton}$, $1 \text{ gm.wt.} = 980 \text{ dyne}$

(unless something else mentioned)

2 The direction of the force :

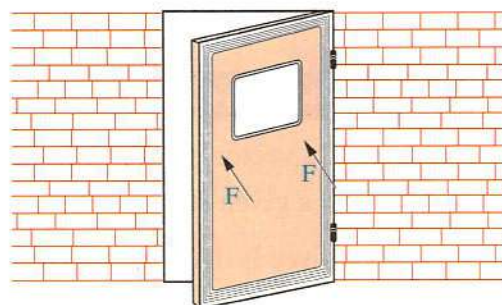
It is the direction of the vector which represents this force and it is determined by the measure of the polar angle of the force vector in the case of the coplanar forces in the same plane.

- The polar angle is the positive directed angle which the vector makes with the positive direction of X-axis.

3 The point of action of the force :

The action of the force is determined by its point of action. If you try to open the door of a room or close it with a force near of the line of hinges , you will find difficult to rotate it.

As you are far from the line of hinges as the difficulty becomes less. As shown in the figure.

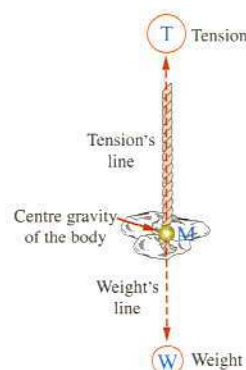


The line of action of the force

The line of action of the force is the line passing through the point of action parallel to the direction of the force.

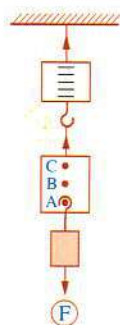
For example :

- The tension line in a string is the string itself.
- The line of action of the weight of a body is the vertical line passing through the center gravity of the body.



Displacing (or translation) of the point of action of the force (force penetration)

If the force \vec{F} acts on a rigid body and A is the point of its action, then we can displace this point to another point on the body "B" or "C" or on the line of action of the force without changing in its influence on the body.



i.e. Any point lying on the line of action of a force can be considered as a point of action of this force.

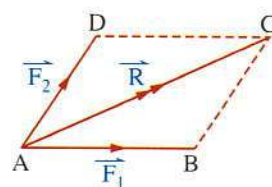
Resultant of two forces meeting at a point

The resultant of two or more forces is a single force has the same effect as the two or more forces.

Finding the resultant of two forces meeting at a point (geometrically)

This method depends on the parallelogram rule to add two forces :

If two forces (\vec{F}_1, \vec{F}_2) meeting at a point are represented in magnitude and direction by two sides of a parallelogram meeting at this point, then their resultant (\vec{R}) is represented in magnitude and direction by the diagonal of the parallelogram which starts from the same point.



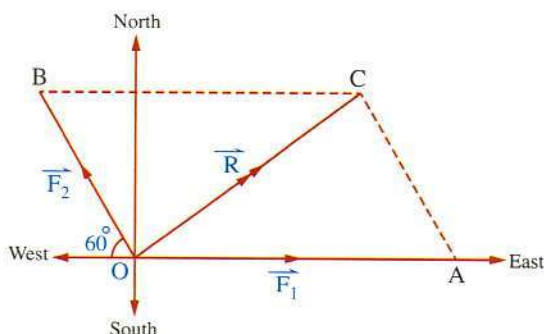
i.e. $\vec{R} = \vec{F}_1 + \vec{F}_2$

Example 1

\vec{F}_1 and \vec{F}_2 are two forces acting on the point O from a solid body, where $F_1 = 500$ newton and acts on the direction of East, $F_2 = 300$ newton and acts in the direction 60° North of west, find their resultant graphically.

Solution

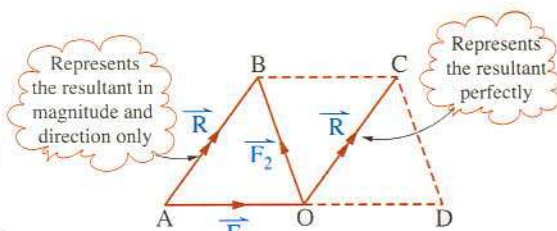
- * We use the drawing scale one cm. per 100 newton
- * Draw \vec{OA} to represent \vec{F}_1 and \vec{OB} to represent \vec{F}_2 where $\|\vec{OA}\| = 5$ cm. , $\|\vec{OB}\| = 3$ cm.
- * Then complete the parallelogram OACB, then \vec{OC} represents the resultant \vec{R}



* By measuring , we find that $\|\vec{OC}\| = 4.4$ cm. approximately , $m(\angle AOC) = 37^\circ$
 $\therefore \vec{R}$ acts at O and its magnitude = 440 newton in the direction 37° North of east approximately.

Remark

If \vec{F}_1 and \vec{F}_2 act at the point O and if they are represented by the two vectors \vec{AO} and \vec{OB} as in the opposite figure , then according to the rule of addition of two vectors , \vec{AB} represents the resultant of these two vectors. But the line of action of the resultant of \vec{F}_1 and \vec{F}_2 must pass through O

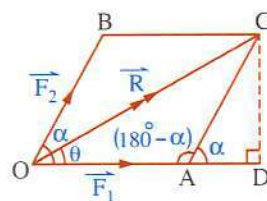


Therefore , we draw from O the directed line segment \vec{OC} equivalent to \vec{AB} which represents the resultant of these two forces perfectly.

Finding the resultant of two forces meeting at a point analytically

Let the two forces \vec{F}_1 and \vec{F}_2 meet at O and α is the measure of the angle between the directions of the two forces.

If \vec{OA} and \vec{OB} represent the two forces \vec{F}_1 and \vec{F}_2 , then \vec{OC} represents the resultant \vec{R}



Let θ be the measure of the angle between the resultant \vec{R} and the force \vec{F}_1 , then from our study of the cosine law in trigonometry , we can get the resultant of the two forces \vec{F}_1 and \vec{F}_2 in magnitude and direction from the following relations :

$$R = \sqrt{(F_1)^2 + (F_2)^2 + 2F_1F_2 \cos \alpha} \quad , \quad \tan \theta = \frac{F_2 \sin \alpha}{F_1 + F_2 \cos \alpha}$$

where F_1 , F_2 and R are the magnitudes of \vec{F}_1 , \vec{F}_2 and \vec{R}

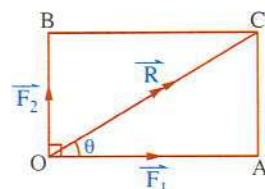
Special cases

1 If the two forces are perpendicular (i.e. $\alpha = 90^\circ$) :

$$\therefore \cos \alpha = 0 \quad , \quad \sin \alpha = 1$$

Substituting in the two previous relations , we get that :

$$R = \sqrt{F_1^2 + F_2^2} \quad , \quad \tan \theta = \frac{F_2}{F_1}$$

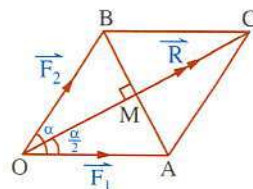


2 If the two forces are equal in magnitude (i.e. $F_1 = F_2 = F$) :

In this case , the parallelogram OACB exchanges to a rhombus , then :

$$R = OC = 2 OM = 2 OA \cos \frac{\alpha}{2}$$

$$= 2 F \cos \frac{\alpha}{2}$$



i.e. $R = 2 F \cos \frac{\alpha}{2}$, $\theta = \frac{\alpha}{2}$ (where \vec{R} bisects the angle between the two forces)

Notice that : $\alpha = 120^\circ$, So $R = F$

3 If the two forces have the same line of action and the same direction (i.e. $\alpha = 0^\circ$) :

$$\therefore \cos \alpha = 1$$

Substituting :

$$\therefore R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2 \times 1} = \sqrt{(F_1 + F_2)^2} = F_1 + F_2$$



i.e. $R = F_1 + F_2$

and the direction of the resultant is the same direction of the line of action of the two forces.

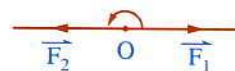
* In this case , R is called the greatest or the maximum value of the resultant.

4 If the two forces have the same line of action but in opposite directions (i.e. $\alpha = 180^\circ$) :

$$\therefore \cos \alpha = -1$$

Substituting :

$$\therefore R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2 \times (-1)} = \sqrt{F_1^2 + F_2^2 - 2F_1F_2} = \sqrt{(F_1 - F_2)^2} = |F_1 - F_2|$$



i.e. $R = |F_1 - F_2|$

and the direction of the resultant is the direction of the greater force in magnitude.

* In this case , R is called the smallest or the minimum value of the resultant.

5 If the two forces are equal in magnitude and have the same line of action but in opposite directions :

In this case : $F_1 = F_2 = F$, $\alpha = 180^\circ$

$$\therefore \cos \alpha = -1$$

$$\therefore R = \sqrt{F^2 + F^2 - 2F^2} = 0$$

$$\therefore R = \text{zero}$$

i.e. The resultant is zero vector.



6 If the resultant is perpendicular to the first force (i.e. $\theta = 90^\circ$) :

$$\therefore \theta = 90^\circ$$

$$\therefore R^2 = F_2^2 - F_1^2 \quad (\text{Pythagoras' theorem})$$

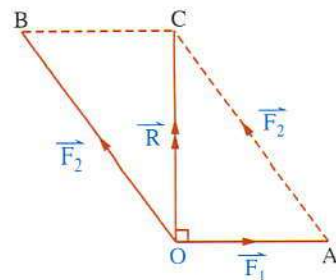
$$\therefore \cot \theta = 0$$

$$\therefore \frac{F_1 + F_2 \cos \alpha}{F_2 \sin \alpha} = 0$$

$$\therefore F_1 + F_2 \cos \alpha = 0$$

$$\therefore \cos \alpha = \frac{-F_1}{F_2}$$

$$\therefore \alpha \text{ is an obtuse angle, } F_1 < F_2$$



i.e. When the resultant is perpendicular to one of the two forces it is perpendicular to the smallest force.

Example 2

Two forces of magnitudes 5 newton and 3 newton act at a point and include an angle of measure 60° , find their resultant in magnitude and direction analytically.

Solution

$$\therefore R = \sqrt{F_1^2 + F_2^2 + 2 F_1 F_2 \cos \alpha}$$

$$\therefore R = \sqrt{25 + 9 + 2 \times 5 \times 3 \times \cos 60^\circ} = 7 \text{ newton.}$$

$$\therefore \tan \theta = \frac{F_2 \sin \alpha}{F_1 + F_2 \cos \alpha}$$

$$\therefore \tan \theta = \frac{3 \sin 60^\circ}{5 + 3 \cos 60^\circ} = \frac{3\sqrt{3}}{13}$$

$$\therefore \theta \approx 21^\circ 47'$$

\therefore The magnitude of \vec{R} is 7 newton and include an angle of measure $21^\circ 47'$ with the first force.

Example 3

Two perpendicular forces act at a point such that $F_1 = 6$ newton and $F_2 = 2.5$ newton. Find their resultant in magnitude and find its direction.

Solution

$$\therefore R = \sqrt{F_1^2 + F_2^2}$$

$$\therefore R = \sqrt{(6)^2 + (2.5)^2} = 6.5 \text{ newton.}$$

$$\therefore \tan \theta = \frac{F_2}{F_1}$$

$$\therefore \tan \theta = \frac{2.5}{6} = \frac{5}{12}$$

$$\therefore \theta = 22^\circ 37'$$

\therefore The magnitude of \vec{R} is 6.5 newton and include an angle of measure $22^\circ 37'$ with the first force.

Example 4

Two forces of magnitudes 50 newton and 100 newton act at a point. Their resultant is perpendicular to the first force. Find the measure of the angle included between the two forces and the magnitude of the resultant.

Solution

$$F_1 = 50 \text{ newton} \quad , \quad F_2 = 100 \text{ newton.}$$

∴ The resultant is perpendicular to the first force.

$$\therefore F_1 + F_2 \cos \alpha = 0$$

$$\therefore 50 + 100 \cos \alpha = 0$$

$$\therefore \cos \alpha = \frac{-50}{100} = -\frac{1}{2}$$

$$\therefore \alpha = 120^\circ$$

$$\therefore R = \sqrt{(F_1)^2 + (F_2)^2 + 2(F_1)(F_2)\cos\alpha}$$

$$\therefore R = \sqrt{(50)^2 + (100)^2 + 2 \times 50 \times 100 \cos 120^\circ} = 50\sqrt{3} \text{ newton.}$$

Another solution :

Let \overrightarrow{OA} represents the force whose magnitude is 50 newton ,

\overrightarrow{OB} represents the force whose magnitude is 100 newton.

, ∴ the resultant is perpendicular to the first force

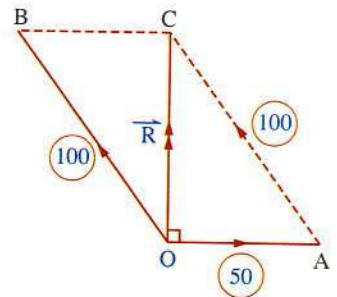
∴ $\triangle OAC$ is right-angled triangle at O

$$\therefore \cos A = \frac{OA}{AC} = \frac{50}{100} = \frac{1}{2}$$

$$\therefore m(\angle A) = 60^\circ$$

∴ $m(\angle AOB) = 120^\circ$ and it is the measure of the angle between the two forces.

$$\therefore R^2 = (100)^2 - (50)^2 \quad \therefore R = \sqrt{(100)^2 - (50)^2} = 50\sqrt{3} \text{ newton.}$$



Example 5

Two forces act at a point. The greatest value of their resultant = 32 kg.wt. and the smallest value of their resultant is 12 kg.wt. Find the magnitude of each of them , then find the magnitude of their resultant if the measure of the included angle between them is 60°

Solution

Let the great force = F_1 and the small force = F_2

$$\therefore F_1 + F_2 = 32 \quad (1) \quad , \quad F_1 - F_2 = 12 \quad (2)$$

From (1) and (2) : $F_1 = 22 \text{ kg.wt.}$ and $F_2 = 10 \text{ kg.wt.}$

$$\text{If } \alpha = 60^\circ , \text{ then } R = \sqrt{(22)^2 + (10)^2 + 2 \times 22 \times 10 \cos 60^\circ} = 2\sqrt{201} \text{ kg.wt.}$$

Example 6

Two forces are equal in magnitude. The magnitude of their resultant is $70\sqrt{3}$ newton and the measure of the angle between them is 60° . Find the magnitude of each of the two forces.

Solution

\therefore The two forces are equal in magnitude.

$$\therefore R = 2 F \cos \frac{\alpha}{2} \qquad \therefore 70\sqrt{3} = 2 F \cos 30^\circ \qquad \therefore F = 70 \text{ newton.}$$

\therefore The two forces are 70 newton and 70 newton.

Example 7

Two forces of magnitude 6 and F kg.wt. act at a particle such that the measure of the angle between them is 135°

Find the magnitude of their resultant if the line of action of the resultant inclines by an angle of measure 45° with the force F

Solution

$$\therefore \tan \theta = \frac{F_1 \sin \alpha}{F_2 + F_1 \cos \alpha}$$

, where θ is the measure of the angle between the resultant and the force F

$$\therefore \tan 45^\circ = \frac{6 \sin 135^\circ}{F + 6 \cos 135^\circ} \qquad \therefore 1 = \frac{3\sqrt{2}}{F - 3\sqrt{2}}$$

$$\therefore F - 3\sqrt{2} = 3\sqrt{2} \qquad \therefore F = 6\sqrt{2} \text{ kg.wt.}$$

$$\therefore R = \sqrt{(F_1)^2 + (F_2)^2 + 2F_1F_2 \cos \alpha}$$

$$\therefore R = \sqrt{(6)^2 + (6\sqrt{2})^2 + 2 \times 6 \times 6\sqrt{2} \cos 135^\circ} = 6 \text{ kg.wt.}$$

Example 8

Two forces acting at a point. If their magnitudes are $4F$ and $3F$, find the measure of the angle between them if the magnitude of the resultant is $\sqrt{13}F$

Solution

$$\therefore R^2 = (F_1)^2 + (F_2)^2 + 2(F_1)(F_2) \cos \alpha$$

$$\therefore (\sqrt{13}F)^2 = (4F)^2 + (3F)^2 + 2 \times 4F \times 3F \times \cos \alpha$$

$$\therefore 13F^2 = 16F^2 + 9F^2 + 24F^2 \cos \alpha \qquad \therefore -12F^2 = 24F^2 \cos \alpha$$

$$\therefore \cos \alpha = \frac{-12F^2}{24F^2} = -\frac{1}{2} \qquad \therefore \alpha = 120^\circ$$

Example 9

Two forces of magnitude 7 kg.wt. and F kg.wt. act at a particle and the measure of the included angle between their directions is 120°

If the magnitude of their resultant is $7\sqrt{3}$ kg.wt.

Find the value of F and the measure of the angle which the resultant makes with the first force.

Solution

$$\therefore R^2 = F_1^2 + F_2^2 + 2 F_1 F_2 \cos \alpha$$

$$\therefore 147 = 49 + F^2 - 7 F$$

$$\therefore (F - 14)(F + 7) = 0$$

$$\therefore \tan \theta = \frac{F_2 \sin \alpha}{F_1 + F_2 \cos \alpha}$$

$$\therefore \cot \theta = \text{zero}$$

$$\therefore (7\sqrt{3})^2 = (7)^2 + F^2 + 2 \times 7 \times F \cos 120^\circ$$

$$\therefore F^2 - 7 F - 98 = 0$$

$$\therefore F = 14 \text{ kg.wt.}$$

$$\therefore \tan \theta = \frac{14 \sin 120^\circ}{7 + 14 \cos 120^\circ} = \frac{7\sqrt{3}}{\text{zero}} \llcorner \text{undefined} \gg$$

$$\therefore \theta = 90^\circ$$

i.e. The resultant is perpendicular to the first force.

Example 10

Two forces of magnitudes 5 and $5\sqrt{2}$ kg.wt. act at a point.

The first towards East, the second is towards Western North. Prove that the magnitude of the resultant = the magnitude of the first force and find the measure of the angle which the resultant makes with each of the two forces.

Solution

$$F_1 = 5 \text{ kg.wt.}, \quad F_2 = 5\sqrt{2} \text{ kg.wt.}$$

From the figure $\alpha = 135^\circ$

$$\therefore R = \sqrt{(F_1)^2 + (F_2)^2 + 2 F_1 F_2 \cos \alpha}$$

$$\therefore R = \sqrt{25 + 50 + 2 \times 5 \times 5\sqrt{2} \times \cos 135^\circ}$$

$$\therefore R = 5 \text{ kg.wt.} = F_1, \quad \therefore \tan \theta = \frac{F_2 \sin \alpha}{F_1 + F_2 \cos \alpha}$$

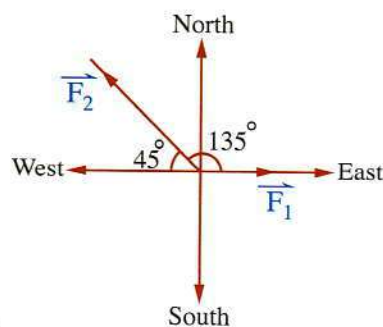
$$\therefore \tan \theta = \frac{5\sqrt{2} \sin 135^\circ}{5 + 5\sqrt{2} \cos 135^\circ} = \frac{5}{\text{zero}} \llcorner \text{undefined} \gg$$

$$\therefore \cot \theta = \text{zero}$$

$$\therefore \theta = 90^\circ$$

$\therefore \vec{R}$ is perpendicular to \vec{F}_1

i.e. Towards North and makes an angle of measure $135^\circ - 90^\circ = 45^\circ$ with \vec{F}_2



Example 11

Two equal forces intersect at a point and the magnitude of their resultant equals 8 newton , if one of them is reversed , then the magnitude of their resultant equals 6 newtons. Find the magnitude of each force.

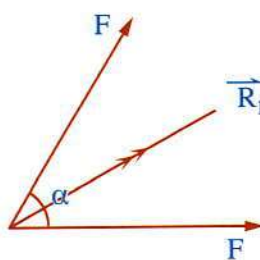
Solution

$$R_1 = 2 F \cos \frac{\alpha}{2} = 8$$

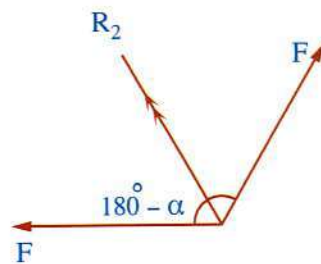
$$\therefore F \cos \frac{\alpha}{2} = 4 \quad (1)$$

$$, R_2 = 2 F \cos \left(\frac{180 - \alpha}{2} \right) = 6$$

$$\therefore F \sin \frac{\alpha}{2} = 3 \quad (2)$$



(First case)



(Second case)

By squaring the two equations (1) , (2) and add :

$$\therefore F^2 \cos^2 \frac{\alpha}{2} + F^2 \sin^2 \frac{\alpha}{2} = 16 + 9 \quad \therefore F^2 \left(\cos^2 \frac{\alpha}{2} + \sin^2 \frac{\alpha}{2} \right) = 25$$

$$\therefore F^2 = 25 \quad \therefore F = 5 \text{ newton}$$

\therefore The magnitude of each force 5 , 5 newton

• Notice that the two equations can be solved as the following :

Dividing equation (2) by equations (1) :

$$\therefore \tan \frac{\alpha}{2} = \frac{3}{4} \quad \therefore \sin \frac{\alpha}{2} = \frac{3}{5}$$

, by substituting in equation (2) :

$$\therefore F \times \frac{3}{5} = 3 \quad \therefore F = 5 \text{ newton.}$$

\therefore The magnitude of each force is 5 newton.

Another solution (Geometrically) :

\therefore The two forces are equal.

$\therefore R_1, R_2$ bisect the angle between the two forces.

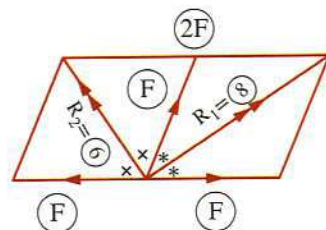
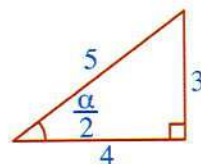
$$\therefore R_1 \perp R_2$$

$$\therefore (R_1)^2 + (R_2)^2 = (2 F)^2$$

$$\therefore 64 + 36 = 4 F^2$$

$$\therefore F^2 = 25 \quad \therefore F = 5$$

\therefore The magnitude of the two forces are 5 , 5 newton.



Lesson

2

Forces resolution into two components



Resolution of a known force into two known directions

Suppose that the force \vec{R} acts at a point O and it is required to resolve \vec{R} into two components \vec{F}_1 and \vec{F}_2 . Let θ_1 and θ_2 be the measure of angles of inclination of \vec{F}_1 and \vec{F}_2 to the direction of \vec{R}

Therefore, we draw using a drawing scale the vector \vec{OC} to represent the force \vec{R} , then we draw from O the two rays \vec{OX} and \vec{OY} making two angles θ_1 and θ_2 with \vec{OC} and in different sides of it.

Then we draw from C two rays one is parallel to \vec{OX} and the other is parallel to \vec{OY} to get the parallelogram OACB as in the shown figure, thus the vector \vec{OA} represents the component \vec{F}_1 and \vec{OB} represents the component \vec{F}_2 and the vector \vec{AC} represents \vec{F}_2 also

By using the sine rule on ΔOAC , where $m(\angle ACO) = \theta_2$

and $\sin(\angle OAC) = \sin[180^\circ - (\theta_1 + \theta_2)] = \sin(\theta_1 + \theta_2)$

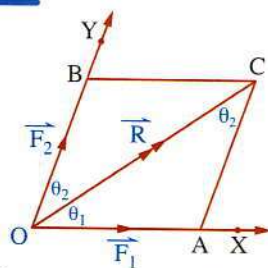
$$\therefore \frac{F_1}{\sin \theta_2} = \frac{F_2}{\sin \theta_1} = \frac{R}{\sin(\theta_1 + \theta_2)}$$

i.e.

$$F_1 \text{ (the magnitude of the component of } \vec{R} \text{, which inclines by } \theta_1 \text{ on } \vec{R}) = \frac{R \sin \theta_2}{\sin(\theta_1 + \theta_2)}$$

And

$$F_2 \text{ (the magnitude of the component of } \vec{R} \text{, which inclines by } \theta_2 \text{ on } \vec{R}) = \frac{R \sin \theta_1}{\sin(\theta_1 + \theta_2)}$$



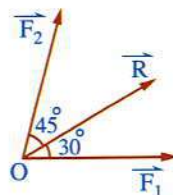
Example 1

Resolve the force of magnitude 20 newton into two components one of them inclined on the given force with an angle of measures 30° and the other force inclined by an angle of measure 45° on the other side of the force, then approximate the answer to the nearest one decimal.

Solution

$$F_1 = \frac{R \sin \theta_2}{\sin (\theta_1 + \theta_2)} = \frac{20 \sin 45^\circ}{\sin 75^\circ} \approx 14.6 \text{ newton.}$$

$$F_2 = \frac{R \sin \theta_1}{\sin (\theta_1 + \theta_2)} = \frac{20 \sin 30^\circ}{\sin 75^\circ} \approx 10.4 \text{ newton.}$$

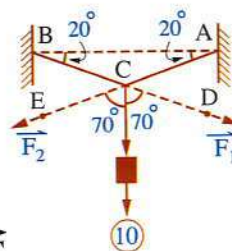
**Example 2**

In the opposite figure :

A light of weight 10 Newton is suspended by two strings \overline{AC} , \overline{BC} fixed in two horizontal points with equal two angles, the measure of each of them is 20°

(1) Resolve the weight of the light in each of the two directions \overline{AC} , \overline{BC}

(2) What happen, if the magnitude of the components of the weight in the two directions of the strings, if its angle decreased with horizontal less than 20° , and what do you deduce to the magnitude of the component of the weight, when the string becomes horizontal.

**Solution**

(1) The weight (10 newton) acts vertically downwards, and from the figure :

$$\frac{W_1}{\sin 70^\circ} = \frac{W_2}{\sin 70^\circ} = \frac{10}{\sin 140^\circ}$$

$$\therefore W_1 = W_2 = \frac{10 \sin 70^\circ}{\sin 140^\circ} \approx 15 \text{ newton.}$$

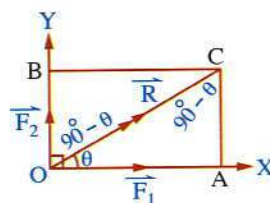
(2) If the measure of the angle decreased with horizontal less than 20° , then the magnitude of the component will increase to become unlimited when the strings are horizontal.

Resolution of the force into two perpendicular directions

Let the force \vec{R} acts at the point O and we want to resolve this force into two perpendicular forces \vec{F}_1 and \vec{F}_2 such that \vec{F}_1 inclines by θ on the direction of \vec{R}

In this case, the parallelogram become a rectangle.

Applying the sine rule on ΔOAC we get :



$$\frac{F_1}{\sin(90^\circ - \theta)} = \frac{F_2}{\sin \theta} = \frac{R}{\sin 90^\circ}$$

$$\frac{F_1}{\cos \theta} = \frac{F_2}{\sin \theta} = \frac{R}{1} = R \quad \text{Thus, } F_1 = R \cos \theta, \quad F_2 = R \sin \theta$$

$\therefore F_1$ (the magnitude of the component in the given direction) = $R \cos \theta$, and F_2 (the magnitude of the component in the perpendicular direction to the given direction) = $R \sin \theta$

The component \vec{F}_1 sometimes is called the projection of \vec{R} in the direction of \vec{OA} and the component \vec{F}_2 is called the projection of \vec{R} in the direction of \vec{OB}

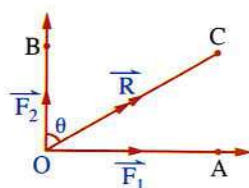
Remarks

- (1) The magnitude of the component adjacent to the given angle = $R \cos$ (this angle), the magnitude of the other perpendicular component to the previous component = $R \sin$ (this angle)

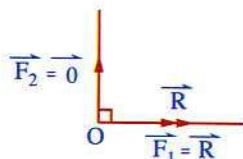
In the opposite figure :

If the component \vec{F}_2 inclines on the direction of \vec{R} by an angle of measure θ , then

$$F_2 = R \cos \theta, \quad F_1 = R \sin \theta$$



- (2) The component of \vec{R} in the same direction of \vec{R} = The same force \vec{R} and its component in the perpendicular direction to its direction = $\vec{0}$



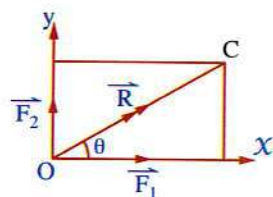
Because in this case, the measure of the angle between \vec{R} and the first component = zero, then the magnitude of the first component = $R \cos 0^\circ = R \times 1 = R$

and the magnitude of the perpendicular component on the previous component = $R \sin 0^\circ = R \times 0 = \text{zero}$

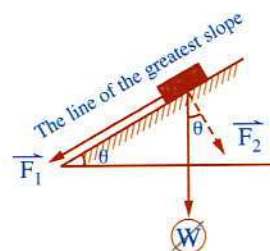
- (3) If \hat{i} and \hat{j} are two perpendicular unit vectors in the directions \vec{OX} and \vec{OY} where O is the origin point.

$$\text{Then } \vec{F}_1 = (R \cos \theta) \hat{i}, \quad \vec{F}_2 = (R \sin \theta) \hat{j}$$

$$\therefore \vec{R} = \vec{F}_1 + \vec{F}_2 = (R \cos \theta) \hat{i} + (R \sin \theta) \hat{j}$$



- (4) If $\vec{F} = (F, \theta)$, then $\vec{F} = F \cos \theta \hat{i} + F \sin \theta \hat{j}$
- (5) If $\theta \in]0, \frac{\pi}{2}[$, then the magnitude of the two components $(R \cos \theta)$, $(R \sin \theta)$ is less than the magnitude of the force (R) because $\theta \in]0, \frac{\pi}{2}[$, thus $0 < \sin \theta < 1$, $0 < \cos \theta < 1$
- (6) If a body of weight (w) is placed on a smooth inclined plane with the horizontal by an angle (θ) , then we can resolve the weight (w) which acts vertically downwards into two components.
- * F_1 (The magnitude of the component in the direction of the line of the greatest slope)
 $= w \sin \theta$
 - * F_2 (The magnitude of the component in the perpendicular direction on the plane)
 $= w \cos \theta$



Example 3

Resolve the force of magnitude $8\sqrt{2}$ newton which acts at the point O in the direction of Eastern North into two components. One of them in the East direction and the other in the North direction.

Solution

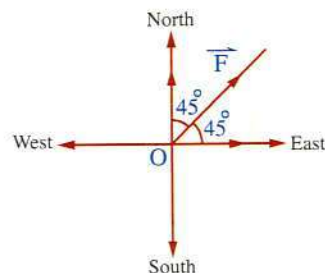
\therefore The two components inclined on the direction of the force by angles of measures 45° and 45° , then they are perpendicular.

\therefore The magnitude of the component which is in the

$$\text{East direction} = F \cos 45^\circ = 8\sqrt{2} \times \frac{1}{\sqrt{2}} = 8 \text{ newton.}$$

The magnitude of the component in North direction

$$= F \sin 45^\circ = 8\sqrt{2} \times \frac{1}{\sqrt{2}} = 8 \text{ newton.}$$



Example 4

A force of magnitude $10\sqrt{3}$ kg.wt. was resolved into two perpendicular components , one of them is of magnitude 15 kg.wt. Find the magnitude of the other component.

Solution

Let the direction of the given component (\vec{F}_1) inclines on the direction of the force by an angle of measure θ

\therefore The magnitude of this component $\vec{F}_1 = 10\sqrt{3} \cos \theta$

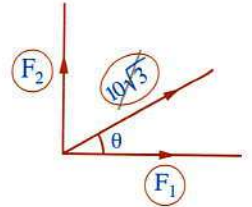
$$\therefore 15 = 10\sqrt{3} \cos \theta \quad \therefore \cos \theta = \frac{15}{10\sqrt{3}} = \frac{\sqrt{3}}{2} \quad \therefore \theta = 30^\circ$$

The magnitude of the other component $F_2 = 10\sqrt{3} \sin 30^\circ = 10\sqrt{3} \times \frac{1}{2} = 5\sqrt{3}$ kg.wt.

Another solution :

$$\therefore R = \sqrt{F_1^2 + F_2^2} \quad \therefore (10\sqrt{3})^2 = (15)^2 + F_2^2$$

$$\therefore F_2^2 = 75 \quad \therefore F_2 = 5\sqrt{3} \text{ kg. wt.}$$



Example 5

A body of weight 50 newton is placed on a smooth inclined plane by 30° with the horizontal. Find the two components of the weight in the direction of the line of the greatest slope and the direction perpendicular to it.

Solution

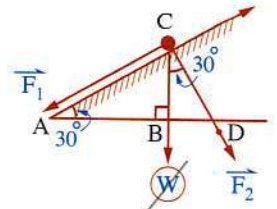
From the figure , we notice that : $m(\angle BCD) = (\angle BAC) = 30^\circ$

$\therefore F_1 =$ (the magnitude of the component in the direction of the line of the greatest slope)

$$= W \sin 30^\circ = 50 \times \frac{1}{2} = 25 \text{ newton.}$$

$\therefore F_2 =$ (the magnitude of the component in the perpendicular direction to the plane)

$$= W \cos 30^\circ = 50 \times \frac{\sqrt{3}}{2} = 25\sqrt{3} \text{ newton.}$$



Lesson

3

The resultant of coplanar forces meeting at a point



1 The geometrical method

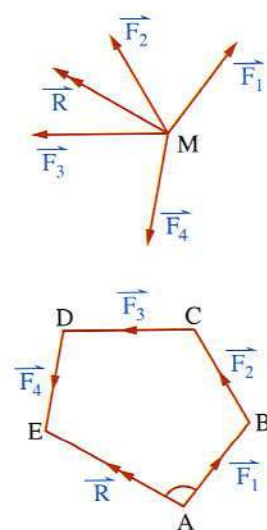
Suppose that the system of coplanar forces $\vec{F}_1, \vec{F}_2, \vec{F}_3, \vec{F}_4$ acts at point M as in the opposite figure.

To find the resultant of this forces :

- * Use an appropriate drawing scale.
- * From any point as A draw the vector \vec{AB} to represent \vec{F}_1 (in magnitude and direction)
- * From point B draw the vector \vec{BC} to represent \vec{F}_2
- * From point C draw the vector \vec{CD} to represent \vec{F}_3
- * At last , from point D draw the vector \vec{DE} to represent \vec{F}_4

Match the first point (A) to the last point (E) to be the vector \vec{AE} which represent the resultant (\vec{R}) in magnitude and direction , where $\vec{R} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4$

- * Find the length of \vec{AE} and $m(\angle EAB)$ assume it is the included angle between the resultant and the first force and using the drawing scale we can find the magnitude and the direction of \vec{R}
- * Then the resultant of the set of forces is a force of magnitude R and acts at point M in direction \vec{AE}



Notice that :

The vector \vec{AE} which represent \vec{R} has an opposite direction to the other directions of vectors which represent the forces and the polygon ABCDE is called "The force polygon"

Remark

If the first and last points are congruent in the force polygon, then $(\vec{R}) = \vec{O}$ and the set of forces are equilibrium.

i.e. The adjusted and sufficient condition to equilibrium a set of concurrent forces is a representing of these forces geometrically by the sides of a closed polygon taken in the same direction.

2 The analytical method

Suppose that the system of coplanar forces $\vec{F}_1, \vec{F}_2, \vec{F}_3, \dots, \vec{F}_n$ meet at the point O and the point O is the origin point of a coplanar cartesian axis.

and $\theta_1, \theta_2, \theta_3, \dots, \theta_n$ are the polar angles of the forces $\vec{F}_1, \vec{F}_2, \vec{F}_3, \dots, \vec{F}_n$ respectively, let \hat{i} and \hat{j} be the fundamental unit vectors in the two directions \vec{OX} and \vec{OY} , then :

$$\vec{F}_1 = (F_1, \theta_1) = F_1 \cos \theta_1 \hat{i} + F_1 \sin \theta_1 \hat{j},$$

$$\vec{F}_2 = (F_2, \theta_2) = F_2 \cos \theta_2 \hat{i} + F_2 \sin \theta_2 \hat{j},$$

$$\vec{F}_3 = (F_3, \theta_3) = F_3 \cos \theta_3 \hat{i} + F_3 \sin \theta_3 \hat{j}$$

$$\text{and so on till : } \vec{F}_n = (F_n, \theta_n) = F_n \cos \theta_n \hat{i} + F_n \sin \theta_n \hat{j},$$

$$\therefore \vec{R} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \dots + \vec{F}_n$$

, then by adding we get :

$$\begin{aligned} \vec{R} &= (F_1 \cos \theta_1 \hat{i} + F_1 \sin \theta_1 \hat{j}) + (F_2 \cos \theta_2 \hat{i} + F_2 \sin \theta_2 \hat{j}) \\ &+ (F_3 \cos \theta_3 \hat{i} + F_3 \sin \theta_3 \hat{j}) + \dots + (F_n \cos \theta_n \hat{i} + F_n \sin \theta_n \hat{j}) \\ &= (F_1 \cos \theta_1 + F_2 \cos \theta_2 + F_3 \cos \theta_3 + \dots + F_n \cos \theta_n) \hat{i} \\ &+ (F_1 \sin \theta_1 + F_2 \sin \theta_2 + F_3 \sin \theta_3 + \dots + F_n \sin \theta_n) \hat{j} \end{aligned}$$

$$\text{i.e. } \vec{R} = \left(\sum_{r=1}^n F_r \cos \theta_r \right) \hat{i} + \left(\sum_{r=1}^n F_r \sin \theta_r \right) \hat{j}$$

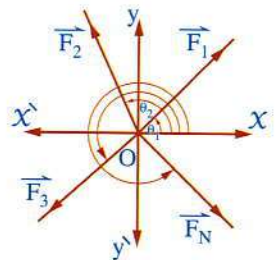
The expression $\left(\sum_{r=1}^n F_r \cos \theta_r \right)$ is called the algebraic sum of the components in the

direction \vec{OX} and is denoted by X and the expression $\left(\sum_{r=1}^n F_r \sin \theta_r \right)$ is called the

algebraic sum of the components in the direction \vec{OY} and is denoted by Y Hence , we can

write the previous equation in the form :

$$\vec{R} = X \hat{i} + Y \hat{j}$$



And let R be the magnitude of \vec{R} and α is the measure of the polar angle of the resultant \vec{R} , then :

$$R = \sqrt{X^2 + Y^2} \text{ and } \tan \alpha = \frac{Y}{X}$$

$$\text{where } \vec{R} = (R, \alpha)$$

Remarks

(1) Notice the difference between X and \hat{i} :

- X is the algebraic sum of the components of forces in the direction of \vec{OX}
- \hat{i} is the fundamental unit vector in the direction of \vec{OX}

(2) If $X = \text{zero}$, then $\vec{R} = Y \hat{j}$
and $\theta = 90^\circ$, if \vec{R} in the direction \vec{OY}
 $\theta = 270^\circ$, if \vec{R} in the direction \vec{OY}

(3) If $Y = \text{zero}$, then $\vec{R} = X \hat{i}$
and $\theta = 0^\circ$, if \vec{R} in the direction \vec{OX}
 $\theta = 180^\circ$, if \vec{R} in the direction \vec{OX}

(4) If $X = \text{zero}$ and $Y = \text{zero}$, then $\vec{R} = \vec{O}$

In this case, the set of coplanar concurrent forces are in equilibrium.

(5) To determine the direction of the resultant, consider that :

X	y	quad.	θ
+	+	1 st	measure of the acute angle
-	+	2 nd	$180^\circ - \text{measure of the acute angle}$
-	-	3 rd	$180^\circ + \text{measure of the acute angle}$
+	-	4 th	$360^\circ - \text{measure of the acute angle}$

(6) The resultant of set of forces $\vec{F}_1, \vec{F}_2, \vec{F}_3$

is $\vec{R} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$, and if $\vec{R} = \vec{O}$, then the set of forces are equilibrium

For Example :

$$\text{If } \vec{F}_1 = 5\hat{i} + 2\hat{j}, \vec{F}_2 = -6\hat{i} + 3\hat{j}$$

$$\text{and } \vec{F}_3 = \hat{i} - 5\hat{j}, \text{ then } \vec{R} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = \vec{O}$$

\therefore The forces are in equilibrium.

Example 1

If the forces $\vec{F}_1 = 5\hat{i} - 4\hat{j}$, $\vec{F}_2 = -6\hat{i} + a\hat{j}$ and $\vec{F}_3 = b\hat{i} + 7\hat{j}$ are meeting at a point and are in equilibrium. Find the value of each of : a and b

Solution

∴ The forces are in equilibrium

$$\therefore (5\hat{i} - 4\hat{j}) + (-6\hat{i} + a\hat{j}) + (b\hat{i} + 7\hat{j}) = \vec{0}$$

$$\therefore -1 + b = 0$$

$$\therefore 3 + a = 0$$

$$\therefore \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = \vec{0}$$

$$\therefore (-1 + b)\hat{i} + (3 + a)\hat{j} = \vec{0}$$

$$\therefore b = 1$$

$$\therefore a = -3$$

Example 2

In each of the following three figures, a set of forces meeting at a point O and their magnitudes are in newton unit.

Determine the magnitude and the direction of the resultant of each of them.

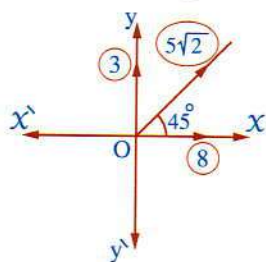


Fig. (1)

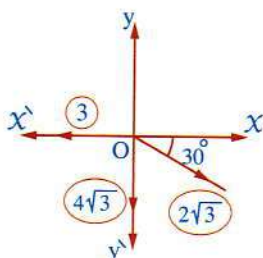


Fig. (2)

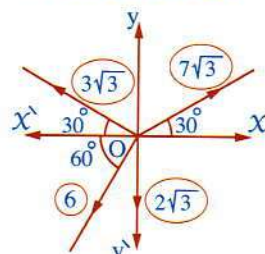


Fig. (3)

Solution

In Fig. (1) : The three forces whose magnitudes are 8, $5\sqrt{2}$ and 3 newton and their polar angles are of measures 0° , 45° and 90° respectively.

- The algebraic sum of the components in the direction of \vec{OX} is

$$\begin{aligned} X &= 8 \cos 0^\circ + 5\sqrt{2} \cos 45^\circ + 3 \cos 90^\circ \\ &= 8 \times 1 + 5\sqrt{2} \times \frac{1}{\sqrt{2}} + 3 \times \text{zero} = 8 + 5 + 0 = 13 \text{ newton.} \end{aligned}$$

- The algebraic sum of the components in the direction of \vec{OY} is

$$\begin{aligned} Y &= 8 \sin 0^\circ + 5\sqrt{2} \sin 45^\circ + 3 \sin 90^\circ \\ &= 8 \times \text{zero} + 5\sqrt{2} \times \frac{1}{\sqrt{2}} + 3 \times 1 = \text{zero} + 5 + 3 = 8 \text{ newton.} \end{aligned}$$

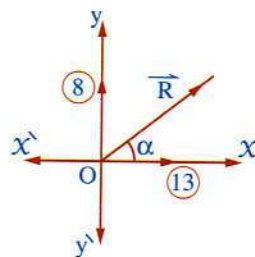
$$\therefore \vec{R} = 13\hat{i} + 8\hat{j}, \text{ then } R = \sqrt{X^2 + Y^2} = \sqrt{169 + 64} = \sqrt{233}$$

$$\therefore R \approx 15.264 \text{ newton, } \tan \alpha = \frac{Y}{X} = \frac{8}{13}$$

$$\therefore X > 0, Y > 0$$

∴ \vec{R} lies in the first quadrant, using the calculator.

$$\therefore \alpha \approx 31^\circ 36'$$



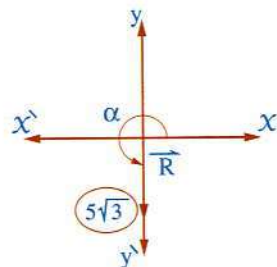
In Fig. (2) : The three forces whose magnitudes are 3 , $4\sqrt{3}$ and $2\sqrt{3}$ newton and their polar angles are of measures 180° , 270° and 330° respectively.

$$\begin{aligned} X &= 3 \cos 180^\circ + 4\sqrt{3} \cos 270^\circ + 2\sqrt{3} \cos 330^\circ \\ &= 3 \times (-1) + 4\sqrt{3} \times 0 + 2\sqrt{3} \times \frac{\sqrt{3}}{2} = -3 + 0 + 3 = \text{zero} \end{aligned}$$

$$\begin{aligned} Y &= 3 \sin 180^\circ + 4\sqrt{3} \sin 270^\circ + 2\sqrt{3} \sin 330^\circ \\ &= 3 \times 0 + 4\sqrt{3} \times (-1) + 2\sqrt{3} \times \left(-\frac{1}{2}\right) \\ &= 0 - 4\sqrt{3} - \sqrt{3} = -5\sqrt{3} \text{ newton.} \end{aligned}$$

$$\therefore \vec{R} = -5\sqrt{3} \hat{j}$$

, then $R = 5\sqrt{3}$ newton and $\alpha = 270^\circ$

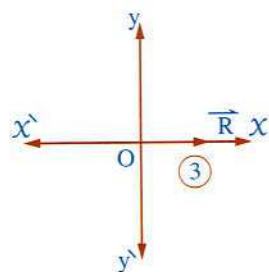


In Fig. (3) : Four forces of magnitudes $7\sqrt{3}$, $3\sqrt{3}$, 6 and $2\sqrt{3}$ newton and their polar angles are of measures 30° , 150° , 240° and 270° respectively.

$$\begin{aligned} X &= 7\sqrt{3} \cos 30^\circ + 3\sqrt{3} \cos 150^\circ + 6 \cos 240^\circ + 2\sqrt{3} \cos 270^\circ \\ &= 7\sqrt{3} \times \frac{\sqrt{3}}{2} + 3\sqrt{3} \times \left(-\frac{\sqrt{3}}{2}\right) + 6 \times \left(-\frac{1}{2}\right) + 2\sqrt{3} \times 0 \\ &= 10.5 - 4.5 - 3 + 0 = 3 \text{ newton ,} \end{aligned}$$

$$\begin{aligned} Y &= 7\sqrt{3} \sin 30^\circ + 3\sqrt{3} \sin 150^\circ + 6 \sin 240^\circ + 2\sqrt{3} \sin 270^\circ \\ &= 7\sqrt{3} \times \frac{1}{2} + 3\sqrt{3} \times \frac{1}{2} + 6 \times \left(-\frac{\sqrt{3}}{2}\right) + 2\sqrt{3} \times (-1) \\ &= \frac{7\sqrt{3}}{2} + \frac{3\sqrt{3}}{2} - 3\sqrt{3} - 2\sqrt{3} = \text{zero} \end{aligned}$$

$$\therefore \vec{R} = 3 \hat{i} \text{ , then } R = 3 \text{ newton , } \alpha = \text{zero}$$



Another solution for the figure (3) :

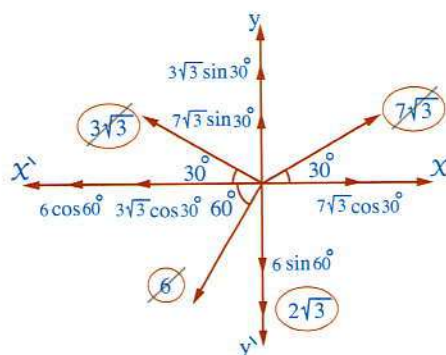
Using the analyzing of the forces into two perpendicular directions :

$$\begin{aligned} \therefore X &= 7\sqrt{3} \cos 30^\circ - 3\sqrt{3} \cos 30^\circ - 6 \cos 60^\circ \\ &= 7\sqrt{3} \times \frac{\sqrt{3}}{2} - 3\sqrt{3} \times \frac{\sqrt{3}}{2} - 6 \times \frac{1}{2} = 3 \text{ newton.} \end{aligned}$$

$$\begin{aligned} , Y &= 3\sqrt{3} \sin 30^\circ + 7\sqrt{3} \sin 30^\circ - 6 \sin 60^\circ - 2\sqrt{3} \\ &= 3\sqrt{3} \times \frac{1}{2} + 7\sqrt{3} \times \frac{1}{2} - 6 \times \frac{\sqrt{3}}{2} - 2\sqrt{3} = 0 \end{aligned}$$

$$\therefore R = \sqrt{(3)^2 + (0)^2} = 3 \text{ newton.}$$

$$\therefore \tan \alpha = \frac{Y}{X} = \frac{0}{3} = 0 \quad \therefore \alpha = 0^\circ$$



Example 3

Five coplanar forces meeting at a point, their magnitudes are 12, 9, $5\sqrt{2}$, $7\sqrt{2}$ and 7 kg.wt., act in the directions: East, North, Western North, Western South and South respectively. Prove that the set of these forces are in equilibrium.

Solution

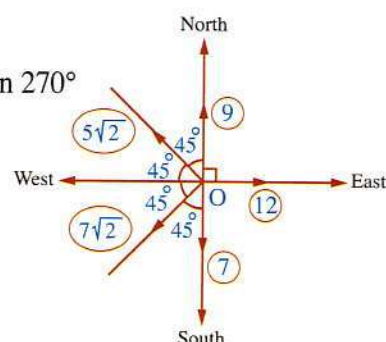
∴ The forces are $(12, 0^\circ)$, $(9, 90^\circ)$, $(5\sqrt{2}, 135^\circ)$, $(7\sqrt{2}, 225^\circ)$, $(7, 270^\circ)$

$$\begin{aligned}\therefore X &= 12 \cos 0^\circ + 9 \cos 90^\circ + 5\sqrt{2} \cos 135^\circ + 7\sqrt{2} \cos 225^\circ + 7 \cos 270^\circ \\ &= 12 \times 1 + \text{zero} + 5\sqrt{2} \times \left(-\frac{1}{\sqrt{2}}\right) + 7\sqrt{2} \times \left(-\frac{1}{\sqrt{2}}\right) + \text{zero} \\ &= 12 - 5 - 7 = \text{zero}\end{aligned}$$

$$\begin{aligned}Y &= 12 \sin 0^\circ + 9 \sin 90^\circ + 5\sqrt{2} \sin 135^\circ + 7\sqrt{2} \sin 225^\circ + 7 \sin 270^\circ \\ &= \text{zero} + 9 + 5\sqrt{2} \times \frac{1}{\sqrt{2}} + 7\sqrt{2} \times \left(-\frac{1}{\sqrt{2}}\right) + 7 \times -1 \\ &= 9 + 5 - 7 - 7 = \text{zero}\end{aligned}$$

$$\therefore X = \text{zero}, Y = \text{zero} \quad \therefore \vec{R} = \vec{O}$$

∴ The set of forces are in equilibrium.



Example 4

Four coplanar forces meeting at a point and their magnitudes are F , $2F$, $3\sqrt{3}F$ and $4F$ kg.wt. The measure of the angle between the first and second forces is 60° and between the second and the third is 90° and between the third and the fourth is 150° . Find the magnitude and the direction of \vec{R} .

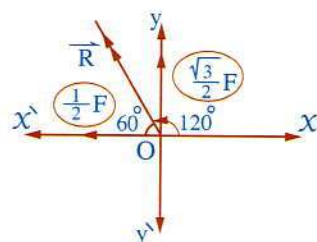
Solution

Let \vec{OX} is the direction of the first force

, then the forces in polar form are $(F, 0^\circ)$, $(2F, 60^\circ)$, $(3\sqrt{3}F, 150^\circ)$, $(4F, 300^\circ)$ respectively.

$$\begin{aligned}\therefore X &= F \cos 0^\circ + 2F \cos 60^\circ + 3\sqrt{3}F \cos 150^\circ + 4F \cos 300^\circ \\ &= F \times 1 + 2F \times \frac{1}{2} + 3\sqrt{3}F \times \left(-\frac{\sqrt{3}}{2}\right) + 4F \times \frac{1}{2} \\ &= F + F - \frac{9}{2}F + 2F = -\frac{1}{2}F,\end{aligned}$$

$$\begin{aligned}Y &= F \sin 0^\circ + 2F \sin 60^\circ + 3\sqrt{3}F \sin 150^\circ + 4F \sin 300^\circ \\ &= F \times 0 + 2F \times \frac{\sqrt{3}}{2} + 3\sqrt{3}F \times \frac{1}{2} + 4F \times \left(-\frac{\sqrt{3}}{2}\right) \\ &= 0 + \sqrt{3}F + \frac{3\sqrt{3}}{2}F - 2\sqrt{3}F = \frac{\sqrt{3}}{2}F\end{aligned}$$



$$\vec{R} = -\frac{1}{2} F \hat{i} + \frac{\sqrt{3}}{2} F \hat{j} \quad \therefore R = \sqrt{\frac{1}{4} F^2 + \frac{3}{4} F^2} = \sqrt{F^2} = F$$

$$\therefore R = F, \tan \alpha = \frac{Y}{X} = \frac{\frac{\sqrt{3}}{2} F}{-\frac{1}{2} F} = -\sqrt{3}$$

$$\therefore X < 0, Y > 0 \quad \therefore \alpha = 180^\circ - 60^\circ = 120^\circ$$

i.e. The resultant magnitude is F and its direction between 2nd and 3rd forces making an angle of measure 30° with the 3rd force.

* Try to solve this example using the analyzing of the forces into two perpendicular directions.

Example 5

Three forces of magnitudes $2F$, $4F$, $6F$ act at a point in directions parallel to the sides of an equilateral triangle in the same cyclic order. Find the magnitude and the direction of the resultant.

Solution

Let the forces act at the point O in the directions

\vec{OX} , \vec{OL} , \vec{OM}

which are parallel to the directions \vec{AB} , \vec{BC} , \vec{CA} in the equilateral triangle ABC

, then the forces in the polar form are :

$(2F, 0^\circ)$, $(4F, 120^\circ)$, $(6F, 240^\circ)$

$$\therefore X = 2F \cos 0^\circ + 4F \cos 120^\circ + 6F \cos 240^\circ$$

$$= 2F \times 1 + 4F \times \left(-\frac{1}{2}\right) + 6F \times \left(-\frac{1}{2}\right) = -3F,$$

$$Y = 2F \sin 0^\circ + 4F \sin 120^\circ + 6F \sin 240^\circ$$

$$= 2F \times 0 + 4F \times \left(\frac{\sqrt{3}}{2}\right) + 6F \times \left(-\frac{\sqrt{3}}{2}\right) = -\sqrt{3}F$$

$$\therefore \vec{R} = -3F \hat{i} - \sqrt{3}F \hat{j}$$

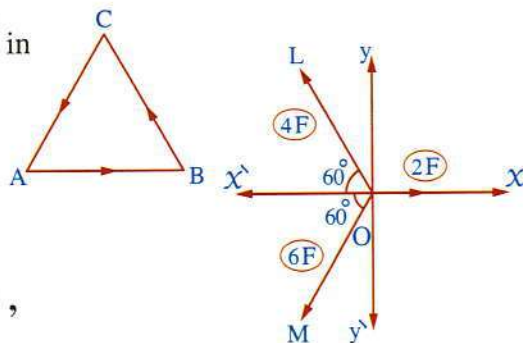
$$\therefore R = \sqrt{X^2 + Y^2} = \sqrt{(-3F)^2 + (-\sqrt{3}F)^2} = \sqrt{12F^2} = 2\sqrt{3}F,$$

$$\tan \alpha = \frac{Y}{X} = \frac{-\sqrt{3}F}{-3F} = \frac{1}{\sqrt{3}}$$

, $\therefore X$ and Y are negative, then $\alpha = 210^\circ$

i.e. Resultant magnitude is $2\sqrt{3}F$ and its direction between the two forces of magnitudes $6F$, $4F$ making an angle of measure 30° with the force $6F$

* Try to solve this example using the resolution of the forces into two perpendicular directions.



Example 6

ABCDEF is a regular hexagon. Forces of magnitudes 6, $2\sqrt{3}$, 6, $2\sqrt{3}$ newton act along \overrightarrow{AB} , \overrightarrow{AC} , \overrightarrow{AD} and \overrightarrow{AE} respectively.

Find the magnitude and the direction of the resultant of these forces.

Solution

Suppose \overrightarrow{OX} is the direction of the first force.

Then the polar form of the forces are $(6, 0^\circ)$, $(2\sqrt{3}, 30^\circ)$, $(6, 60^\circ)$, $(2\sqrt{3}, 90^\circ)$

$$\therefore X = 6 \cos 0^\circ + 2\sqrt{3} \cos 30^\circ + 6 \cos 60^\circ + 2\sqrt{3} \cos 90^\circ$$

$$= 6 \times 1 + 2\sqrt{3} \times \left(\frac{\sqrt{3}}{2}\right) + 6 \times \frac{1}{2} + 2\sqrt{3} \times 0 = 12 \text{ newton},$$

$$Y = 6 \sin 0^\circ + 2\sqrt{3} \sin 30^\circ + 6 \sin 60^\circ + 2\sqrt{3} \sin 90^\circ$$

$$= 6 \times 0 + 2\sqrt{3} \times \frac{1}{2} + 6 \times \frac{\sqrt{3}}{2} + 2\sqrt{3} \times 1 = 6\sqrt{3} \text{ newton.}$$

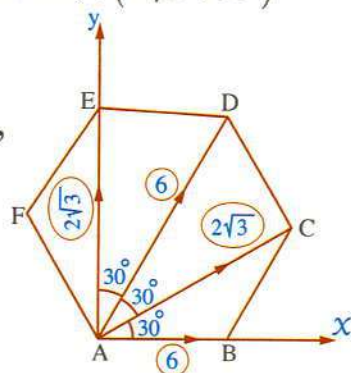
$$\therefore \vec{R} = 12\vec{i} + 6\sqrt{3}\vec{j} \quad \therefore R = \sqrt{X^2 + Y^2}$$

$$R = \sqrt{(12)^2 + (6\sqrt{3})^2} = 6\sqrt{7} \text{ newton}, \tan \alpha = \frac{Y}{X} = \frac{6\sqrt{3}}{12} = \frac{\sqrt{3}}{2}$$

$$\therefore X > 0, Y > 0 \quad \therefore \alpha = 40^\circ 53' 36''$$

i.e. The resultant magnitude is $6\sqrt{7}$ N.

and its direction between \overrightarrow{AC} and \overrightarrow{AD} making an angle of measure $10^\circ 53' 36''$ with \overrightarrow{AC}



Another solution :

Using the resolution of the forces into two perpendicular directions :

$$\therefore X = 6 \cos 60^\circ + 2\sqrt{3} \cos 30^\circ + 6$$

$$= 6 \times \frac{1}{2} + 2\sqrt{3} \times \frac{\sqrt{3}}{2} + 6 = 12 \text{ newton.}$$

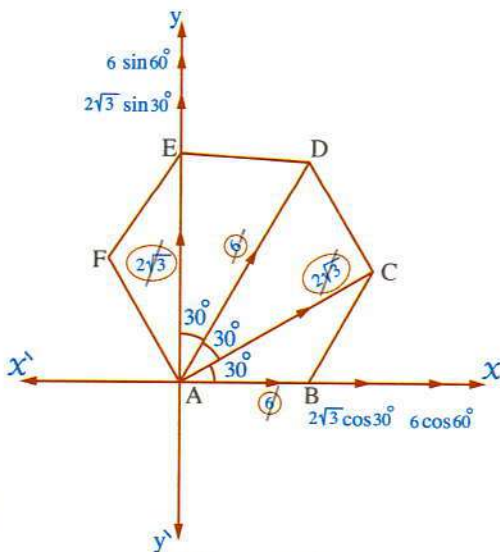
$$Y = 6 \sin 60^\circ + 2\sqrt{3} \sin 30^\circ + 2\sqrt{3}$$

$$= 6 \times \frac{\sqrt{3}}{2} + 2\sqrt{3} \times \frac{1}{2} + 2\sqrt{3} = 6\sqrt{3} \text{ newton.}$$

$$\therefore R = \sqrt{(12)^2 + (6\sqrt{3})^2} = 6\sqrt{7} \text{ newton.}$$

$$\tan \alpha = \frac{Y}{X} = \frac{6\sqrt{3}}{12} = \frac{\sqrt{3}}{2} \quad \therefore \alpha = 40^\circ 53' 36''$$

i.e. The resultant magnitude is $6\sqrt{7}$ N. and its direction between \overrightarrow{AC} and \overrightarrow{AD} making an angle of measure $10^\circ 53' 36''$ with \overrightarrow{AC}



Example 7

Four coplanar forces meeting at a point their magnitudes are $F_1, 6\sqrt{2}, 8\sqrt{2}, F_2$ gm.wt. The first force is in the East direction, the second force is in the direction of Eastern North, the third is in the direction of Western North and the fourth acts in South direction.

If their resultant is 7 gm.wt. in magnitude and acts in the East direction, find the value of F_1 and F_2

Solution

The magnitude of the resultant is 7 gm. wt. and acts towards East

$$\therefore X = 7 \text{ and } Y = \text{zero}$$

$$\therefore F_1 \cos 0^\circ + 6\sqrt{2} \cos 45^\circ + 8\sqrt{2} \cos 135^\circ + F_2 \cos 270^\circ = 7$$

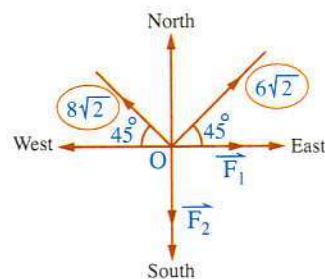
$$\therefore F_1 \times 1 + 6\sqrt{2} \times \frac{1}{\sqrt{2}} + 8\sqrt{2} \times -\frac{1}{\sqrt{2}} + F_2 \times 0 = 7$$

$$\therefore F_1 + 6 - 8 + 0 = 7 \quad \therefore F_1 = 9 \text{ gm. wt.}$$

$$F_1 \sin 0^\circ + 6\sqrt{2} \sin 45^\circ + 8\sqrt{2} \sin 135^\circ + F_2 \sin 270^\circ = 0$$

$$\therefore F_1 \times 0 + 6\sqrt{2} \times \frac{1}{\sqrt{2}} + 8\sqrt{2} \times \frac{1}{\sqrt{2}} + F_2 \times (-1) = 0$$

$$\therefore 6 + 8 - F_2 = 0 \quad \therefore F_2 = 14 \text{ gm. wt.}$$



* Try to solve this example using the resolution of the forces into two perpendicular directions.

Example 8

Five coplanar forces meeting at a point their magnitudes are $F, 9, 5\sqrt{2}, 7\sqrt{2}, K$ (kg.wt.)

The measure of the angle between the first force and the second force is 90° , between the second and the third is 45° , between the third and the fourth is 90° and between the fourth and the fifth 45° . If the system of forces is in equilibrium, find the value of F and K

Solution

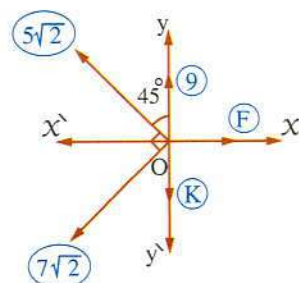
Let \overrightarrow{OX} is the direction of the first force

, then the forces in the polar form are :

$$(F, 0^\circ), (9, 90^\circ), (5\sqrt{2}, 135^\circ), (7\sqrt{2}, 225^\circ), (K, 270^\circ)$$

, \therefore the forces are in equilibrium

$$\therefore X = Y = 0$$



$$\therefore X = F \cos 0^\circ + 9 \cos 90^\circ + 5\sqrt{2} \cos 135^\circ + 7\sqrt{2} \cos 225^\circ + K \cos 270^\circ$$

$$\therefore F + 9 \times 0 + 5\sqrt{2} \times -\frac{1}{\sqrt{2}} + 7\sqrt{2} \times -\frac{1}{\sqrt{2}} + K \times 0 = 0 \quad \therefore F = 12$$

$$\therefore Y = F \sin 0^\circ + 9 \sin 90^\circ + 5\sqrt{2} \sin 135^\circ + 7\sqrt{2} \sin 225^\circ + K \sin 270^\circ$$

$$\therefore 12 \times 0 + 9 \times 1 + 5\sqrt{2} \times \frac{1}{\sqrt{2}} + 7\sqrt{2} \times -\frac{1}{\sqrt{2}} + K \times -1 = 0 \quad \therefore K = 7$$

Example 9

ABCD is a rectangle in which AB = 8 cm. , BC = 6 cm. , $F \in \overline{CD}$ where FD = 6 cm. The forces of magnitudes 6, 20, $13\sqrt{2}$ and 2 newton act along \overrightarrow{AB} , \overrightarrow{CA} , \overrightarrow{AF} , \overrightarrow{AD} respectively. Find the magnitude and the direction of the resultant of these forces.

Solution

$$\text{In } \triangle ABC : \therefore (AC)^2 = 6^2 + 8^2 = 100 \quad \therefore AC = 10 \text{ cm.}$$

$$\therefore \sin \theta = \frac{6}{10} = \frac{3}{5}, \cos \theta = \frac{8}{10} = \frac{4}{5}$$

$\therefore \triangle AFD$ is an isosceles triangle

$$\therefore m(\angle AFD) = 45^\circ \quad \text{i.e. } \alpha = 45^\circ$$

Suppose \overrightarrow{AB} is the direction of the first force and in the direction of \hat{i}

\therefore The measures of the polar angles of the forces are 0° , $180^\circ + \theta$, α , 90° respectively

$$\therefore X = 6 \cos 0^\circ + 20 \cos (180^\circ + \theta) + 13\sqrt{2} \cos \alpha + 2 \cos 90^\circ$$

$$= 6 \times \cos 0^\circ + 20 (-\cos \theta) + 13\sqrt{2} \times \cos 45^\circ + 2 \cos 90^\circ$$

$$= 6 \times 1 - 20 \times \frac{4}{5} + 13\sqrt{2} \times \frac{1}{\sqrt{2}} + 2 \times 0 = 6 - 16 + 13 = 3 \text{ newton.}$$

$$\therefore Y = 6 \times \sin 0^\circ + 20 \sin (180^\circ + \theta) + 13\sqrt{2} \sin \alpha + 2 \sin 90^\circ$$

$$= 6 \times \sin 0^\circ - 20 \sin \theta + 13\sqrt{2} \sin 45^\circ + 2 \sin 90^\circ$$

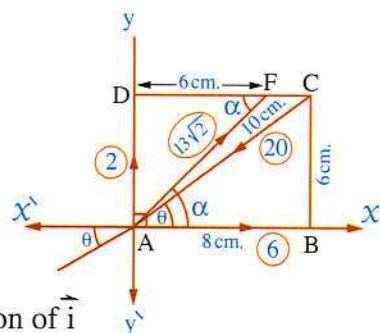
$$= 6 \times 0 - 20 \times \frac{3}{5} + 13\sqrt{2} \times \frac{1}{\sqrt{2}} + 2 \times 1 = 3 \text{ newton.}$$

$$\therefore R = \sqrt{X^2 + Y^2} = \sqrt{(3)^2 + (3)^2} = 3\sqrt{2} \text{ newton ,}$$

$\tan \ell = \frac{Y}{X} = \frac{3}{3} = 1$ where ℓ is the measure of the polar angle of \vec{R} in this example

$$\therefore X > 0, Y > 0 \quad \therefore \ell = 45^\circ$$

i.e. \vec{R} is in the direction of \overrightarrow{AF}



Another solution :

Using the analyzing of the forces into two perpendicular directions :

From Pythagoras' theorem :

$$AC = 10 \text{ cm.}$$

$$\therefore \sin \theta = \frac{6}{10} = \frac{3}{5}, \cos \theta = \frac{8}{10} = \frac{4}{5}$$

, $\therefore \triangle AFD$ is an isosceles triangle

$$\therefore m(\angle \alpha) = 45^\circ$$

$$\begin{aligned} \therefore X &= 13\sqrt{2} \cos \alpha + 6 - 20 \cos \theta \\ &= 13\sqrt{2} \times \frac{1}{\sqrt{2}} + 6 - 20 \times \frac{4}{5} = 3 \text{ newton.} \end{aligned}$$

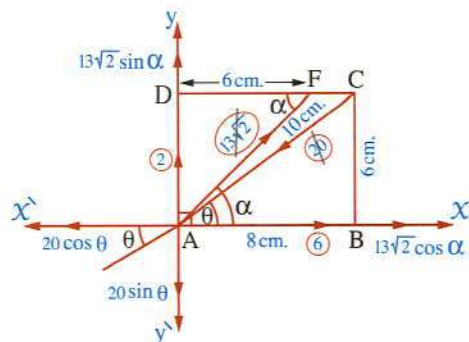
$$\begin{aligned} , Y &= 13\sqrt{2} \sin \alpha + 2 - 20 \sin \theta \\ &= 13\sqrt{2} \times \frac{1}{\sqrt{2}} + 2 - 20 \times \frac{3}{5} = 3 \text{ newton.} \end{aligned}$$

$$\therefore R = \sqrt{(3)^2 + (3)^2} = 3\sqrt{2} \text{ newton.}, \tan \ell = \frac{3}{3} = 1$$

$$, \therefore X > 0, Y > 0$$

$$\therefore \ell = 45^\circ$$

i.e. The resultant in direction of \overrightarrow{AF}



Lesson

4

Equilibrium of a rigid body under the effect of two forces / three forces meeting at a point
(The triangle of forces rule - Lami's rule)



First

Equilibrium of a rigid body under the action of two forces :

The conditions of equilibrium of a rigid body under the action of two forces

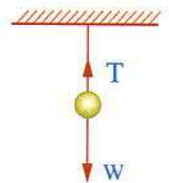
The rigid body is in equilibrium under the action of two forces only , if the two forces :

- (1) Are equal in magnitude.
- (2) Are opposite in direction.
- (3) Their lines of action are on the same straight line.

★ Examples on the equilibrium of a body under the action of two forces :

(1) A body suspended by a light string :

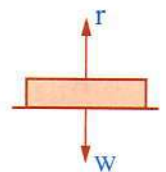
If a weight (W) is suspended by a light string. It balances under the action of two forces which are : weight (W) acting vertically downwards and the tension in the string (T) acting vertically upwards therefore : $T = W$



(2) A body of weight W placed on a horizontal smooth plane :

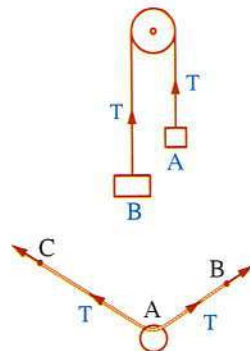
If a body of weight (W) is placed on a smooth horizontal plane. It balances under the action of two forces which are : weight (W) acting vertically downwards and the reaction of the horizontal smooth plane (r) acting vertically upwards as shown in the figure.

, we deduce that : $r = W$



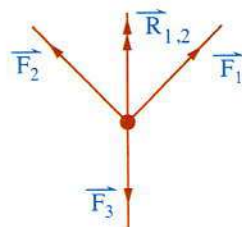
Remarks

- (1) If a rigid body is acted upon by two forces equal in magnitude, opposite in direction and their lines of action are on the same straight line, they have no effect on the body neither in case of rest nor motion.
- (2) **In the opposite figure :** If a string passes over a smooth pulley. and two bodies A and B are suspended from its two terminals such that the string is tensioned, then the two tensions in the two terminals of the string are equal in magnitude.
- (3) **In the opposite figure :** If a string passes through a smooth ring to be suspended freely in it, then the tensions in each of the two branches of the string \overline{AB} , \overline{AC} are equal in magnitude.



Second Equilibrium of a rigid body under the action of three forces acting at a point :

If three coplanar forces as \vec{F}_1 , \vec{F}_2 and \vec{F}_3 are acting at a point and they are in equilibrium as shown in the figure, and if $\vec{R}_{1,2}$ is the resultant of the two forces \vec{F}_1 and \vec{F}_2 , then the two forces, $\vec{R}_{1,2}$ and \vec{F}_3 are balanced. Then from the conditions of the equilibrium of two forces, we deduce that $\vec{R}_{1,2}$ and \vec{F}_3 are equal in magnitude, opposite in direction and they have the same line of action.



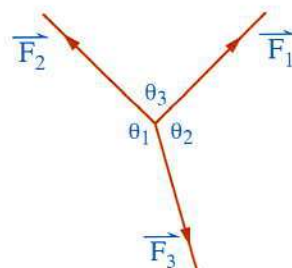
Generally : If three forces acting at a point are in equilibrium, then the resultant of any two forces of them is equal in magnitude to the third force and acts in the opposite direction of it and they have the same line of action.

Example 1

\vec{F}_1 , \vec{F}_2 and \vec{F}_3 are three coplanar forces meeting at a point, their magnitudes are 12, $12\sqrt{3}$ and 24 newton respectively. If these forces are balanced, find the measures of the angles among the three lines of action of the three forces.

Solution

Suppose the measure of the angle between the lines of action of \vec{F}_1 and \vec{F}_2 be θ_3
 \therefore The three forces are balanced.
 $\therefore R_{1,2} = F_3 = 24$ and act in the opposite direction.
 $\therefore (R_{1,2})^2 = F_1^2 + F_2^2 + 2 F_1 F_2 \cos \theta_3$



$$\therefore (24)^2 = (12)^2 + (12\sqrt{3})^2 + 2 \times 12 \times 12\sqrt{3} \cos \theta_3$$

$$\therefore 576 = 144 + 432 + 288\sqrt{3} \cos \theta_3$$

$$\therefore \cos \theta_3 = \text{zero} \quad \therefore \theta_3 = 90^\circ$$

Similarly, suppose the measure of the angle between the lines of action of \vec{F}_2 and \vec{F}_3 is θ_1 ,
 \therefore the three forces are balanced.

$$\therefore R_{2,3} = F_1 = 12 \text{ and act in the opposite direction.}$$

$$\therefore (R_{2,3})^2 = F_2^2 + F_3^2 + 2 F_2 F_3 \cos \theta_1$$

$$\therefore 144 = 432 + 576 + 2 \times 12\sqrt{3} \times 24 \cos \theta_1 \quad \therefore \cos \theta_1 = -\frac{\sqrt{3}}{2}$$

$$\therefore \theta_1 = 150^\circ$$

$$\therefore \theta_2 \text{ (the measure of the angle between the lines of action of } \vec{F}_1 \text{ and } \vec{F}_3 \text{)}$$

$$= 360^\circ - (90^\circ + 150^\circ) = 120^\circ$$

* We know that, the adjusted and sufficient condition to equilibrium of a rigid body under acting of a set of concurrent forces is a representing of these forces geometrically by the sides of a closed polygon, then we can deduce the following rule.

Rule (1)

If three forces are acting at a point and can be represented by the sides of a triangle taken in the same cyclic order, then the forces are in equilibrium.

In the opposite figure :

If \vec{F}_1 , \vec{F}_2 and \vec{F}_3 are three coplanar forces meeting at A and the vectors \vec{AB} , \vec{BC} and \vec{CA} represent these forces in magnitude and direction.

$$\therefore \vec{AC} = \vec{AB} + \vec{BC}$$

$$\therefore \vec{AC} \text{ represents the resultant of the two forces } \vec{F}_1 \text{ and } \vec{F}_2$$

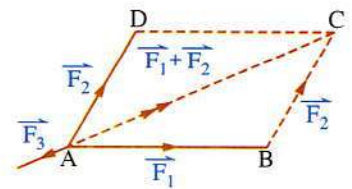
$$\text{but } \vec{CA} \text{ represents } \vec{F}_3,$$

$$\therefore \vec{AC} + \vec{CA} = \vec{O}$$

$$\therefore \vec{F}_3 \text{ equals in magnitude and opposite in direction to the resultant of } \vec{F}_1 \text{ and } \vec{F}_2$$

i.e. \vec{F}_3 is balanced with the resultant of the two forces \vec{F}_1 and \vec{F}_2

\therefore The three forces are in equilibrium



Remark

The three coplanar non collinear forces acting at a point. In order to be in equilibrium , their magnitude should be formed to be lengths of sides of a triangle.

i.e. The greatest magnitude of these forces should be less than the sum of the other two magnitudes of the other two forces because in any triangle , the longest side should be less than the sum of two lengths of the other two sides.

For example :

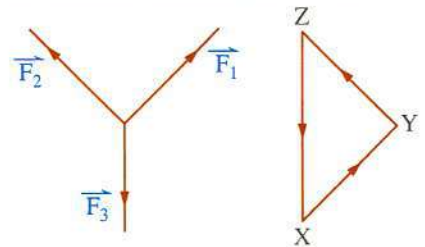
The three forces whose magnitudes are 3 , 4 and 9 force unit cannot be in equilibrium because the numbers 3 , 4 and 9 cannot be lengths of sides of any triangle because $9 > 3 + 4$ but the forces whose magnitudes are 4 , 7 , 8 could be in equilibrium , but we can not say that they are in equilibrium because that is depending on their magnitudes and their directions also.

- Three forces act at a point equilibrate if the magnitude of the greatest force equals the sum of the magnitudes of the other two forces in case they act on the same straight line.

Rule (2) The triangle of forces rule

If a rigid body is in equilibrium under the action of three forces acting at a point and a triangle is drawn whose sides are parallel to the lines of action of the forces and taken in the same cyclic order , then the lengths of the sides of the triangle are proportional to the magnitudes of the corresponding forces.

And if we symbolized to the forces' magnitude by F_1 , F_2 and F_3 , and ΔABC was the triangle whose sides are parallel to the lines of action of three forces then \overrightarrow{AB} , \overrightarrow{BC} and \overrightarrow{CA} represents the forces \vec{F}_1 , \vec{F}_2 and \vec{F}_3 respectively in the magnitude and the direction where the body is in



equilibrium , then $\frac{F_1}{AB} = \frac{F_2}{BC} = \frac{F_3}{AC}$ and ΔABC is called "the triangle of forces".

It is noted that , we can draw an infinite number of similar triangles each of them is called the triangle of forces.

Example 2

Three forces of magnitudes F_1 , F_2 and 96 kg.wt. act on a particle. They are represented by the directed line segments \overrightarrow{AB} , \overrightarrow{BC} and \overrightarrow{CA} respectively in ΔABC , where $AB = 8$ cm. , $BC = 10$ cm. and $CA = 12$ cm. Find the value of each of F_1 and F_2

Solution

\therefore The forces are represented by the directed line segments of a triangle taken in one direction.

\therefore The three forces are balanced , then using the triangle of forces rule we get :

$$\frac{F_1}{AB} = \frac{F_2}{BC} = \frac{96}{CA} \quad \therefore \frac{F_1}{8} = \frac{F_2}{10} = \frac{96}{12}$$

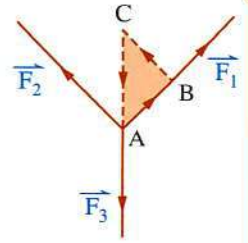
$\therefore F_1 = 64$ kg. wt. , $F_2 = 80$ kg.wt.

Important remarks :

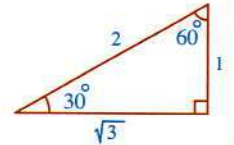
- (1) It is possible to draw the triangle of forces, such that two of its sides are on the line of action of two forces and the third side is parallel to the line of action of the third force.

As in the opposite figure :

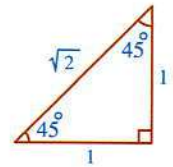
$\triangle ABC$ is the triangle of forces.



- (2)* If the triangle of forces of three equilibrium forces is a right-angled triangle and it has an angle of measure 30° , then the ratio among its side lengths is $1 : 2 : \sqrt{3}$



- * If the triangle of forces is isosceles right-angled triangle, then the ratio among its side lengths is $1 : 1 : \sqrt{2}$



Enrich knowledge

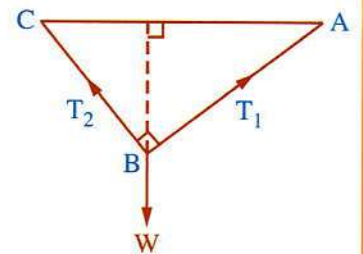
If a triangle is drawn in which its sides are perpendicular to the directions of the equilibrium, then the ratio between each force and the length of the side perpendicular to it is equal.

In the opposite figure :

$$\vec{W} \perp \overline{AC}, \vec{T}_1 \perp \overline{BC}, \vec{T}_2 \perp \overline{AB}$$

$$\therefore \frac{W}{AC} = \frac{T_1}{BC} = \frac{T_2}{AB}$$

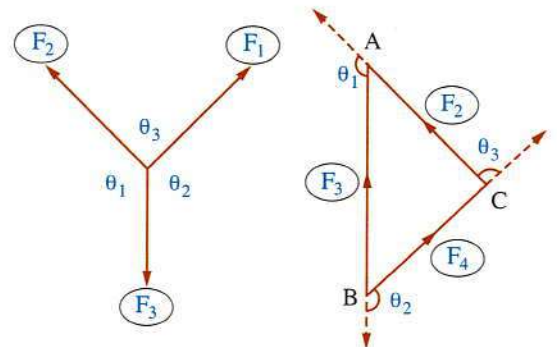
This rule is called perpendicular of forces triangle.



Rule (3) - Lami's rule

If three coplanar forces meeting at a point and acting up on a particle are in equilibrium, then the magnitude of each force is proportional to the sine of the angle between the two other forces.

If the symbols of the magnitudes of the forces are F_1 , F_2 and F_3 , and θ_1 , θ_2 and θ_3 are the measures of the opposite angles for them respectively as shown in the opposite figure :
Then $\triangle BCA$ is the triangle of forces



$$\therefore \frac{F_1}{BC} = \frac{F_2}{CA} = \frac{F_3}{AB} \quad (1)$$

and from sin law :

$$\therefore \frac{BC}{\sin (180 - \theta_1)} = \frac{CA}{\sin (180 - \theta_2)} = \frac{AB}{\sin (180 - \theta_3)}$$

$$\text{i.e. } \frac{BC}{\sin \theta_1} = \frac{CA}{\sin \theta_2} = \frac{AB}{\sin \theta_3} \quad (2)$$

From (1) , (2) we deduce that : $\frac{F_1}{\sin \theta_1} = \frac{F_2}{\sin \theta_2} = \frac{F_3}{\sin \theta_3}$

Example 3

Three coplanar forces of magnitudes F_1 , F_2 and 18 newton meeting at a particle in balance. If the measure of the angle between the line of action of 1st and 2nd forces is 90° and between the 2nd and the 3rd is 120° Find the value of each of F_1 and F_2

Solution

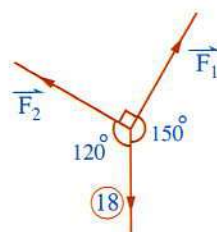
The measure of the angle between the 1st and 3rd forces = $360^\circ - (90^\circ + 120^\circ) = 150^\circ$

Due to Lami's rule we get :

$$\therefore \frac{F_1}{\sin 120^\circ} = \frac{F_2}{\sin 150^\circ} = \frac{F_3}{\sin 90^\circ} \quad \therefore \frac{F_1}{\frac{\sqrt{3}}{2}} = \frac{F_2}{\frac{1}{2}} = \frac{18}{1}$$

$$\therefore F_1 = 18 \times \frac{\sqrt{3}}{2} = 9\sqrt{3} \text{ newton}$$

$$, F_2 = 18 \times \frac{1}{2} = 9 \text{ newton.}$$



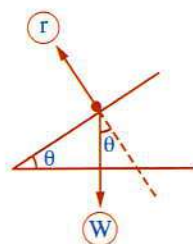
Equilibrium of a body placed on a smooth inclined plane

If a body of weight (W) is placed on a smooth inclined plane which inclines by an angle of measure θ with the horizontal , then the body will be under the action of two forces :

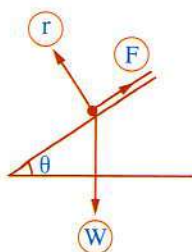
- (1) The weight force (\vec{W}) acting vertically downwards.
- (2) The reaction force (\vec{r}) of the inclined plane and it acts in direction perpendicular to the plane.

These two forces cannot be in equilibrium because they have two different lines of action. Therefore , in order to be in equilibrium , a third force must act on the body.

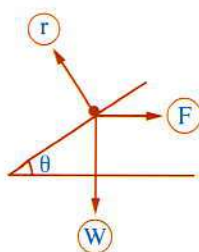
It may be in one of the following forms :



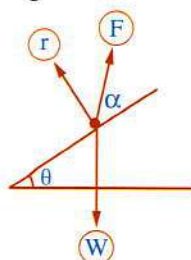
(1) The force is in the direction of the line of the greatest slope upwards.



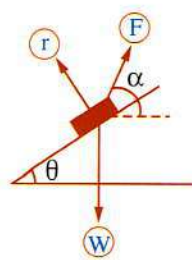
(2) The force acts horizontally.



(3) The force inclines by α with the plane upwards.



(4) The force inclines by α with the horizontal upward.



Remark

The reaction of the smooth plane (\vec{r}) is perpendicular to the plane.

Example 4

A body of weight 5 kg.wt. is placed on a smooth plane inclined to the horizontal at an angle of measure 30° . It is pulled up the plane under the action of a force of magnitude F which its line of action coincides the line of the greatest slope up the plane. Find F and the reaction of the plane.

Solution

The body is in equilibrium under the action of forces of magnitudes F , r and 5 kg.wt. as shown in the figure.

and the measure of the angle between the two lines of action of the two first forces = 90°

and between the 2nd and the 3rd = $180^\circ - 30^\circ = 150^\circ$

and between the 3rd and the 1st = $90^\circ + 30^\circ = 120^\circ$

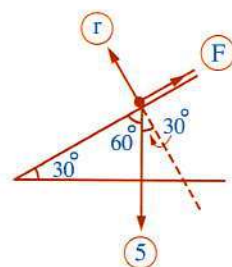
Applying Lami's rule we get :

$$\frac{F}{\sin 150^\circ} = \frac{r}{\sin 120^\circ} = \frac{5}{\sin 90^\circ}$$

$$\text{i.e. } \frac{F}{\frac{1}{2}} = \frac{r}{\frac{\sqrt{3}}{2}} = \frac{5}{1}$$

$$\therefore F = 5 \times \frac{1}{2} = 2 \frac{1}{2} \text{ kg.wt.}$$

$$, r = 5 \times \frac{\sqrt{3}}{2} = \frac{5\sqrt{3}}{2} \text{ kg.wt.}$$



Example 5

A body of weight 20 kg.wt. is placed on a smooth plane inclined to the horizontal with an angle of measure α where $\cos \alpha = \frac{4}{5}$

The body is kept in equilibrium by a horizontal force of magnitude F

Find F and the reaction of the plane.

Solution

\therefore The weight is in equilibrium under the action of forces of magnitudes F , r and 20 kg.wt. therefore, the measure of the angle between the 1st and 2nd forces = $90^\circ + \alpha$ and between the 2nd and 3rd = $180^\circ - \alpha$ and between the 3rd and the first = 90°

Applying Lami's rule we get :

$$\frac{F}{\sin (180^\circ - \alpha)} = \frac{r}{\sin 90^\circ} = \frac{20}{\sin (90^\circ + \alpha)}$$

$$\text{i.e. } \frac{F}{\sin \alpha} = \frac{r}{1} = \frac{20}{\cos \alpha}$$

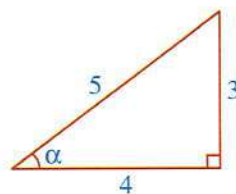
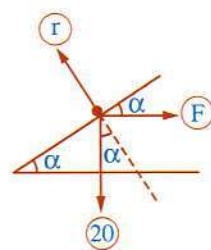
$$\therefore \cos \alpha = \frac{4}{5}$$

$$\therefore \frac{F}{\frac{3}{5}} = \frac{r}{1} = \frac{20}{\frac{4}{5}}$$

$$\therefore r = 20 \times \frac{5}{4} = 25 \text{ kg.wt.}$$

$$\therefore \sin \alpha = \frac{3}{5}$$

$$\therefore F = 20 \times \frac{3}{5} \times \frac{5}{4} = 15 \text{ kg.wt.}$$



General examples on the equilibrium of three coplanar concurrent forces

Example 6

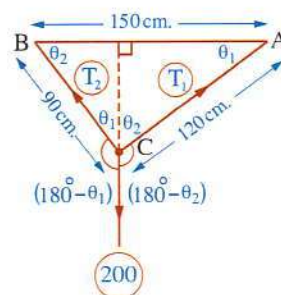
A weight of 200 gm.wt. is suspended by two strings of lengths 90 cm. and 120 cm. fixed in two horizontal points, the distance between them is 150 cm. Find the value of the tension in each of the two strings in case of equilibrium.

Solution

$$\therefore (90)^2 + (120)^2 = (150)^2$$

$\therefore \triangle ABC$ is right-angled at C , from the figure we get :

$$\sin \theta_1 = \frac{90}{150} = \frac{3}{5}, \sin \theta_2 = \frac{120}{150} = \frac{4}{5}$$



Using Lami's rule :

$$\therefore \frac{T_1}{\sin (180^\circ - \theta_1)} = \frac{T_2}{\sin (180^\circ - \theta_2)} = \frac{200}{\sin 90^\circ}$$

$$\therefore \frac{T_1}{\sin \theta_1} = \frac{T_2}{\sin \theta_2} = \frac{200}{\sin 90^\circ} \quad \therefore \frac{T_1}{\frac{3}{5}} = \frac{T_2}{\frac{4}{5}} = \frac{200}{1}$$

$$\therefore T_1 = \frac{200}{1} \times \frac{3}{5} = 120 \text{ gm.wt.}, T_2 = \frac{200}{1} \times \frac{4}{5} = 160 \text{ gm.wt.}$$

Another solution :

By using the triangle of forces rule : Draw $\overline{DF} \parallel \overline{CB}$

, then ΔDFC is the triangle of forces

$$\therefore \frac{T_1}{CF} = \frac{T_2}{FD} = \frac{200}{DC}$$

$$\therefore T_1 = 200 \times \frac{CF}{DC} = 200 \sin \theta_1 = 200 \times \frac{3}{5} = 120 \text{ gm.wt.}$$

$$, T_2 = 200 \times \frac{FD}{DC} = 200 \sin \theta_2 = 200 \times \frac{4}{5} = 160 \text{ gm.wt.}$$

Third solution : (By resolution) :

\therefore The three forces are in equilibrium.

\therefore The resultant of the two tensions = The third force and in the opposite direction

$\therefore \vec{T}_1$ and \vec{T}_2 are the two components of \vec{R} and they are perpendicular

$$\therefore T_1 = R \cos \theta_2 = 200 \times \frac{3}{5} = 120 \text{ gm.wt.},$$

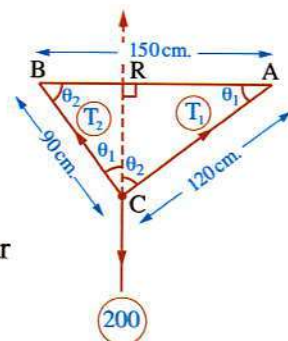
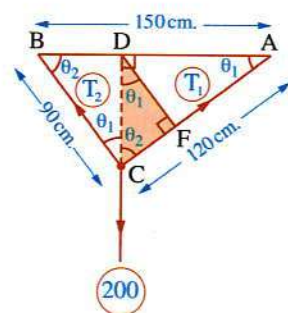
$$T_2 = R \sin \theta_2 = 200 \times \frac{4}{5} = 160 \text{ gm.wt.}$$

Fourth solution : (By perpendicular of forces triangle) :

$\therefore \Delta ABC$ is perpendicular of forces triangle.

$$\therefore \frac{T_1}{90} = \frac{200}{150} = \frac{T_2}{120}$$

$$\therefore T_1 = 120 \text{ gm. wt.}, T_2 = 160 \text{ gm. wt.}$$



Example 7

A weight of 80 gm.wt. is suspended by a string fixed in a vertical wall. The weight is pulled by a force perpendicular to the string till it becomes in equilibrium when it is inclined on the wall by an angle of measure 30° , find in case of equilibrium, the magnitude of the force and the tension in the string.

Solution

Due to Lami's rule we get : $\frac{F}{\sin 150^\circ} = \frac{T}{\sin 120^\circ} = \frac{80}{\sin 90^\circ}$

$$\therefore \frac{F}{\frac{1}{2}} = \frac{T}{\frac{\sqrt{3}}{2}} = 80$$

$$\therefore F = 80 \times \frac{1}{2} = 40 \text{ gm.wt.}, \quad T = 80 \times \frac{\sqrt{3}}{2} = 40\sqrt{3} \text{ gm.wt.}$$

Another solution : (By triangle of forces) :

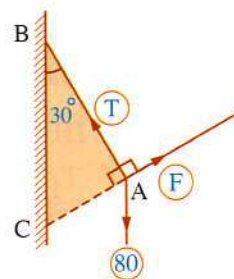
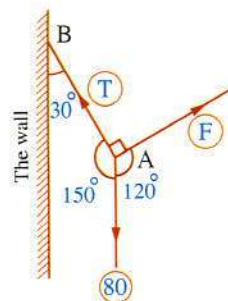
Draw the line of action of \vec{F} to meet the wall at C

, then $\triangle CAB$ is the triangle of forces.

Due to the triangle of forces rule we get : $\frac{F}{CA} = \frac{T}{AB} = \frac{80}{BC}$

In $\triangle CAB : \therefore CA : AB : BC = 1 : \sqrt{3} : 2$

$$\therefore \frac{F}{1} = \frac{T}{\sqrt{3}} = \frac{80}{2} \quad \therefore F = 40 \text{ gm.wt.}, \quad T = 40\sqrt{3} \text{ gm.wt.}$$



Example 8

A light string \overline{AB} of length 8 cm. Its terminal A is fixed at a point. A weight of 300 gm.wt. is suspended at the other terminal B. Find the magnitude of the needed force to keep the weight in equilibrium at a distance of 4 cm. From the horizontal line passing through A, also find the tension in the string in each of the two cases.

(1) If the force is horizontal. (2) If the direction of the force is perpendicular to \overline{AB}

Solution

The first case :

If the force is horizontal, we can take $\triangle ABC$ as the triangle of forces.

$$\therefore \frac{T}{AB} = \frac{F}{AC} = \frac{300}{BC}$$

$$\therefore AC = \sqrt{(8)^2 - (4)^2} = 4\sqrt{3} \text{ cm.}$$

$$\therefore \frac{T}{8} = \frac{F}{4\sqrt{3}} = \frac{300}{4}$$

$$\therefore T = 600 \text{ gm.wt.}, \quad F = 300\sqrt{3} \text{ gm.wt.}$$

The second case :

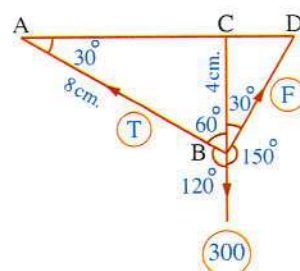
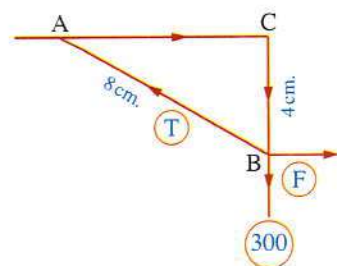
If the direction of the force is perpendicular to \overline{AB}

Due to Lami's rule :

$$\therefore \frac{T}{\sin 150^\circ} = \frac{F}{\sin 120^\circ} = \frac{300}{\sin 90^\circ}$$

$$\therefore T = \frac{300 \sin 150^\circ}{\sin 90^\circ}$$

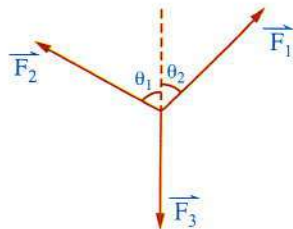
$$\therefore T = 150 \text{ gm.wt.}, \quad F = \frac{300 \sin 120^\circ}{\sin 90^\circ} = 150\sqrt{3} \text{ gm.wt.}$$



Remark

If the line of action of one force of three equilibrium forces is extended to divide the angle between the two lines of action of the other two forces into two angles, then we can apply Lami's rule as follows :

$$\frac{F_1}{\sin \theta_1} = \frac{F_2}{\sin \theta_2} = \frac{F_3}{\sin (\theta_1 + \theta_2)}$$



Example 9

A body of weight 18 kg.wt. is placed on a smooth plane inclined to the horizontal by an angle of measure 30° . It is pulled upwards under the action of a force (F) inclines with the line of greatest slope of the plane by an angle of measure 30° . Find the magnitude of this force and the reaction of the plane.

Solution

The body is in equilibrium under the action of the three forces of magnitudes F , r and 18 kg.wt. where :

The measure of the angle between the 1st and 2nd forces is 60°

and between the 2nd and the 3rd is 150°

and between the 3rd and the 1st is $30^\circ + 90^\circ + 30^\circ = 150^\circ$ also.

Applying Lami's rule we get :

$$\frac{F}{\sin 150^\circ} = \frac{r}{\sin 150^\circ} = \frac{18}{\sin 60^\circ} \quad \text{i.e.} \quad \frac{F}{\frac{1}{2}} = \frac{r}{\frac{1}{2}} = \frac{18}{\frac{\sqrt{3}}{2}}$$

$$\therefore F = r = 18 \times \frac{1}{2} \div \frac{\sqrt{3}}{2} = 6\sqrt{3} \text{ kg.wt.}$$

Another solution :

\therefore The line of action of the weight \vec{W} which is the line of action of the resultant of the two forces \vec{F} and \vec{r} and it bisects the angle between them.

$$F = r ,$$

$$\therefore R = 2 F \cos \frac{\alpha}{2}$$

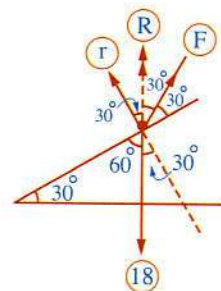
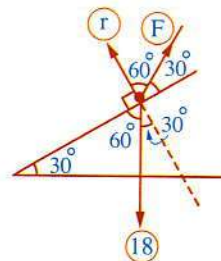
$$\therefore 18 = 2 F \cos 30^\circ$$

$$\therefore F = \frac{18}{\sqrt{3}} = 6\sqrt{3} \text{ kg.wt.}$$

$$\therefore 18 = 2 F \times \cos \frac{60^\circ}{2}$$

$$\therefore 18 = 2 F \times \frac{\sqrt{3}}{2}$$

$$\therefore F = r = 6\sqrt{3} \text{ kg.wt.}$$



Example 10

A light string is fastened from its terminals at two points B and C such that \overline{BC} is equilibrium horizontal. A small smooth ring of weight 20 gm.wt. slides on the string till the angle between the two branches of the string in equilibrium becomes 90° in measure. Prove that the lengths of the two branches of the string are equal, then find the value of the tension in each of them.

Solution

\therefore The ring is smooth.

\therefore The values of tension in the two branches of the string are equal.

i.e. Tension in the branch \overline{AB} = tension in the branch \overline{AC} = T

Using Lami's rule we get :

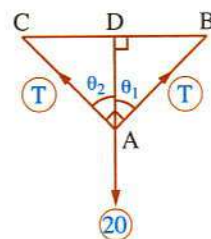
$$\frac{T}{\sin \theta_2} = \frac{T}{\sin \theta_1} = \frac{20}{\sin 90^\circ}$$

$$\therefore \theta_1 = \theta_2 = \frac{90^\circ}{2} = 45^\circ$$

$$\therefore T = 10\sqrt{2}$$

$$\therefore \overline{AD} \perp \overline{BC} \quad , \quad \theta_1 = \theta_2 = 45^\circ \quad \therefore AB = AC$$

\therefore The lengths of the two branches of the string are equal.



Example 11

A body of weight (W) newton is suspended by two strings. The first inclines on the vertical by an angle of measure 30° and passes over a fixed smooth pulley and carries at its free end a weight $16\sqrt{3}$ newton. The second string inclines on the vertical by an angle of measure θ and passes over another fixed smooth pulley and carries at its terminal a body of weight 16 newton. Find in equilibrium case the value of W and the value of θ

Solution

Using Lami's rule :

$$\therefore \frac{T_1}{\sin \theta} = \frac{T_2}{\sin 30^\circ} = \frac{W}{\sin (\theta + 30^\circ)}$$

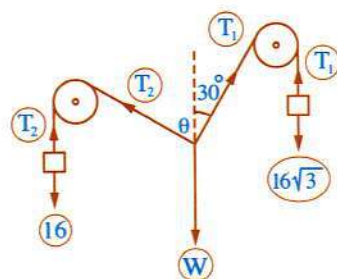
$$\therefore \frac{16\sqrt{3}}{\sin \theta} = \frac{16}{\sin 30^\circ} = \frac{W}{\sin (\theta + 30^\circ)}$$

$$\therefore \sin \theta = \frac{16\sqrt{3} \sin 30^\circ}{16} = \frac{\sqrt{3}}{2}$$

$$\therefore \frac{16}{\sin 30^\circ} = \frac{W}{\sin (60^\circ + 30^\circ)}$$

$$\therefore \theta = 60^\circ$$

$$\therefore W = 32 \text{ newton.}$$



Lesson

5

Follow : The equilibrium

(Meeting lines of action of three equilibrium forces)



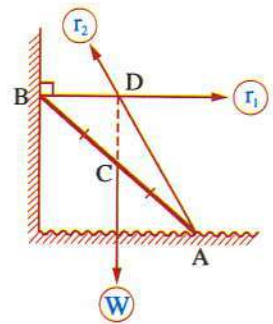
Rule (4)

If a rigid body is in equilibrium under the action of three coplanar non parallel forces , then the lines of action of these forces meet at a point.

For example , in the opposite figure :

If a uniform rod of weight (W) is in equilibrium on a smooth vertical wall and on rough horizontal ground then :

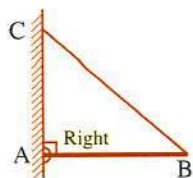
- (1) The weight of the rod acts vertically downwards at its midpoint. (centre of gravity)
- (2) The reaction of the smooth vertically wall (r_1) which is perpendicular to the wall in direction of \overrightarrow{BD}
- (3) The reaction of the rough ground (r_2) with unknown direction , and to determine its direction , we draw \overrightarrow{AD} passes through the point D (The point of intersection between \overrightarrow{w} , $\overrightarrow{r_1}$)



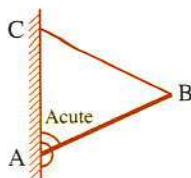
Remarks

- (1) The weight of the uniform sphere acts at its geometric centre (centre of gravity)
- (2) If \overline{AB} is a rod , its end A is attached to a hinge fixed at a vertical wall and the other end B is attached to a string fixed at the point C which lies above A exactly and we notice that :

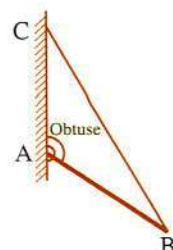
$(BC)^2 = (AB)^2 + (AC)^2$,
then $\angle BAC$ is right-angled.



$(BC)^2 < (AB)^2 + (AC)^2$,
then $\angle BAC$ is acute.

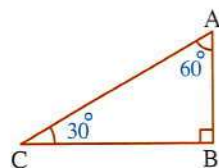


$(BC)^2 > (AB)^2 + (AC)^2$,
then $\angle BAC$ is obtuse.



(3) If $\triangle ABC$ is a $30^\circ - 60^\circ$ triangle

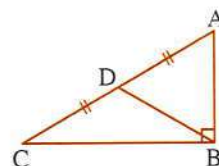
$$\therefore AB = \frac{1}{2} AC, BC = \frac{\sqrt{3}}{2} AC$$



(4) If $\triangle ABC$ is right-angled at B,

\overline{BD} is a median of it

$$\text{, then } BD = \frac{1}{2} AC$$



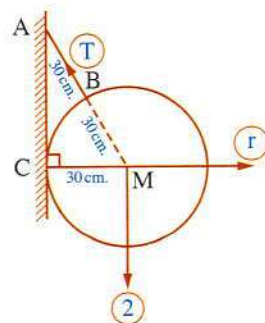
Example 1

A metallic sphere of weight 2 kg.wt. and of radius length 30 cm. is suspended at a point B on its surface by a string of length 30 cm., its other end A is fixed at a point in a vertical wall to be in equilibrium as it rests on the wall. Find the magnitude of the tension in the string and the magnitude of the reaction of the wall.

Solution

The sphere is at rest under the action of three forces :

- (1) The weight of the sphere whose magnitude is 2 kg.wt. acts vertically downwards at its centre M
- (2) The reaction of the wall of magnitude r , acts at the point of touch of the sphere with the wall (C) in the direction perpendicular to the wall, hence it passes through the centre of the sphere M



(3) The tension in the string of magnitude T acts in the direction of \overrightarrow{BA}

∴ The lines of action of the weight force and the reaction force meet at M

∴ The line action of the tension force in the string should pass through the point M

i.e. \overrightarrow{AB} passes through the point M , then $\triangle MAC$ is the triangle of forces where

$$MA = MB + BA = 60 \text{ cm.}, \quad CM = 30 \text{ cm.}$$

∴ $\triangle AMC$ is a right-angled triangle of $(30^\circ - 60^\circ)$ angles.

$$\therefore AC = 30\sqrt{3} \text{ cm.} \quad \therefore \frac{T}{60} = \frac{r}{30} = \frac{2}{30\sqrt{3}}$$

$$\therefore T = \frac{4\sqrt{3}}{3} \text{ kg.wt.}, \quad r = \frac{2\sqrt{3}}{3} \text{ kg.wt.}$$

Drill Try to solve the previous problem by Lami's rule.

Example 2

\overline{AB} is a uniform rod of length 60 cm., and its weight 24 kg.wt., acting at (D) the midpoint of \overline{AB} , the end A of the rod is attached to a hinge fixed at a vertical wall. The other end B is attached to a light string, its other end is fixed at the point C on a vertical wall above A at a distance 80 cm. from A . If the rod is in equilibrium horizontally. Find the magnitude of the tension in the string and the magnitude and direction of the reaction of the hinge at A

Solution

$$\text{From } \triangle ABC : BC = \sqrt{(60)^2 + (80)^2} = 100 \text{ cm.}$$

The rod is in equilibrium under the effect of three forces :

(1) Its weight 24 kg.wt., acts vertically downwards at the point D (the midpoint of \overline{AB})

(2) The tension in the string (T) acts in the direction of \overrightarrow{BC}

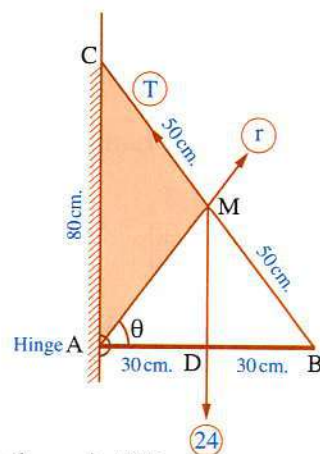
(3) The reaction of the hinge at A (its magnitude = r)

∴ The two lines of action of the weight and the tension meet at the point M

∴ The line of action of the reaction of the hinge passes through the point M also ,

∴ D is the midpoint of \overline{AB} , $\overline{MD} \parallel \overline{AC}$

∴ M is the midpoint of \overline{BC}



$\therefore \overline{AM}$ is a median of $\triangle ABC$ which is right at A

$$\therefore AM = \frac{1}{2} BC = 50 \text{ cm.}$$

$$\therefore \triangle AMC \text{ is the triangle of forces} \quad \therefore \frac{T}{MC} = \frac{r}{AM} = \frac{24}{CA}$$

$$\text{i.e. } \frac{T}{50} = \frac{r}{50} = \frac{24}{80} \quad \therefore r = T = 24 \times \frac{50}{80} = 15 \text{ kg.wt. ,}$$

$$\text{From } \triangle AMD : \tan \theta = \frac{40}{30} = \frac{4}{3} \quad \therefore \theta \approx 53^\circ \hat{8}$$

\therefore The reaction of the hinge at A inclines to the rod with an angle of measure $53^\circ \hat{8}$

Example 3

A uniform rod of length 50 cm. and weight 120 gm. wt. , is suspended at its two ends freely by two strings , their other two ends are fixed at one point.

If the lengths of the two strings are 30 cm. and 40 cm. respectively.

Find the magnitude of the tension in each of the two strings.

Solution

The rod is in equilibrium under the effect of three forces :

their magnitudes are T_1 , T_2 and 120 gm.wt , where \overline{AC} is the line of action of T_1 and \overline{BC} is the line of action of T_2 they are meeting at the point C

\therefore The action line of the weight of the rod should pass through C also

$$\therefore (AB)^2 = 2500 , (AC)^2 + (CB)^2 = 900 + 1600 = 2500$$

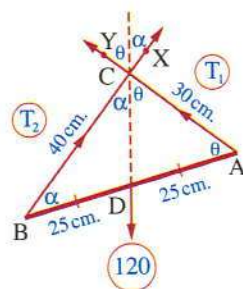
\therefore D is the midpoint of the hypotenuse \overline{AB}

$$\therefore m(\angle ACD) = m(\angle DAC) , m(\angle DCB) = m(\angle DBC)$$

$$\text{Applying Lami's rule we get : } \frac{T_1}{\sin(\alpha)} = \frac{T_2}{\sin(\theta)} = \frac{120}{\sin 90^\circ}$$

$$\therefore \frac{T_1}{30} = \frac{T_2}{40} = 120$$

$$\therefore T_1 = 120 \times \frac{30}{50} = 72 \text{ gm.wt. , } T_2 = 120 \times \frac{40}{50} = 96 \text{ gm.wt.}$$



Example 4

\overline{AB} is a uniform rod of length 140 cm. and weight 480 gm.wt. , its end A is attached to a hinge fixed on a vertical wall. A force \vec{F} acts horizontally at the other end B to make the rod at rest at a position in which the rod inclines to the horizontal at an angle of measure 30°

Find the magnitude of \vec{F} , and the magnitude and the direction of the reaction of the hinge at A

Solution

The rod is at rest under the effect of three forces :

(1) Its weight 480 gm.wt. acts vertically downwards at the point D (the midpoint of \overline{AB})

(2) The horizontal force \vec{F} at B

(3) The reaction of the hinge at A of magnitude r

\therefore The lines of action of the forces of weight and the horizontal force are meeting at the point M (*i.e.* in the direction of \overrightarrow{MA})

\therefore The line of action of the reaction of the hinge should pass through the point M also

$\therefore \triangle CMA$ is the triangle of forces.

$$\therefore \frac{F}{CM} = \frac{r}{MA} = \frac{480}{AC}$$

$$\therefore m(\angle ABC) = 30^\circ$$

$$\therefore AC = \frac{1}{2} AB = 70 \text{ cm.}, \quad BC = \frac{\sqrt{3}}{2} AB = 70\sqrt{3} \text{ cm.}$$

\therefore D is the midpoint of \overline{AB} , $\overrightarrow{DM} \parallel \overrightarrow{AC}$

\therefore M is the midpoint of \overline{BC}

$$\therefore MC = 35\sqrt{3} \text{ cm.}$$

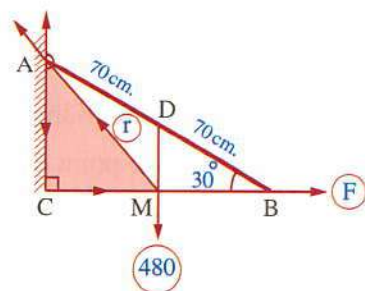
$$\therefore AM = \sqrt{(70)^2 + (35\sqrt{3})^2} = 35\sqrt{7} \text{ cm.} \quad \therefore \frac{F}{35\sqrt{3}} = \frac{r}{35\sqrt{7}} = \frac{480}{70}$$

$$\therefore F = 240\sqrt{3} \text{ gm.wt.}, \quad r = 240\sqrt{7} \text{ gm.wt.}$$

$$\therefore \triangle AMC \text{ is right-angled triangle at C} \quad \therefore \tan(\angle AMC) = \frac{AC}{MC} = \frac{70}{35\sqrt{3}} = \frac{2}{\sqrt{3}}$$

$$\therefore m(\angle AMC) \approx 49^\circ \hat{6}$$

\therefore The reaction of the hinge makes an angle of measure $49^\circ \hat{6}$ with the horizontal.



Example 5

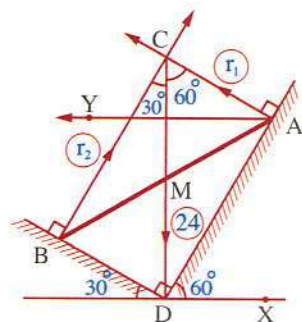
A uniform rod rests with its two ends on two smooth planes incline to the horizontal at two angles of measures 60° and 30° . Find the measure of the angle which the rod makes with the horizontal at the equilibrium position and if the weight of the rod equals 24 newton. Determine the magnitudes of the two reactions of the two planes.

Solution

The rod is in equilibrium under the action of three forces :

(1) The weight 24 newton acts vertically downwards at the point M (the midpoint of \overline{AB})

(2) The reaction of the first plane r_1



(3) The reaction of the second plane r_2

\therefore The lines of action of the two forces of reactions meet at the point C

\therefore The line of action of the weight of the rod passes through the same point "C" either.

If D is the point of meeting the two planes, then $\angle A$, $\angle D$ and $\angle B$ are right-angles

\therefore The figure ACBD is a rectangle.

If M is the midpoint of \overline{AB}

\therefore M is the point of intersection of the two diagonals of the rectangle.

$\therefore \overline{CD}$ is a diagonal in the rectangle passing through M

$\therefore \overline{CD}$ is vertical

$$\therefore m(\angle CDX) = 90^\circ$$

$$\therefore m(\angle MDA) = 30^\circ$$

$$\therefore MD = MA$$

$$\therefore m(\angle DAM) = 30^\circ$$

$$\therefore m(\angle YAD) = 60^\circ$$

$$\therefore m(\angle YAM) = 30^\circ$$

\therefore The rod makes an angle of measure 30° with the horizontal.

From $\triangle ADC$: $\therefore m(\angle ACD) = 60^\circ$

$$\text{Applying Lami's rule we get : } \therefore \frac{r_1}{\sin 150^\circ} = \frac{r_2}{\sin 120^\circ} = \frac{24}{\sin 90^\circ}$$

$$\therefore r_1 = 12 \text{ newton} \quad , \quad r_2 = 12\sqrt{3} \text{ newton.}$$

Example 6

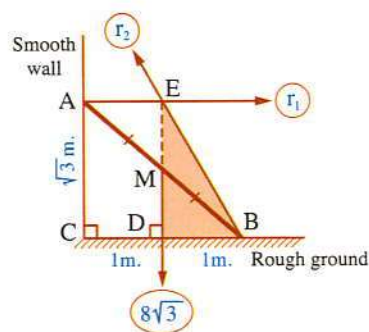
\overline{AB} is a uniform ladder of weight $8\sqrt{3}$ kg.wt. , its upper end A rests on a smooth vertical wall and its lower end B rests on rough horizontal ground such that the upper end is far from the surface of the ground by $\sqrt{3}$ metre and the lower end is far from the wall by 2 metre. Find the magnitude of pressure on each of the wall and the ground in case of equilibrium.

Solution

The ladder is in equilibrium under the effect of three forces :

- (1) The weight of the ladder of magnitude $8\sqrt{3}$ kg.wt. , acts vertically downwards at the midpoint of the ladder (M)
- (2) The reaction of the smooth vertical wall of magnitude r_1 which is perpendicular to the wall at A
- (3) The reaction of the rough ground of magnitude r_2

\therefore The lines of action of the two forces of the weight and the reaction of the wall meet at the point E



∴ The line of action of the reaction of the ground must pass through the point E

, then $\triangle DBE$ is the triangle of forces where :

$$DE = AC = \sqrt{3} \text{ metre.}, \quad BD = \frac{1}{2} BC = 1 \text{ metre}$$

$$BE = \sqrt{(1)^2 + (\sqrt{3})^2} = 2 \text{ metre}$$

Applying the triangle of forces rule we get :

$$\frac{r_1}{DB} = \frac{r_2}{BE} = \frac{8\sqrt{3}}{ED} \quad \therefore \frac{r_1}{1} = \frac{r_2}{2} = \frac{8\sqrt{3}}{\sqrt{3}}$$

$$\therefore r_1 = 8 \text{ kg.wt.}, \quad r_2 = 16 \text{ kg.wt.}$$

∴ The pressure on the wall = 8 kg.wt. , the pressure on the ground = 16 kg.wt.

Remark

The pressure of the two ends of the ladder on the floor and the wall equals in magnitude the reactions of the floor and the wall on the two ends of the ladder.

Example 7

\overline{AB} is a uniform rod of length 120 cm. and weight 15 kg.wt. , its end A is attached to a hinge fixed at a point on a vertical wall.

The rod is kept in equilibrium horizontally by attaching it at a point C on it where $AC = 80$ cm. by a string. The other end of the string is fixed at a point D on the vertical wall above A and at a distance $80\sqrt{3}$ cm. from it. Calculate the magnitude of each of the tension in the string and the reaction of the hinge.

Solution

The rod is kept in equilibrium under the effect of three forces :

(1) Its weight 15 kg.wt. , acts vertically downwards at

M (the midpoint of \overline{AB})

(2) The tension force in the string (T)

(3) The reaction of the hinge (r)

∴ The lines of action of the weight and the tension are meeting at the point E

∴ The line of action of the reaction of the hinge passes through the point E also.

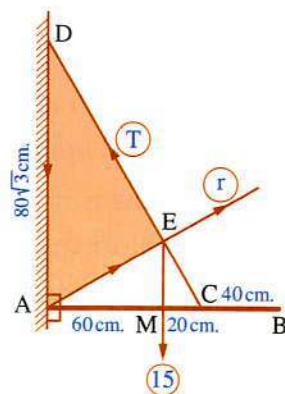
∴ $\triangle AED$ is the triangle of forces

$$\therefore CD = \sqrt{(80)^2 + (80\sqrt{3})^2} = 160 \text{ cm.}$$

$$\therefore \triangle CME \sim \triangle CAD : \quad \therefore \frac{CM}{CA} = \frac{CE}{CD} = \frac{ME}{AD}$$

$$\therefore \frac{20}{80} = \frac{CE}{160} = \frac{ME}{80\sqrt{3}} \quad \therefore CE = 40 \text{ cm.}, \quad ME = 20\sqrt{3}, \quad ED = 120 \text{ cm.}$$

$$\therefore AE = \sqrt{(60)^2 + (20\sqrt{3})^2} = 40\sqrt{3} \text{ cm.}$$



Applying the triangle of forces rule : $\therefore \frac{r}{40\sqrt{3}} = \frac{T}{120} = \frac{15}{80\sqrt{3}}$
 $\therefore r = 7.5 \text{ kg.wt.}$ $\therefore T = 7.5\sqrt{3} \text{ kg.wt.}$

Example 8

A string of length 24 cm. is fixed from its ends at two pins A and B in a horizontal line, the distance between them is 12 cm. the string passes inside a smooth ring to be suspended in it, its weight is 144 dyne, then the ring is pulled horizontally by a force \vec{F} till it becomes down B directly.

Find the magnitude of tension in each of the two branches of the string, find also the magnitude of \vec{F}

Solution

Supposing that $BM = l$ cm.

$$\therefore MA = (24 - l) \text{ cm.}$$

$$\therefore (24 - l)^2 = l^2 + (12)^2$$

$$\therefore l = 9 \text{ cm.}$$

Let $m(\angle MAB) = \theta$

$$\therefore \cos \theta = \frac{4}{5}, \sin \theta = \frac{3}{5}$$

\therefore The ring is smooth.

\therefore The tension in the two branches \overline{MB} and \overline{MA} are equal in magnitude.

\therefore The ring is in equilibrium under the effect of four forces which are :

$$(F, 0^\circ), (T, 90^\circ), (T, 180^\circ - \theta), (144, 270^\circ)$$

$$\therefore X = 0$$

$$\therefore F \cos 0^\circ + T \cos 90^\circ + T \cos (180^\circ - \theta) + 144 \cos 270^\circ = \text{zero}$$

$$\therefore F \times 1 + T \times 0 + T \times -\cos \theta + 144 \times 0 = \text{zero}$$

$$\therefore F - \frac{4}{5} T = 0 \quad (1)$$

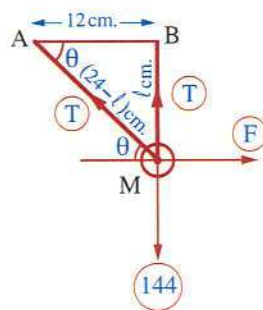
$$\therefore Y = 0$$

$$\therefore F \sin 0^\circ + T \sin 90^\circ + T \sin (180^\circ - \theta) + 144 \times \sin 270^\circ = \text{zero}$$

$$\therefore F \times 0 + T \times 1 + T \sin \theta + 144 \times -1 = \text{zero} \quad \therefore T + \frac{3}{5} T - 144 = \text{zero}$$

$$\therefore T \left(1 + \frac{3}{5}\right) = 144 \quad \therefore T = 90 \text{ dyne.}$$

$$\text{Substituting in (1) : } \therefore F = \frac{4}{5} \times 90 = 72 \text{ dyne.}$$



Notice that

The ring is in equilibrium under the effect of four forces, so it will be solved by using the method you studied in lesson 3
 $X = 0, Y = 0$

Another solution :

∴ The weight and the vertical tension

on the same straight line

∴ Their resultant can be calculated as $(144 - T)$

Using lami's rule :

$$\therefore \frac{F}{\sin (90^{\circ} + \theta)} = \frac{T}{\sin 90^{\circ}} = \frac{144 - T}{\sin (180^{\circ} - \theta)}$$

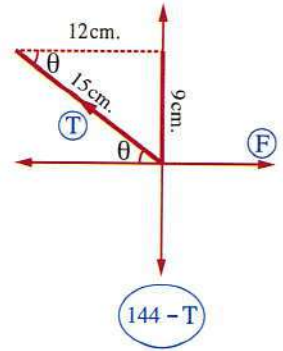
$$\therefore \frac{F}{\cos \theta} = \frac{T}{1} = \frac{144 - T}{\sin \theta}$$

$$\therefore \frac{F}{\left(\frac{4}{5}\right)} = T = \frac{144 - T}{\left(\frac{3}{5}\right)}$$

$$\therefore \frac{3}{5} T = 144 - T$$

$$\therefore \frac{8}{5} T = 144$$

$$\therefore T = 90 \text{ dyne} \quad , \quad F = 90 \times \frac{4}{5} = 72 \text{ dyne}$$



Third solution :

∴ The weight and the vertical tension on the same straight line

∴ Their resultant can be calculated as $(144 - T)$

and the ΔABM becomes the triangle of forces

$$\therefore \frac{144 - T}{9} = \frac{T}{15} = \frac{F}{12}$$

$$\therefore \frac{144 - T}{3} = \frac{T}{5}$$

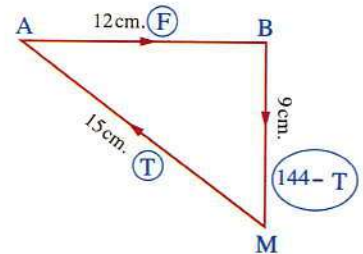
$$\therefore 3 T = 720 - 5 T$$

$$\therefore 8 T = 720$$

$$\therefore T = 90 \text{ dyne}$$

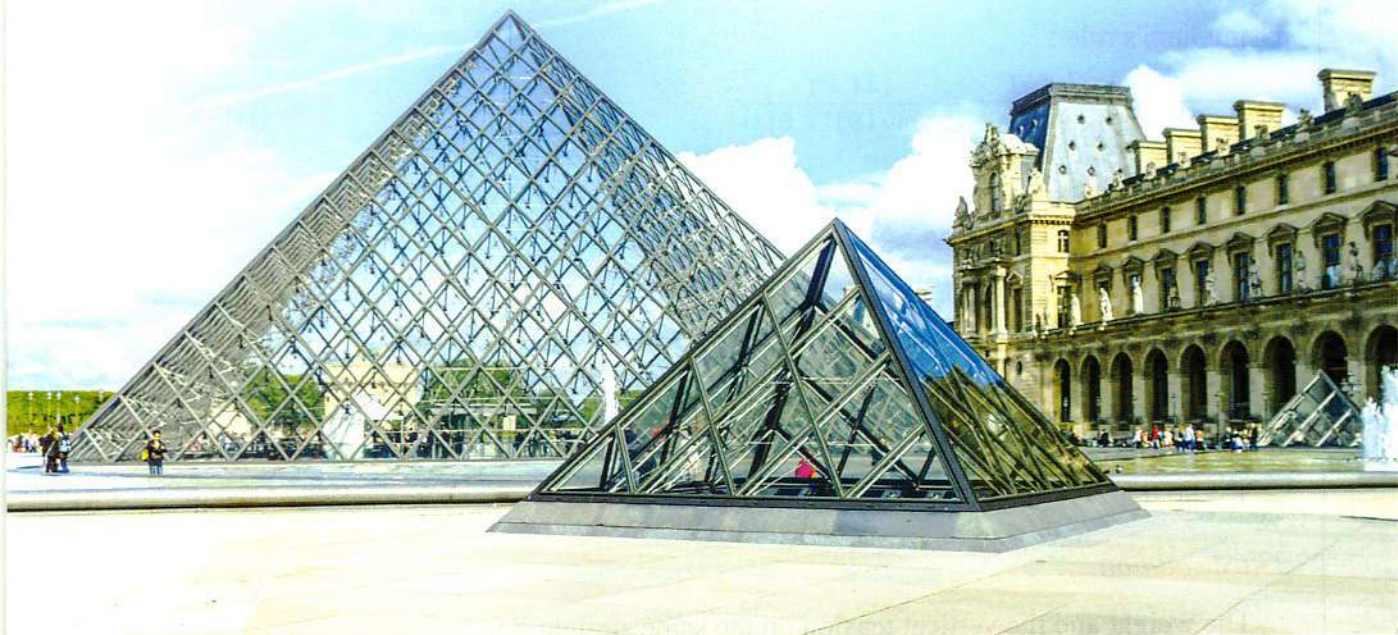
$$\therefore \frac{90}{15} = \frac{F}{12}$$

$$\therefore F = 72 \text{ dyne}$$



Unit Two

Geometry and Measurement



Lesson

1

The straight lines and the planes in the space.

Lesson

2

The pyramid.

Lesson

3

The cone.

Lesson

4

The circle.

Lesson

1

The straight lines and the planes in the space



Geometrical concepts and axioms

1 The straight line :

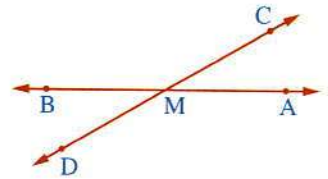
Is an infinite set of points , and we can determine it exactly if we know any two different points on it.

For example :

In the opposite figure :

The two points "A , B" passing through one and only one straight line which is \overleftrightarrow{AB} while the two points "C , D" passing through another straight line which is \overleftrightarrow{CD}

i.e. The straight line is determined by two different points on it.



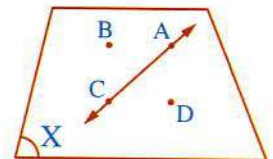
Remark

$$\overline{AB} \subset \overleftrightarrow{AB} \subset \overline{AB}$$

2 The plane :

Is an infinite set of points represents a surface with no ends where any straight line passing through two points on it lies completely on that surface and we denote it by capital letters as X , Y or and we can denote it using at least three non-collinear points on the plane as : ABC

i.e. The plane is determined by three distinct non-collinear points.



Remarks

- (1) The geometrical shapes as the triangle , the square , the circle and ... is an infinite set of points and these shapes are called planed geometrical shapes because each of them is a subset of its plane.
- (2) Where the plane extends into infinity in all directions , we can represent it using a planed geometrical shape on it as a square or a circle or a parallelogram or ...

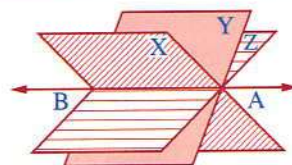
3 The space :

Is an infinite set of points , it contains all the geometric figures and planes , surfaces and solids while the solids as the sphere and the cylinder and the cube , ... etc. are a set of infinite points but we can't contain it in one plane , but we can contain it in the space and the faces of these solids formed from some planed parts as the cube or non-planed as the sphere.



Remarks

- Any point in the space passing through it an infinite number of the straight lines.
- Any point in the space passing through it an infinite number of the planes.
- Any two points in the space passing through them one and only one straight line.
- Any two points in the space passing through them an infinite number of the planes.



Determination of the plane in the space

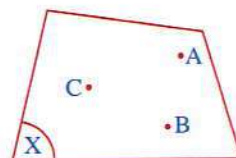
The plane is determined in each of the following cases :

1 Three distinct non-collinear points :

In the opposite figure :

The points A , B , C are non-collinear so that we can determine the plane (X) or ABC from that we can deduce :

There is one and only one plane which passes through three non-collinear points.



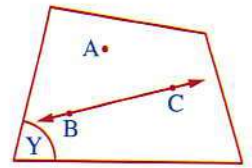
2 A straight line and a point not belonging to it :

In the opposite figure :

$A \notin \overleftrightarrow{BC}$, then the point A and

the straight line \overleftrightarrow{BC}

determine the plane (Y) or ABC



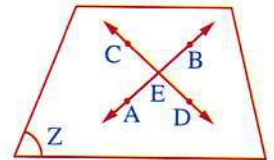
3 Two intersecting straight lines :

In the opposite figure :

$\overleftrightarrow{AB} \cap \overleftrightarrow{CD} = \{E\}$

, then \overleftrightarrow{AB} and \overleftrightarrow{CD}

determine the plane (Z)



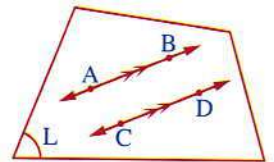
4 Two parallel and non-coincident straight lines :

In the opposite figure :

$\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$, $\overleftrightarrow{AB} \cap \overleftrightarrow{CD} = \emptyset$

, then \overleftrightarrow{AB} and \overleftrightarrow{CD}

determine the plane (L)



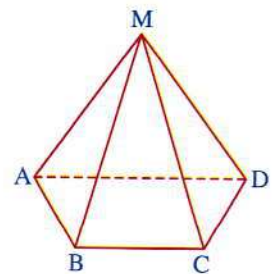
Example 1

In the opposite figure :

If $M \notin$ the plane ABCD

Find :

- (1) Four straight lines passing through the point (A)
- (2) Three planes passing through the point (A)
- (3) The straight lines passing through the two points A and B together.
- (4) Two planes each of them passing through the two points A and B together.
- (5) Four planes passing through the point (M)
- (6) The number of the planes which determine the solid in the figure.



Solution

(1) \overleftrightarrow{AB} , \overleftrightarrow{AC} , \overleftrightarrow{AD} , \overleftrightarrow{AM}

(2) \overleftrightarrow{AB}

(5) MAB, MBC, MCD, MAD

(2) ABCD, ABM, ADM

(4) ABCD, ABM

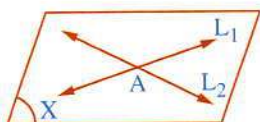
(6) Five planes.

Relative positions of lines and planes in the space

1 The relative positions of two different straight lines in the space :

Intersecting straight lines

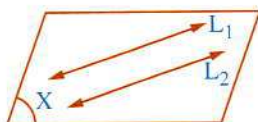
Two straight lines lie on the same plane and have one point in common.



- L_1, L_2 are intersecting.
- $L_1 \cap L_2 = \{A\}$
- They lie in one plane.

Parallel straight lines

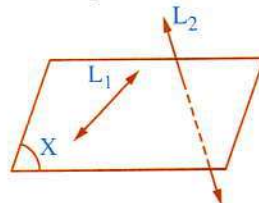
Two straight lines lie on the same plane and doesn't have any common point.



- $L_1 \parallel L_2$
- $L_1 \cap L_2 = \emptyset$
- They lie in one plane.

Skew straight lines

Two straight lines can't be lie in one plane.



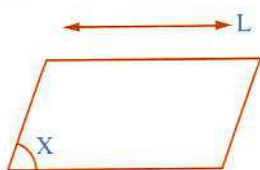
- L_1, L_2 are skew.
- $L_1 \cap L_2 = \emptyset$
- They do not lie in one plane.

Notice that

The two skew straight lines are not parallel and not intersecting because they are not in one plane.

2 The relative positions for the straight line and the plane in the space :

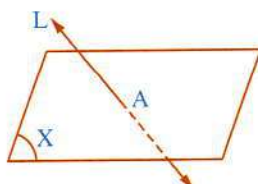
The straight line is parallel to the plane



- The straight line $L \parallel$ the plane X

i.e. $L \cap X = \emptyset$

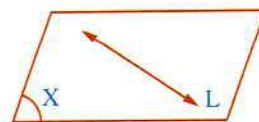
The straight line intersects the plane



- The straight line L intersects the plane X in one point.

i.e. $L \cap X = \{A\}$

The straight line lies completely in the plane



- The straight line L lies completely in the plane X ($L \subset X$)

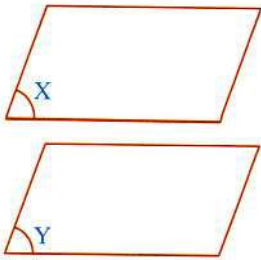
i.e. $L \cap X = L$

Notice that

If a straight line intersects a plane in more than one point, then the straight line lies completely in the plane.

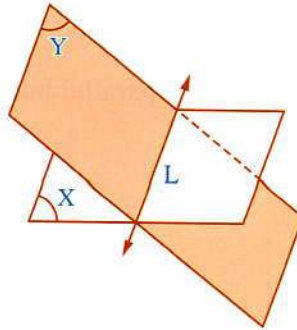
3 The relative positions for two different planes in the space :

Two parallel planes



The plane $X \parallel$ the plane Y
i.e. $X \cap Y = \emptyset$

Two intersecting planes



The two planes intersecting
 at a straight line L
i.e. $X \cap Y = L$

Two coincident planes



The two planes are in
 common in all points
 (Coincide)
i.e. $X \cap Y = X = Y$

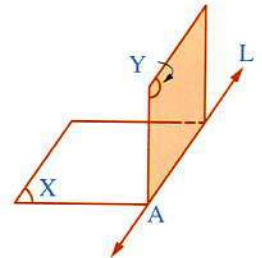
Remarks

- (1) If there are two planes have a common point, then they have in a common straight line passing through this point.

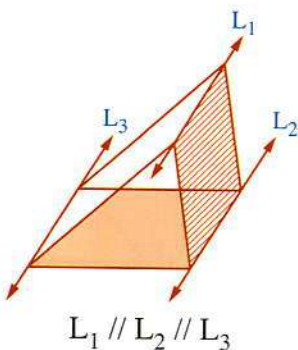
In the opposite figure :

The two planes X and Y have a common point (A) then :
 the two planes X and Y have a common straight line (L)

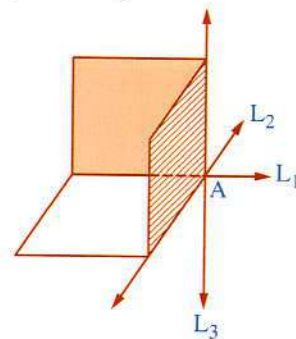
i.e. $X \cap Y = L$ where $A \in L$



- (2) If there are three planes are intersected "each two with each other", then their intersected straight lines will be parallel or intersecting at one point.



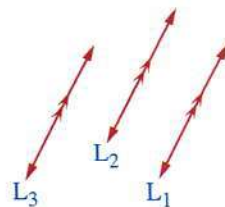
$L_1 \parallel L_2 \parallel L_3$



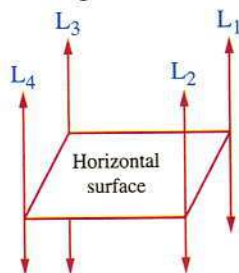
$L_1 \cap L_2 \cap L_3 = \{A\}$

- (3) If two straight lines are parallel to a third in the space, then they all are parallel.

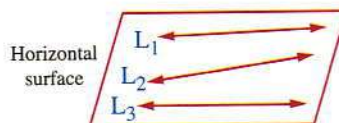
i.e. If $L_1 \parallel L_3$, $L_2 \parallel L_3$, then $L_1 \parallel L_2$



- (4) All vertical straight lines in the space are parallel but not all horizontal straight lines in the space are parallel.

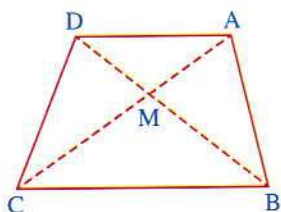


All the vertical straight lines L_1 , L_2 , L_3 and L_4 are parallel.

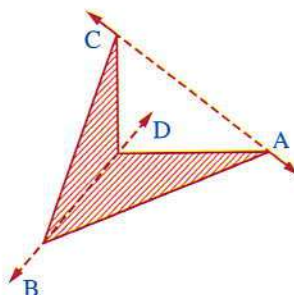


The horizontal straight lines L_1 , L_2 and L_3 are not parallel.

- (5) If the straight lines contain the diagonals of a quadrilateral intersected at a point, then all its sides lie in one plane.



The sides of the quadrilateral ABCD lie in one plane, because $\overleftrightarrow{AC} \cap \overleftrightarrow{BD} = \{M\}$



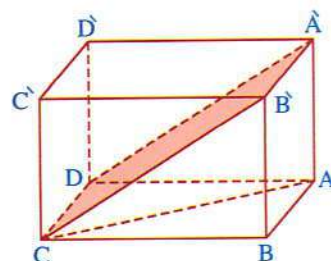
The sides of the quadrilateral ABCD doesn't lie in one plane, because $\overleftrightarrow{AC} \cap \overleftrightarrow{BD} = \emptyset$
(\overleftrightarrow{AC} , \overleftrightarrow{BD} are skew)

Example 2

In the opposite figure :

ABCD $\hat{=}$ B' C' D' is a cuboid, complete the following :

- (1) $\overleftrightarrow{BC} \parallel$ the plane
- (2) \overleftrightarrow{AB} and are skew.
- (3) The plane AB B' A' \parallel the plane



- (4) The plane $AB \hat{B} \hat{A} \cap$ the plane $\hat{A} \hat{B} \hat{C} D = \dots\dots\dots$
 (5) The plane $ABC \cap$ the plane $\hat{A} \hat{B} \hat{C} D = \dots\dots\dots$
 (6) The plane $\hat{A} \hat{A} \hat{D} \hat{D} \cap$ the plane $AB \hat{B} \hat{A} \cap$ the plane $ABCD = \dots\dots\dots$

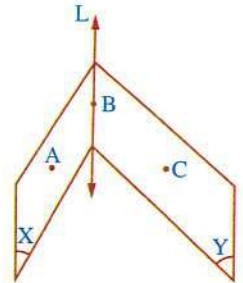
Solution

- | | | |
|--|---|--------------------------|
| (1) $\hat{A} \hat{A} \hat{D} \hat{D}$ or $\hat{A} \hat{B} \hat{C} \hat{D}$ | (2) \overleftrightarrow{DD} or \overleftrightarrow{AD} (Find other answers) | (3) $DC \hat{C} \hat{D}$ |
| (4) \overleftrightarrow{AB} | (5) \overleftrightarrow{DC} | (6) $\{A\}$ |

Example 3

In the opposite figure :

The plane $X \cap$ the plane $Y =$ the straight line L
 $A \in X, C \in Y, B \in L$



Choose the correct answer from those given :

- (1) The plane $ABC \cap$ the plane $X = \dots\dots\dots$
 (2) The plane $ABC \cap$ the plane $Y = \dots\dots\dots$
 (3) The plane $ABC \cap$ the plane $X \cap$ the plane $Y = \dots\dots\dots$

- $(\overleftrightarrow{AB}, \overleftrightarrow{BC}, \overleftrightarrow{CA}, \{B\})$
 $(\overleftrightarrow{AB}, \overleftrightarrow{BC}, \overleftrightarrow{CA}, \{B\})$
 $(\overleftrightarrow{AB}, \overleftrightarrow{BC}, \overleftrightarrow{CA}, \{B\})$

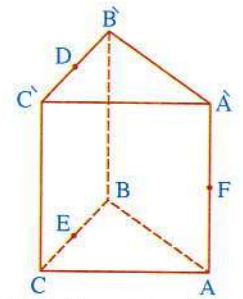
Solution

- (1) \overleftrightarrow{AB} (2) \overleftrightarrow{BC} (3) $\{B\}$

Example 4

In the opposite figure :

If the plane $ABC \parallel$ the plane $\hat{A} \hat{B} \hat{C}$, D and E are the midpoints of \overleftrightarrow{BC} and \overleftrightarrow{BC} respectively, $F \in \overleftrightarrow{AA}$



- 1 Find four planes passing through the point (A)
 2 Choose the correct answer from those given :

- | | |
|--|--|
| (1) A $\dots\dots\dots$ the plane DFE | $(\in, \notin, \subset, \supset)$ |
| (2) \overleftrightarrow{AA} $\dots\dots\dots$ the plane DFE | $(\in, \notin, \subset, \supset)$ |
| (3) $\overleftrightarrow{AA}, \overleftrightarrow{BC}$ are two $\dots\dots\dots$ straight lines. (parallel, skew, intersecting, perpendicular) | $(\overleftrightarrow{AD}, \overleftrightarrow{AE}, \overleftrightarrow{DE}, \overleftrightarrow{FE})$ |
| (4) The plane $DFE \cap$ The plane $\hat{A} \hat{B} \hat{C} = \dots\dots\dots$ | $(\overleftrightarrow{AA}, \overleftrightarrow{AE}, \overleftrightarrow{AD}, \overleftrightarrow{DE})$ |
| (5) The plane $DEF \cap$ The plane $AB \hat{B} \hat{A} = \dots\dots\dots$ | $(\overleftrightarrow{DE}, \overleftrightarrow{AE}, \overleftrightarrow{AD}, \overleftrightarrow{AA})$ |
| (6) The plane $AE\hat{A} \cap$ The plane $\hat{B} \hat{B} \hat{C} \hat{C} = \dots\dots\dots$ | |

Solution

- 1 The planes are : $\hat{A} \hat{A} \hat{B} \hat{B}, \hat{A} \hat{A} \hat{C} \hat{C}, \hat{A} \hat{A} \hat{E}, ABC$
 2 (1) \in (2) \subset (3) skew
 (4) \overleftrightarrow{AD} (5) \overleftrightarrow{AA} (6) \overleftrightarrow{DE}

Lesson

2

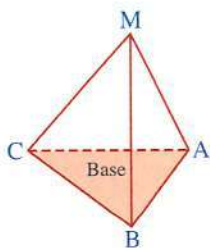
The pyramid



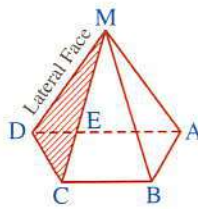
The definition of the pyramid

Is a solid has one base as a polygon and all its other faces are triangles with a common vertex and the pyramid is called a triangular , quadrilateral , pentagonal or according to the number of sides of the polygon of its base.

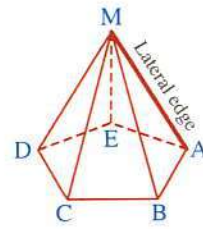
For example :



triangular pyramid,
its base is a triangle



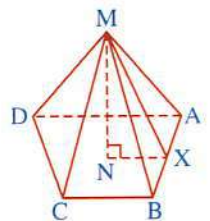
quadrilateral pyramid,
its base is a quadrilateral



pentagonal pyramid,
its base is a pentagon

By using the opposite figure : we can explain some concepts of the pyramid :

- MABCD is a quadrilateral pyramid.
its lateral faces are the triangles MAB, MBC, MCD, MAD, and its base is a polygon ABCD
- **The lateral faces of the pyramid :** are always triangles but the base could be a triangle or a quadrilateral or a pentagon or
- **The vertex of the pyramid :** Is the common point for all lateral faces of the pyramid.
In the figure : The point "M" is the vertex of the pyramid MABCD



- **The lateral edge of the pyramid :** Is a line segment joining between the vertex of the pyramid and any vertex of its base vertices. as $(\overline{MA}, \overline{MB}, \overline{MC}, \overline{MD})$, as in the figure)
- **The height of the pyramid :** Is the distance between the vertex of the pyramid and its base surface.
i.e. Is the length of the perpendicular from the vertex of the pyramid to its base surface.
(MN is the height of the pyramid as in the figure)
- **The slant height of the pyramid :** Is the distance between the vertex of the pyramid and one of its base sides.
i.e. Is the length of the perpendicular line segment from the vertex to one of the base sides of the pyramid.
(\overline{MX} is a slant height of the pyramid $MABCD$ where $\overline{MX} \perp \overline{AB}$)

Remarks

- The perpendicular straight line to a plane is perpendicular to any straight line in that plane, then the perpendicular straight line to the base of the pyramid is perpendicular to any straight line in it.
- The regular polygon is a polygon in which its sides are equal in length and its angles are equal in measure.
- The geometrical centre of any regular polygon is the centre of inscribed circle or the circumcircle for it.
- The geometrical centre of the parallelogram and its special cases is the point of intersection of the diagonals.
- The geometrical centre of the triangle is the point of intersection of its medians.

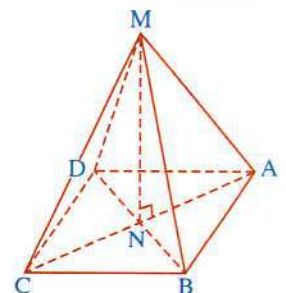
Special cases of the pyramid

1 The right pyramid :

The pyramid is right if the position of perpendicular from the vertex of the pyramid to the base is passing through its geometrical centre.

For example :

- In the pyramid $MABCD$ as in the figure : If N is the geometrical centre of the base $ABCD$ and $\overline{MN} \perp$ the plane of the base $ABCD$, then the pyramid $MABCD$ is called a right pyramid.



2 The regular pyramid :

Is the pyramid in which its base is a regular polygon whose centre is the position of the perpendicular from the vertex of the pyramid to the base.

i.e. A right pyramid of a base as a regular polygon.

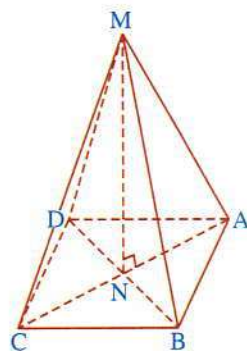
For example :

In the pyramid MABCD as in the figure :

if N is the geometrical centre of the regular base ABCD “square” and $\overline{MN} \perp$ the plane of the base , then the pyramid MABCD is a regular pyramid.

Properties of the regular pyramid :

- (1) Its lateral edges are equal in length.
- (2) Its slant heights are equal in length.
- (3) Its lateral faces are congruent isosceles triangles.



Remarks

- Every regular pyramid is a right pyramid but not vice verse.
- Not necessary that the lateral edges of the right pyramid are equal in length.
- Not necessary that the slant heights of the right pyramid are equal in length.
- The regular triangular pyramid is called a triangular pyramid of regular faces if its all faces are equilateral triangles and any one of them is its base.

Example 1

MABCD is a regular quadrilateral pyramid , the length of its base side is 12 cm. and its height length equals 8 cm. Find the length of its slant height.

Solution

Let X is a midpoint of \overline{AB}

\therefore MABCD is a regular quadrilateral pyramid

$\therefore MA = MB \quad \therefore \overline{MX} \perp \overline{AB}$

$\therefore \overline{MX}$ is the slant height of the pyramid

In $\triangle ABC$: X is a midpoint of \overline{AB} , N is a midpoint of \overline{AC}

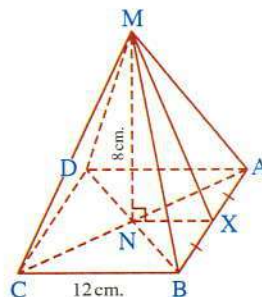
$\therefore XN = \frac{1}{2} BC \quad \therefore XN = 6 \text{ cm.}$

$\therefore \overline{MN} \perp$ The plane ABCD $\therefore \overline{MN} \perp \overline{XN}$

$\therefore \triangle MXN$ is right at N

$\therefore (XM)^2 = (XN)^2 + (MN)^2 \quad \therefore (XM)^2 = 36 + 64 = 100$

$\therefore XM = 10 \text{ cm.}$



Example 2

MABC is a regular triangular pyramid with base $\triangle ABC$, the length of its base length is 6 cm. and the length of its height is 4 cm. Find the length of its edge and its slant height.

Solution

Let X is the midpoint of \overline{AB}

\therefore MABC is a regular triangular pyramid

$\therefore \triangle ABC$ is an equilateral triangle

\therefore X is the midpoint of \overline{AB}

$$\therefore \overline{CX} \perp \overline{AB}$$

$\therefore \triangle BXC$ is a right-angled triangle at X

$$\therefore (XC)^2 = (BC)^2 - (BX)^2 = 36 - 9 = 27$$

$$\therefore XC = \sqrt{27} = 3\sqrt{3} \text{ cm.}$$

\therefore N is the centre of $\triangle ABC$

\therefore N is the point of intersection of the medians of $\triangle ABC$

$$\therefore NX : NC = 1 : 2$$

$$\therefore NX = \sqrt{3} \text{ cm.}, NC = 2\sqrt{3} \text{ cm.}$$

$\therefore \overline{MN} \perp$ The plane ABC

$$\therefore \overline{MN} \perp \overline{XC}$$

$\therefore \triangle MNC$ is right-angled triangle at N

$$\therefore (MC)^2 = (MN)^2 + (NC)^2 = 16 + 12 = 28$$

$$\therefore MC = \sqrt{28} = 2\sqrt{7} \text{ cm.}$$

$$\therefore \text{The length of the edge of the pyramid} = 2\sqrt{7} \text{ cm.}$$

$\therefore \triangle MNC$ is right-angled triangle at N

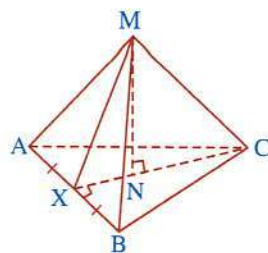
$$\therefore (MX)^2 = (MN)^2 + (NX)^2 = 16 + 3 = 19$$

$$\therefore MX = \sqrt{19} \text{ cm.}$$

\therefore X is a midpoint of \overline{AB}

$$\therefore \overline{MX} \perp \overline{AB}$$

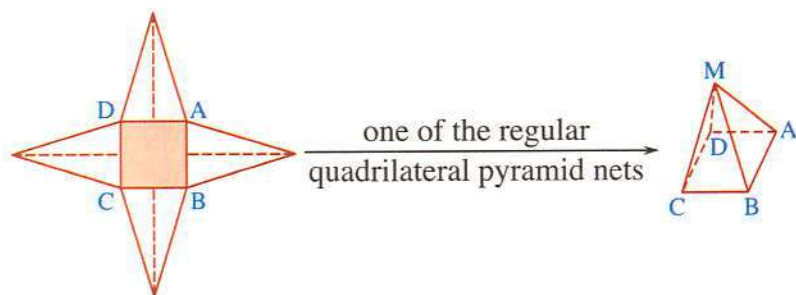
$\therefore \overline{MX}$ is the slant height of the pyramid $= \sqrt{19} \text{ cm.}$



Solids net

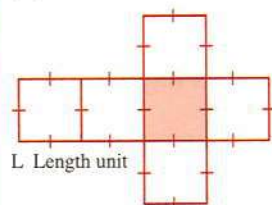
The solids net is used to make the solids by tracking the shape of the solid on the surface of the plane and folding this plane to form the solid.

The pyramid net :

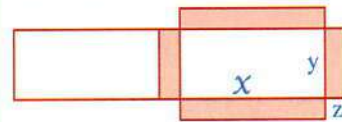


Remember

(1) One of the cube nets



(2) One of the cuboid nets



From the net of the regular quadrilateral pyramid we note that :

- (1) It has 5 faces , four of them are lateral faces and one face to the base.
- (2) It has 8 edges , four of them are lateral edges.
- (3) It has 5 vertices , one of them (M) is called the vertex of the pyramid.

Enrich your knowledge

Euler relation : For any solid in which its base as a polygon , then :

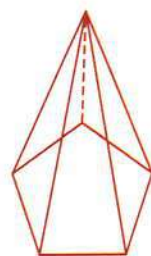
(Number of its faces + number of its vertices = number of its edges + 2)

For example : In the **pentagonal pyramid** : number of its faces = 6 faces ,
number of its vertices = 6 vertices , number of its edges = 10 edges)

i.e. Number of its faces + number of its vertices = $6 + 6 = 12$

, number of its edges + 2 = $10 + 2 = 12$

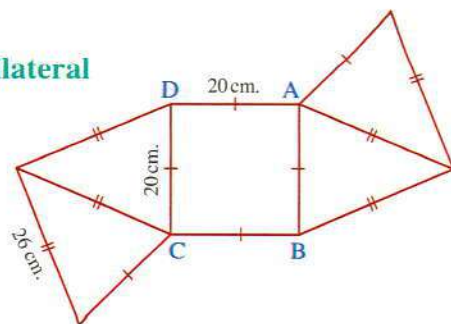
\therefore Number of faces + number of vertices = number of edges + 2



Example 3

The opposite net represents a net of a regular quadrilateral pyramid. Find :

- (1) Height of the pyramid.
- (2) The slant height of the pyramid.



Solution

The net is representing a regular quadrilateral pyramid its base ABCD is a square, its vertex M, and its height \overline{MN} , where N is the point of intersection of the diagonals of the base.

∴ ABCD is a square

∴ The length of its diagonal = its side length $\times \sqrt{2}$

$$\therefore AC = 20 \times \sqrt{2} = 20\sqrt{2} \text{ cm.}$$

$$\therefore AN = 10\sqrt{2} \text{ cm.}$$

∴ MABCD is a right quadrilateral pyramid

∴ $\overline{MN} \perp$ The plane of the base ABCD

$$\therefore \overline{MN} \perp \overline{AN}$$

∴ $\triangle ANM$ in which $m(\angle ANM) = 90^\circ$

$$\therefore (MN)^2 = (AM)^2 - (AN)^2 = (26)^2 - (10\sqrt{2})^2 = 476$$

$$\therefore MN = \sqrt{476} = 2\sqrt{119} \text{ cm.}$$

∴ The height of the pyramid = $2\sqrt{119} \text{ cm.}$

Let X is a midpoint of \overline{AB}

$$\therefore AX = 10 \text{ cm.}$$

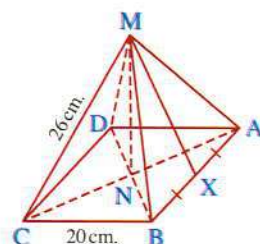
, ∴ $MA = MB$

$$\therefore \overline{MX} \perp \overline{AB}$$

∴ $\triangle AXM$ in which $m(\angle AXM) = 90^\circ$

$$\therefore (MX)^2 = (AM)^2 - (AX)^2 = (26)^2 - (10)^2 = 576 \quad \therefore MX = \sqrt{576} = 24 \text{ cm.}$$

∴ The slant height of the pyramid = 24 cm.



The lateral area of regular pyramid - the total area of pyramid - the volume of pyramid :

- * The lateral area of the pyramid = the sum of areas of the lateral faces
- * The lateral area of the regular pyramid = $\frac{1}{2}$ base perimeter \times slant height
- * The total surface area of the pyramid = lateral area + area of the base
- * The volume of the pyramid = $\frac{1}{3}$ base area \times height

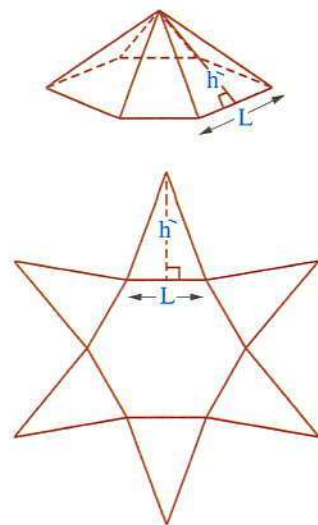
Finding the lateral area of regular pyramid

If the side length of the base in regular pyramid is L and the number of its base sides is n and its lateral height h , then from the net of this pyramid we find it has n congruent faces each one is an isosceles triangle and the area of each triangle $= \frac{1}{2} \times L \times h$
 \therefore The lateral area of the regular pyramid

$$= \frac{1}{2} L \times h \times n ,$$

, \therefore perimeter of the base $= n \times L$

\therefore The lateral area of the regular pyramid
 $= \frac{1}{2} \times \text{base perimeter} \times \text{slant height}$



Finding the volume of the pyramid

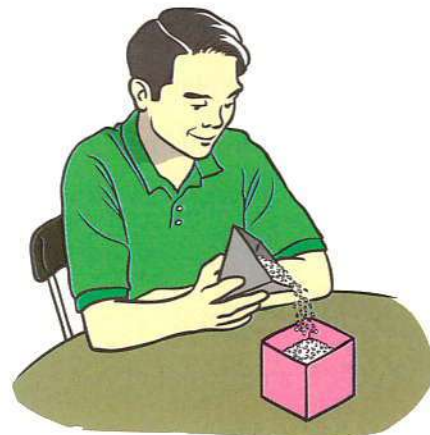
Experiment activity

- * Bring a hollow vessel in the shape of a right prism , and another one in the shape of a right pyramid where be their bases are congruent and they have the same height as in the opposite figure.
- * Fill the pyramid vessel with **grains** of rice or sand then put it in the prism.
- * Repeat this process three times and you will note that : The prism will filled completely with the **grains** and that means :

Volume of the pyramid $= \frac{1}{3}$ volume of the prism has the same base and height.

\therefore The volume of the prism $= \text{the base area} \times \text{height}$

\therefore Volume of the pyramid $= \frac{1}{3} \times \text{base area} \times \text{height}$



Remarks

- (1) In the triangular pyramid of regular faces :
double the square of its edge length $= 3$ times the square of its height.

i.e. $2L^2 = 3h^2$ Where L = edge length , h = the height

- (2) The total surface area to the triangular pyramid of regular faces = $L^2\sqrt{3}$ where L is the length of edge.
- (3) The volume of the triangular pyramid of regular faces = $\frac{\sqrt{2}}{12} L^3$ where L is the edge length.

Example 4

A regular quadrilateral pyramid the length of its base diagonal is $60\sqrt{2}$ cm. and its slant height is 50 cm. Find :

- (1) The height of the pyramid.
- (2) L.S.A and T.S.A of the pyramid.
- (3) Volume of the pyramid.

Solution

* Let MABCD is a regular quadrilateral pyramid , the diagonals of its base intersected at N

, the length of the base side = $\frac{60\sqrt{2}}{\sqrt{2}} = 60$ cm.

, E is the midpoint of \overline{AB}

- (1) \therefore The quadrilateral pyramid is regular.

\therefore Its base as a square shape.

, $\overline{MN} \perp$ the plane ABCD $\therefore \overline{MN} \perp \overline{EN}$

$\therefore \Delta MEN$ is right angled at N ,

\therefore E is the midpoint of \overline{AB} , N is the midpoint of \overline{AC}

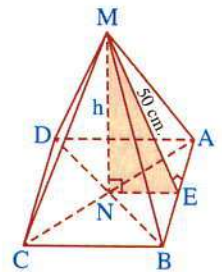
$\therefore EN = \frac{1}{2} BC = 30$ cm. $\therefore h = \sqrt{50^2 - 30^2} = 40$ cm.

- (2) The L.S.A of the pyramid = $\frac{1}{2} \times \text{base perimeter} \times \text{slant height}$
 $= \frac{1}{2} \times (60 \times 4) \times 50 = 6000 \text{ cm}^2$.

\therefore The area of the base = $60 \times 60 = 3600 \text{ cm}^2$.

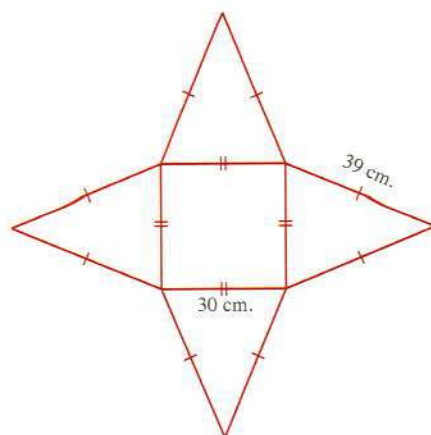
\therefore The T.S.A of the pyramid = L.S.A + the base area = $6000 + 3600 = 9600 \text{ cm}^2$.

- (3) Volume of the pyramid = $\frac{1}{3} \text{ base area} \times \text{height} = \frac{1}{3} \times 3600 \times 40 = 48000 \text{ cm}^3$.



Example 5

Use the opposite net to describe the formed solid , then find its total surface area and its volume.



Solution

The net represents a regular quadrilateral pyramid , its base as a square of side length = 30 cm. and the length of its lateral edge = 39 cm.

, let the pyramid MABCD , N is the point of intersection of the diagonals of the base ,
E is the midpoint of \overline{AB}

, \therefore the lateral face MAB is an isosceles triangle.

$\therefore \overline{ME}$ is slant height.

, $AE = 15$ cm.

$\therefore \triangle AEM$ which is right - angled at E :

$$\therefore \overline{ME} = \sqrt{(MA)^2 - (AE)^2} = \sqrt{(39)^2 - (15)^2} = 36 \text{ cm.}$$

$\therefore \overline{MN} \perp$ the plane ABCD $\therefore \overline{MN} \perp \overline{EN}$

, $\therefore EN = \frac{1}{2} BC = 15$ cm.

\therefore In $\triangle MEN$ which is right - angled at N :

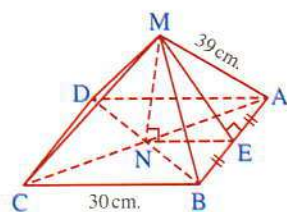
$$MN = \sqrt{(ME)^2 - (EN)^2} = \sqrt{(36)^2 - (15)^2} = 3\sqrt{119} \text{ cm.}$$

$$\begin{aligned} \therefore \text{The L.S.A. of the pyramid} &= \frac{1}{2} \times \text{base perimeter} \times \text{slant height} \\ &= \frac{1}{2} \times (30 \times 4) \times 36 = 2160 \text{ cm}^2. \end{aligned}$$

, area of the base = $30 \times 30 = 900 \text{ cm}^2$.

$$\begin{aligned} \therefore \text{The T.S.A. of the pyramid} &= \text{L.S.A} + \text{the base area} \\ &= 2160 + 900 = 3060 \text{ cm}^2. \end{aligned}$$

$$\begin{aligned} \therefore \text{the volume of the pyramid} &= \frac{1}{3} \text{ base area} \times \text{height} \\ &= \frac{1}{3} \times 900 \times 3\sqrt{119} = 900\sqrt{119} \text{ cm}^3 \end{aligned}$$



Example 6

MABCD is a regular quadrilateral pyramid, its total surface area = 360 cm^2 and its slant height = 13 cm. Find the length of its base edge, then find its volume.

Solution

Let the edge length of the squared base = x cm.

, \therefore the T.S.A of the pyramid = 360 cm^2 .

\therefore The base area + L.S.A = 360

$$\therefore x \times x + \frac{1}{2} \times 4x \times 13 = 360$$

$$\therefore x^2 + 26x - 360 = 0$$

$$\therefore (x + 36)(x - 10) = 0$$

$$\therefore x = -36 \text{ (refused)} \text{ or } x = 10$$

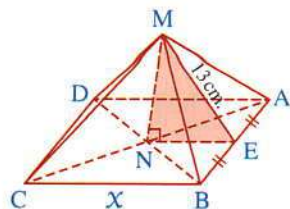
\therefore The edge length of the pyramid base = 10 cm.

$$\therefore EN = \frac{1}{2} BC = 5 \text{ cm.}$$

, $\therefore \triangle MEN$ is right-angled at N

$$\therefore MN = \sqrt{13^2 - 5^2} = 12 \text{ cm.}$$

$$\therefore \text{Volume of the pyramid} = \frac{1}{3} \text{ base area} \times \text{height} = \frac{1}{3} \times (10)^2 \times 12 = 400 \text{ cm}^3$$



Example 7

A regular quadrilateral pyramid of volume 48 cm^3 and the length of its base edge = 6 cm, find its total surface area.

Solution

Let MABCD is a regular quadrilateral pyramid

, N is the intersection point of its base diagonal

, E is the midpoint of \overline{AB}

\therefore Volume of the pyramid = 48 cm^3 .

$$\therefore \frac{1}{3} \times \text{the base area} \times \text{the height} = 48$$

$$\therefore \frac{1}{3} \times 6 \times 6 \times h = 48 ,$$

$$\therefore h = 4 \text{ cm.}$$

\therefore The height of the pyramid = $MN = 4 \text{ cm.}$

$$\therefore EN = \frac{1}{2} BC = 3 \text{ cm.},$$

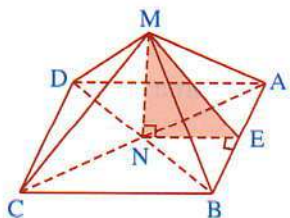
$\therefore \triangle MEN$ is right-angled at N

$$\therefore ME = \sqrt{4^2 + 3^2} = 5 \text{ cm.}$$

\therefore The T.S.A of the pyramid = L.S.A. + the base area

$$= \frac{1}{2} \text{ base perimeter} \times \text{slant height} + \text{base area}$$

$$= \frac{1}{2} \times (4 \times 6) \times 5 + 6 \times 6 = 96 \text{ cm}^2$$



Example 8

MABC is triangular pyramid of regular faces, the length of each edge of its edges equals $8\sqrt{3}$ cm. Find :

- (1) The slant height of the pyramid.
- (2) Height of the pyramid.
- (3) T.S.A of the pyramid.
- (4) Volume of the pyramid.

Solution

\therefore The triangular pyramid is a regular faces.

\therefore Each face is an equilateral triangle.

\therefore The slant height of the pyramid

$$= MD = AD = 8\sqrt{3} \sin 60^\circ = 12 \text{ cm.}$$

\therefore E is the point of intersection of the medians of $\triangle ABC$

$$\therefore AE = \frac{2}{3} AD = \frac{2}{3} \times 12 = 8 \text{ cm.}$$

$\therefore \overline{ME} \perp$ the plane ABC

$\therefore \overline{ME} \perp \overline{AD}$

$\therefore \triangle MAE$ is right-angled at E

$$\therefore ME = \sqrt{(8\sqrt{3})^2 - (8)^2} = 8\sqrt{2} \text{ cm.}$$

\therefore The height of the pyramid $= 8\sqrt{2}$ cm.

\therefore L.S.A. of the pyramid

$$= \frac{1}{2} \text{ base perimeter} \times \text{slant height}$$

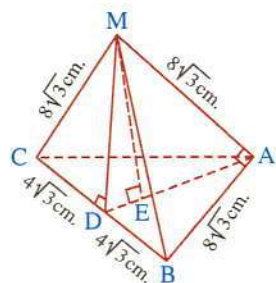
$$= \frac{1}{2} \times (3 \times 8\sqrt{3}) \times 12 = 144\sqrt{3} \text{ cm}^2$$

$$\therefore \text{Area of the base} = \frac{1}{2} \times 8\sqrt{3} \times 8\sqrt{3} \sin 60^\circ = 48\sqrt{3} \text{ cm}^2$$

$$\therefore \text{T.S.A. of the pyramid} = 144\sqrt{3} + 48\sqrt{3} = 192\sqrt{3} \text{ cm}^2$$

$$\therefore \text{volume of the pyramid} = \frac{1}{3} \text{ base area} \times \text{height}$$

$$= \frac{1}{3} \times 48\sqrt{3} \times 8\sqrt{2} = 128\sqrt{6} \text{ cm}^3$$



Notice that

\therefore The triangular pyramid is a regular faces.

$$\therefore 2L^2 = 3h^2$$

$$\therefore 2 \times (8\sqrt{3})^2 = 3h^2$$

$$\therefore h = 8\sqrt{2}$$

$$\therefore \text{Height of the pyramid} = 8\sqrt{2} \text{ cm.}$$

Example 9

A regular hexagonal pyramid in which the sum of areas of its lateral faces is seven times its base area.

Prove that : The volume of the pyramid $= 8r^3$

Where (r) is the radius of the inscribed circle of the base.

Solution

Let the edge length of the base of the pyramid = L cm.

, height of the pyramid = h , and its slant height = \hat{h}

\therefore The sum of areas of its lateral faces = $7 \times$ base area

$\therefore \frac{1}{2} \times \text{base perimeter} \times \text{slant height} = 7 \times \text{base area}$

$\therefore \frac{1}{2} \times 6L \times \hat{h} = 7 \times \frac{1}{2} \times L \times r \times 6$

$\therefore 3L\hat{h} = 21Lr$

$\therefore \hat{h} = 7r$

, $\therefore \overline{MN} \perp$ the plane $ABCDEF$

$\therefore \overline{MN} \perp \overline{NY}$

$\therefore \triangle MNY$ is right-angled at N

$\therefore h = \sqrt{\hat{h}^2 - r^2} = \sqrt{(7r)^2 - r^2} = 4\sqrt{3}r$

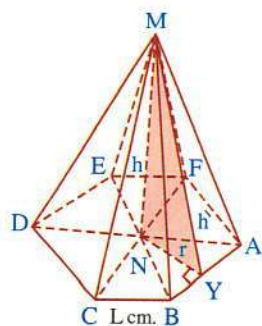
, \therefore volume of the pyramid = $\frac{1}{3} \times \text{base area} \times \text{height}$

$$= \frac{1}{3} \times \frac{1}{2} \times L \times r \times 6 \times 4\sqrt{3}r = 4\sqrt{3}Lr^2$$

, $\therefore r = L \sin 60^\circ$

$$\therefore L = \frac{2}{\sqrt{3}}r$$

\therefore Volume of the pyramid = $4\sqrt{3} \times \frac{2}{\sqrt{3}}r \times r^2 = 8r^3$



Example 10

$\triangle ABC$ is a triangular pyramid its vertex M is at distance $4\sqrt{5}$ cm. from the base ABC where $AB = 7$ cm. , $BC = 8$ cm. , $AC = 9$ cm. Find the volume of the pyramid.

Solution

\therefore The perimeter of $\triangle ABC$

$$= 7 + 8 + 9 = 24 \text{ cm.}$$

\therefore Half the perimeter = 12 cm.

\therefore The area of $\triangle ABC$

$$= \sqrt{12(12-7)(12-8)(12-9)}$$

$$= 12\sqrt{5} \text{ cm}^2.$$

\therefore The volume of the pyramid = $\frac{1}{3} \times \text{base area} \times \text{height}$

$$= \frac{1}{3} \times 12\sqrt{5} \times 4\sqrt{5} = 80 \text{ cm}^3.$$

Remember that

The area of $\triangle ABC = \sqrt{S(S-AB)(S-BC)(S-AC)}$
where : S is half the perimeter of $\triangle ABC$

Lesson

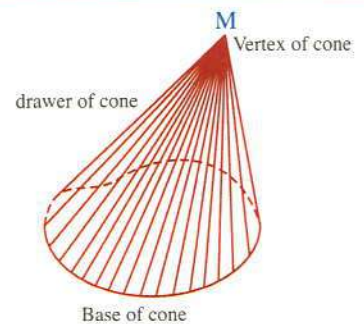
3

The cone



The definition of the cone

Is a solid has only one base as a closed curve and one vertex , and its lateral surface formed from line segments drawn from its vertex to its curved base and each of them is called drawer of the cone.

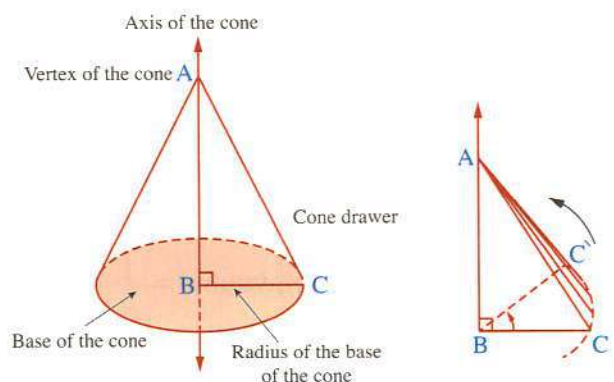


The right circular cone

Is the solid formed from the rotation of a right -angled triangle with complete rotation about one of its right sides as an axis or is the space formed from the folding of a circular sector where their two radii coincide on each other.

In the opposite figure :

$\triangle ABC$ is right angled triangle at B , if it is rotated about the axis \overline{AB} with a full turn , the formed solid is called right circular cone , and the point A is called vertex of the cone , \overline{AC} is the drawer of the cone , \overline{AB} is axis of the cone , surface of circle B is the base of the cone.



Properties of the right circular cone :

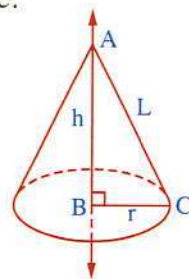
- (1) The axis of the right circular cone is perpendicular to the plane of the base.

i.e. $\overline{AB} \perp$ the plane of circle B

- (2) The height of the right circular cone is the length of the line segment joining between the vertex of the cone and the centre of its base and its length is always less than the length of the drawer of the cone.

If the length of $\overline{AB} = h$ length unit, the length of $\overline{AC} = L$ length unit.

Then the height of the cone $(h) = \sqrt{L^2 - r^2}$, then : $h < L$

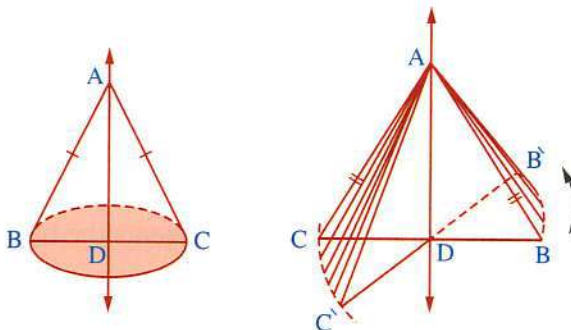


Remark

The right circular cone can be formed by the rotation of an isosceles triangle about its axis of symmetry by a half turn.

In the opposite figure :

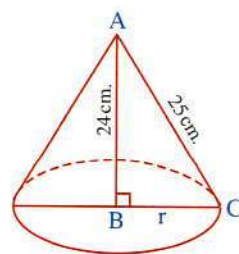
If $\triangle ABC$ is isosceles in which $AB = AC$, \overline{AD} is the axis of symmetry of $\triangle ABC$ and the triangle ABC is rotated about \overline{AD} by a half turn, then the formed solid is a right circular cone its base is the circle D, and its drawer is \overline{AB} or \overline{AC} , its height is \overline{AD} and its vertex is the point A



Example 1

A right circular cone, the length of its drawer is 25 cm. and its height is 24 cm.

Find the perimeter and the area of the base of the cone. $(\pi = \frac{22}{7})$



Solution

$$\therefore \overline{AB} \perp \text{the plane of circle B}$$

$$\therefore \overline{AB} \perp \overline{BC}$$

$$\therefore m(\angle ABC) = 90^\circ$$

$$\therefore (BC)^2 = (AC)^2 - (AB)^2 = (25)^2 - (24)^2 = 49$$

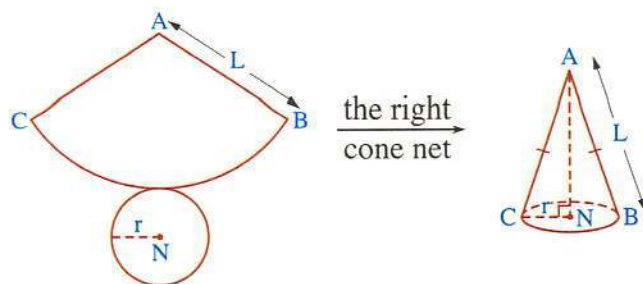
$$\therefore BC = 7 \text{ cm.}$$

$$\therefore r \text{ (the radius of the base)} = 7 \text{ cm.}$$

$$\therefore \text{The perimeter of the base} = 2\pi r = 2 \times \frac{22}{7} \times 7 = 44 \text{ cm.}$$

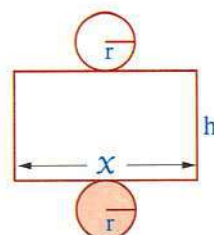
$$\therefore \text{the area of the base} = \pi r^2 = \frac{22}{7} \times 49 = 154 \text{ cm}^2$$

The right cone net :



Remember

One of the right circular cylinder nets



From the net of the right cone we note that :

- (1) $AB = AC = L$, where L is the length of drawer of the cone.
- (2) The circular sector ABC represents the lateral surface of the cone and the length of $\widehat{BC} =$ perimeter of the circle $N = 2 \pi r$
- (3) Surface of the circle N represents the base of the cone.

Remember that

The circular sector is a part of the surface of a circle bounded by two radii and an arc of the circle.

* Area of the circular sector $= \frac{1}{2} L r$

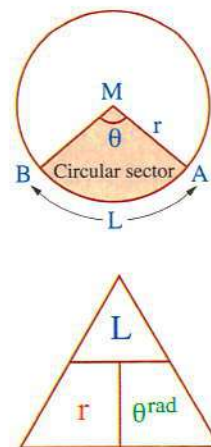
(where L is the length of the arc of the sector)

* Area of the circular sector $= \frac{1}{2} \theta^{\text{rad}} r^2$ (where θ^{rad} is the radian measure of the sector angle)

* Area of the circular sector $= \frac{\mathcal{X}^{\circ}}{360^{\circ}} \times \pi r^2 = \frac{\mathcal{X}^{\circ}}{360^{\circ}} \times \text{area of the circle}$

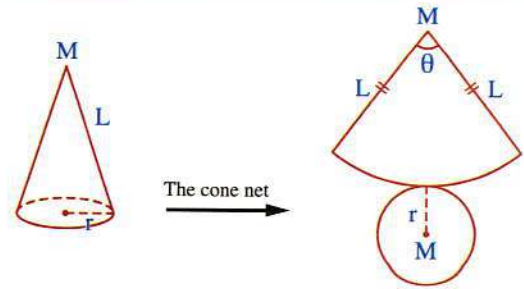
(where \mathcal{X}° is degree measure of the sector angle)

* Perimeter of the sector $= 2 r + L$ length unit.

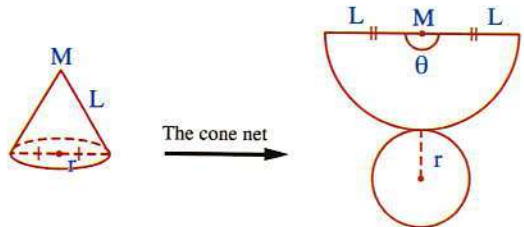


Important remarks

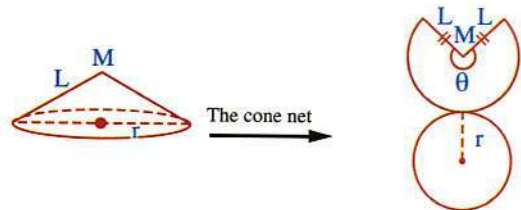
- (1) If $L > 2r$, then the cone net as shown
 $, 0^\circ < \theta < 180^\circ$



- (2) If $L = 2r$, then the cone net as shown
 $, \theta = 180^\circ$



- (3) If $L < 2r$, then the cone net as shown
 $, 180^\circ < \theta < 360^\circ$

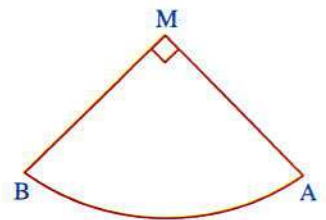


Example 2

In the opposite figure :

A piece of paper as a circular sector , the area of its surface = $25\pi \text{ cm}^2$.

and the measure of its central angle equals 90° folded to touch \overline{MA} and \overline{MB} and formed a cone. Find the height of the cone to nearest tenth.



Solution

$$\therefore \text{Area of the sector} = \frac{1}{2} \theta^{\text{rad}} r^2$$

$$\therefore \frac{1}{2} \theta^{\text{rad}} r^2 = 25\pi$$

$$\therefore \frac{1}{2} \times \frac{\pi}{2} \times r^2 = 25\pi$$

$$\therefore r = 10 \text{ cm.}$$

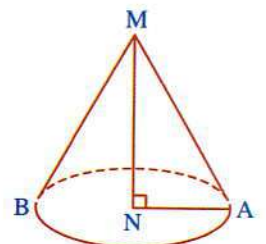
$$\therefore \text{area of the sector} = \frac{1}{2} L r$$

$$\therefore \frac{1}{2} \times L \times 10 = 25\pi$$

$$\therefore r^2 = 100$$

$$\therefore MA = 10 \text{ cm.}$$

$$\therefore L = 5\pi \text{ cm.}$$



∴ The length of $\widehat{AB} = 5\pi$ cm.

∴ The circumference of the circle N = 5π cm.

$$\therefore 2\pi r_N = 5\pi$$

$$\therefore r_N = 2.5 \text{ cm.}$$

∴ $\triangle ANM$ in which $m(\angle ANM) = 90^\circ$, $NA = 2.5$ cm.

, $MA = 10$ cm.

$$\therefore MN = \sqrt{(MA)^2 - (NA)^2} = \sqrt{(10)^2 - (2.5)^2} \approx 9.7 \text{ cm.}$$

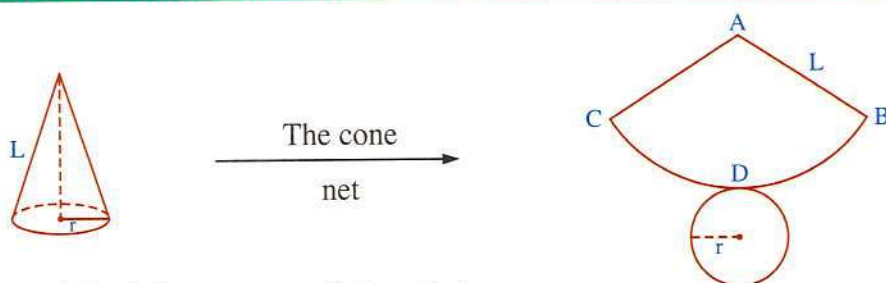
∴ Height of the cone = 9.7 cm.

The lateral area - total area - volume of a right cone :

If (r) is the radius of the cone base , (L) is the cone drawer , (h) is the height , then :

- The lateral surface area (L.S.A.) of the right cone = $\pi L r$
- The total surface area (T.S.A.) of the right cone = $\pi r (L + r)$
- Volume of the right cone = $\frac{1}{3} \pi r^2 h$

Finding the lateral surface area and total surface area of the right cone



From the net of the right cone , we deduce that :

The lateral surface area of the right cone = the area of sector ABC

$$= \frac{1}{2} \times \text{length of } \widehat{BC} \times AB$$

$$= \frac{1}{2} \times \text{perimeter of the cone base} \times AB$$

$$= \frac{1}{2} \times 2\pi r \times L$$

$$= \pi L r$$

$$\therefore \text{The lateral surface area of the right cone} = \pi L r$$

, ∴ the total surface area of the right cone = the lateral surface area + the base area

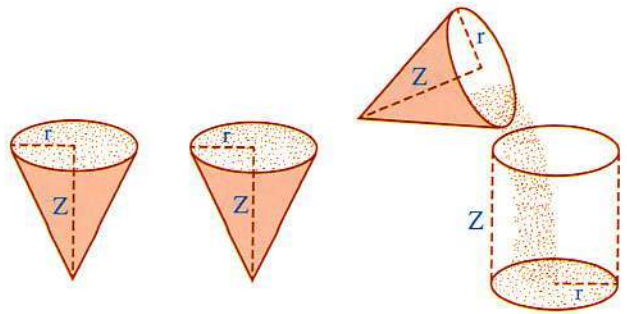
$$= \pi L r + \pi r^2$$

$$\therefore \text{The total surface area of the right cone} = \pi r (L + r)$$

Finding the volume of the right cone

Experiment activity

- * Bring a hollow vessel as a right circular cylinder and another one as a right circular cone where their bases are congruent and they have the same height as in the opposite figure.



- * Fill the cone vessel with grains of rice or sand then empties it in the cylinder vessel.
- * Repeat this process three times and you will note that : the cylinder will filled completely with the grains and that means :

The volume of the cone = $\frac{1}{3}$ volume of the cylinder has the same base and height

, \therefore the volume of the cylinder = base area \times height

$$\begin{aligned}\therefore \text{The volume of the right cone} &= \frac{1}{3} \text{ base area} \times \text{height} \\ &= \frac{1}{3} \pi r^2 h\end{aligned}$$

Example 3

A right circular cone , the length of its base diameter is 12 cm. and its height is 8 cm. , find :

(1) The L.S.A.

(2) The T.S.A.

(3) The volume.

Solution

$\therefore \overline{AM} \perp$ the circle plane

$\therefore \overline{AM} \perp \overline{MB}$

$\therefore \triangle MAB$ is right-angled at M

, $\therefore r = \frac{12}{2} = 6$ cm.

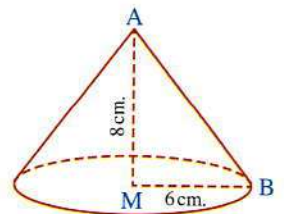
$\therefore AB = \sqrt{(8)^2 + (6)^2} = 10$ cm. $\therefore L = 10$ cm.

\therefore The lateral surface area (L.S.A.) = $\pi r L = \pi \times 6 \times 10 = 60 \pi$ cm²

, area of the base = $\pi r^2 = 36 \pi$ cm²

\therefore The total surface area (T.S.A) = $60 \pi + 36 \pi = 96 \pi$ cm²

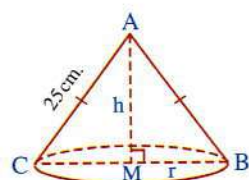
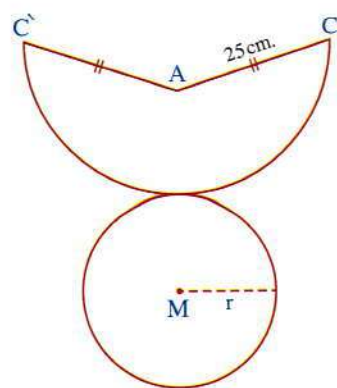
, volume of the cone = $\frac{1}{3} \pi r^2 h = \frac{1}{3} \times \pi \times 36 \times 8 = 96 \pi$ cm³



Example 4

Use the opposite net to describe the formed solid and if the length of the arc $\widehat{CC} = 30\pi$ cm.

Find the volume of this solid and its total surface area.



Solution

The net represents a right cone

, \therefore the length of $\widehat{CC} = 30\pi$

$$\therefore 2\pi r = 30\pi$$

$$\therefore r = 15 \text{ cm.}$$

, $\therefore \triangle ABM$ is right-angled at M

$$\therefore h = \sqrt{25^2 - 15^2} = 20 \text{ cm.}$$

\therefore The volume = $\frac{1}{3}$ base area \times height

$$= \frac{1}{3} \times \pi \times (15)^2 \times 20 = 1500\pi \text{ cm}^3$$

, the total surface area = $\pi r (L + r) = \pi \times 15 \times (15 + 25)$

$$= 600\pi \text{ cm}^2$$

Example 5

A flask in the shape of a cone of capacity 6.16 litres. and height 30 cm.

Find the length of the radius of its base. $\left(\pi \approx \frac{22}{7}\right)$

Solution

\therefore The capacity of the flask = 6.16 litres.

\therefore Volume of the right cone = $6.16 \times 1000 \text{ cm}^3$

$$\therefore \frac{1}{3} \times \frac{22}{7} \times r^2 \times 30 = 6160$$

$$\therefore r = 14 \text{ cm.}$$

Remember that

$$\begin{aligned} 1 \text{ litre} &= 1000 \text{ millilitre} \\ &= 1000 \text{ cm}^3 = 1 \text{ dm}^3 \end{aligned}$$

Example 6

A pure gold alloy as a right cone of radius length 3 cm. and lateral surface area = $15\pi \text{ cm}^2$. Find the gold density if the mass of the alloy = 727 gm. " $\pi = 3.14$ "

Solution

\therefore The L.S.A. of the cone = 15π

$$\therefore \pi L r = 15\pi$$

$$\therefore \pi \times L \times 3 = 15\pi$$

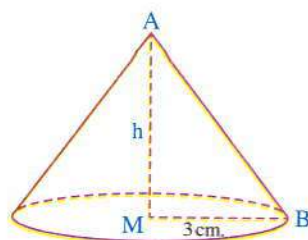
$$\therefore L = 5 \text{ cm.}$$

$\therefore \triangle ABM$ is right-angled at M

$$\therefore h = AM = \sqrt{5^2 - 3^2} = 4 \text{ cm.}$$

$$\begin{aligned} \therefore \text{Volume of the cone} &= \frac{1}{3} \pi r^2 h \\ &= \frac{1}{3} \times \pi \times 3^2 \times 4 = 12\pi = 37.68 \text{ cm}^3 \end{aligned}$$

$$\therefore \text{The density} = \frac{\text{mass}}{\text{volume}} = \frac{727}{37.68} \approx 19.3 \text{ gm./cm}^3$$

**Remember that**

$$\text{The density} = \frac{\text{mass}}{\text{volume}}$$

Example 7

A regular octagonal pyramid of silver, its base side length is 6 cm. and its height 30 cm., melted and convert into a circular right cone whose length of its base radius is 9 cm. if 10% of the silver was missed through the melting process. Find the height of the cone to nearest one decimal place.

Solution

$$\begin{aligned} \therefore \text{The area of the regular octagon} &= \frac{8}{4} \times^2 \cot \frac{\pi}{8} \\ &= 2 \times (6)^2 \cot 22^\circ 30' \approx 173.82 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \therefore \text{Volume of the pyramid} &= \frac{1}{3} \text{ base area} \times \text{height} \\ &= \frac{1}{3} \times 173.82 \times 30 = 1738.2 \text{ cm}^3 \end{aligned}$$

$$\begin{aligned} \therefore \text{Volume of the silver in the cone} \\ &= \frac{90}{100} \times 1738.2 \approx 1564.4 \text{ cm}^3 \end{aligned}$$

$$\therefore \frac{1}{3} \pi \times (9)^2 \times h \approx 1564.4$$

$$\therefore h \approx 18.4 \text{ cm.}$$

Remember that

Area of the regular polygon whose number of sides = n , and the length of its side \times equals $\frac{n}{4} \times^2 \cot \frac{\pi}{n}$

Example 8

$\triangle ABC$ is right-angled at A, $AB = 15$ m., $AC = 20$ m., if the triangle is rotated a complete rotation around \overline{BC} , describe the formed solid, then find the cost of painting this solid with a material resistant to erosion, if the cost of one square metre = 10 pounds and find the volume of this solid. " $\pi = \frac{22}{7}$ "

Solution

The solid will be as a two right cones with the same base.

From the figure :

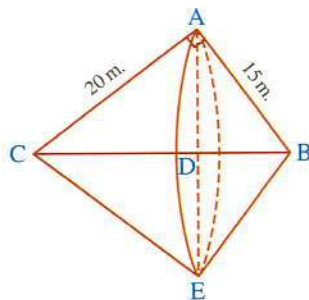
$\triangle ABC$ is right angled at A, $\overline{AD} \perp \overline{BC}$

$$\therefore BC = \sqrt{(15)^2 + (20)^2} = 25 \text{ metres.}$$

$$\therefore AD = \frac{15 \times 20}{25} = 12 \text{ metres.}$$

$$\therefore BD = \sqrt{(15)^2 - (12)^2} = 9 \text{ metres.}$$

$$\therefore CD = 25 - 9 = 16 \text{ metres.}$$



According to the first cone whose vertex is B :

$$L = 15 \text{ m.}, r = 12 \text{ m.}, h = 9 \text{ m.}$$

$$\therefore \text{The L.S.A.} = \pi L r = \pi \times 15 \times 12 = 180 \pi \text{ m}^2$$

$$\therefore \text{the volume} = \frac{1}{3} \pi r^2 h = \frac{1}{3} \pi \times (12)^2 \times 9 = 432 \pi \text{ m}^3$$

According to the second cone whose vertex is C :

$$L = 20 \text{ m.}, r = 12 \text{ m.}, h = 16 \text{ m.}$$

$$\therefore \text{The L.S.A.} = \pi L r = \pi \times 20 \times 12 = 240 \pi \text{ m}^2$$

$$\therefore \text{the volume} = \frac{1}{3} \pi r^2 h = \frac{1}{3} \pi \times (12)^2 \times 16 = 768 \pi \text{ m}^3$$

$$\begin{aligned} \therefore \text{The total painting area} &= \text{the sum of the lateral surface areas of the two cones} \\ &= 180 \pi + 240 \pi = 420 \pi = 420 \times \frac{22}{7} = 1320 \text{ m}^2 \end{aligned}$$

$$\therefore \text{The cost of the painting} = 1320 \times 10 = 13200 \text{ L.E.}$$

$$\begin{aligned} \therefore \text{volume of the solid} &= \text{sum of volumes of the two cones} = 432 \pi + 768 \pi = 1200 \pi \\ &= 1200 \times \frac{22}{7} = 3771 \frac{3}{7} \text{ m}^3. \end{aligned}$$

Lesson

4

The circle

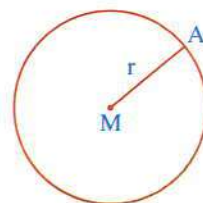


Definition of the circle

It is the set of points of the plane which are at a constant distance from a fixed point in the same plane.

- The fixed point is called "the centre of the circle". (M)
- The constant distance is called "the radius length of the circle". (r)
- The circle is usually denoted by (C)

, where $C = \{A : MA = r, r > 0\}$

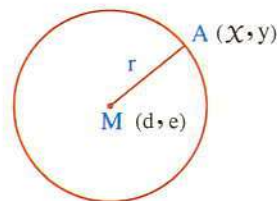


First Equation of the circle (In terms of its centre coordinates and radius length)

If $A = (X, y)$ is a point on the circle whose centre $M (d, e)$, and the length of its radius $= r$, in the perpendicular cartesian coordinates using "the distance between two points" rule we find :

$$\sqrt{(X-d)^2 + (y-e)^2} = r$$

i.e. $(X-d)^2 + (y-e)^2 = r^2$ "Equation of the circle"



Remarks

- (1) If the centre of the circle is the origin point $(0, 0)$, then the equation of the circle is :

$$x^2 + y^2 = r^2$$

- (2) The position of the point (x_1, y_1) in respect to the circle

$$C : (x - d)^2 + (y - e)^2 = r^2$$

* If $(x_1 - d)^2 + (y_1 - e)^2 = r^2$, then the point lies on the circle.

* If $(x_1 - d)^2 + (y_1 - e)^2 > r^2$, then the point lies outside the circle.

* If $(x_1 - d)^2 + (y_1 - e)^2 < r^2$, then the point lies inside the circle.

- (3) Two circles are congruent if the lengths of their radii are equal.

For example : If the circle equation of C_1 is : $x^2 + y^2 = 49$

, the circle equation of C_2 is $(x - 3)^2 + (y - 4)^2 = 49$

, then $r_1 = r_2 = \sqrt{49} = 7$ length unit, then the two circles are congruent, and the circle C_2 is the image of the circle C_1 by translation $(3, 4)$

Where the image of point (x, y) by translation (a, b) is : $(x + a, y + b)$

Second The general form of the circle equation

The general form of the circle equation is :

$$x^2 + y^2 + 2Lx + 2Ky + C = 0$$

Where the centre $(M) = (-L, -K) = (-\frac{1}{2} \text{ coefficient of } x, -\frac{1}{2} \text{ coefficient of } y)$

$$, r = \sqrt{L^2 + K^2 - C}, L^2 + K^2 - C > 0$$

For example : The circle whose equation is : $x^2 + y^2 + 8x - 4y - 16 = 0$

its centre $= (-\frac{1}{2} \text{ coefficient of } x, -\frac{1}{2} \text{ coefficient of } y) = (-4, 2)$

$$, r = \sqrt{L^2 + K^2 - C} = \sqrt{16 + 4 - (-16)} = 6 \text{ length unit.}$$

• We can deduce the general form of the circle equation as follows :

We know that : The circle whose centre (d , e) , the length of its radius = r is :

$$(X - d)^2 + (y - e)^2 = r^2$$

$$\text{i.e. } X^2 + y^2 - 2dX - 2ey + d^2 + e^2 - r^2 = 0$$

$$\text{Let } M(D , K) = (-L , -K)$$

$$\therefore X^2 + y^2 + 2LX + 2Ky + L^2 + K^2 - r^2 = 0$$

, $\therefore L , K$ and r are constants.

$$\therefore L^2 + K^2 - r^2 = C \text{ (constant)}$$

\therefore The general form of the circle equation is :

$$X^2 + y^2 + 2LX + 2Ky + C = 0$$

Remarks

(1) The general form of the circle equation $X^2 + y^2 + 2LX + 2Ky + C = 0$ is :

* An equation of 2nd degree in X , y

* Free of the term Xy *i.e.* Coefficient of $Xy = \text{zero}$

* Coefficient of $X^2 = \text{coefficient of } y^2 = 1$

(2) To be the equation of the 2nd degree in X , y represents a circle , it must satisfy the three conditions in the previous remark and $L^2 + K^2 - C > 0$

(3) To identify the centre or the radius length of a circle using the general form must be the coefficient of $X^2 = \text{the coefficient of } y^2 = 1$, so we need to divide by this coefficient if is not equals 1

Special cases

1 Equation of the circle passing through the origin point :

$$X^2 + y^2 + 2LX + 2Ky = 0 \quad \text{The equation is free of the absolute term i.e. (C = 0)}$$

2 Equation of the circle whose centre lies on X-axis :

$$X^2 + y^2 + 2LX + C = 0 \quad \text{The equation is free of the term containing y i.e. (K = 0)}$$

3 Equation of the circle whose centre lies on y-axis :

$$X^2 + y^2 + 2Ky + C = 0 \quad \text{The equation is free of the term containing X i.e. (L = 0)}$$

4 Equation of the circle touching X-axis :

If the circle whose centre $(-L, -K)$

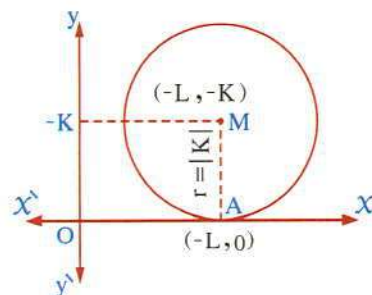
touches X-axis then :

the point of tangency A is : $(-L, 0)$ and $r = |K|$

$$\therefore C = L^2 + K^2 - r^2 = L^2 + K^2 - K^2 = L^2$$

Then the equation of the circle becomes :

$$x^2 + y^2 + 2Lx + 2Ky + L^2 = 0$$



5 Equation of the circle touching y-axis :

If the circle whose centre $(-L, -K)$ touches y-axis

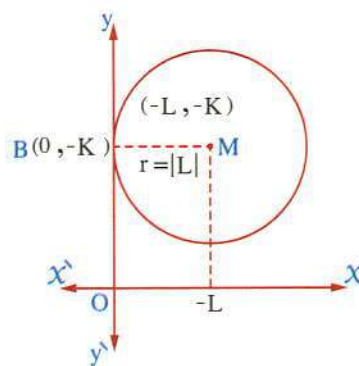
, then the point of tangency B is

$(0, -K)$ and $r = |L|$

$$\therefore C = L^2 + K^2 - r^2 = L^2 + K^2 - L^2 = K^2$$

, then the equation of the circle becomes :

$$x^2 + y^2 + 2Lx + 2Ky + K^2 = 0$$



6 Equation of the circle touching the two coordinates :

If the circle whose centre is $(-L, -K)$

touches the two coordinates axes then $r = |L| = |K|$

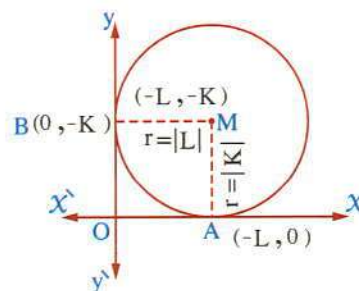
$$\therefore C = L^2 + K^2 - r^2 = r^2 + r^2 - r^2 = r^2$$

$$\therefore C = L^2 = K^2 = r^2$$

and the equation of the circle becomes :

$$x^2 + y^2 + 2Lx + 2Ky + C = 0$$

$$\text{where } |L| = |K| = r, C = L^2 = K^2 = r^2$$



Remember that :

(1) The position of a straight line with respect to a circle (D) whose centre (M)

and let $\overline{MC} \perp L$ and intersects it at C

* If $MC < r$, then L is a secant to the circle at two points.

* If $MC = r$, then L is a tangent to the circle.

* If $MC > r$, then L is outside the circle and doesn't intersect it at any point.

- (2) If M, N are two circles of radii r_1, r_2 respectively (where $r_1 > r_2$)

If the two circles M and N	Then
(1) Distant	$MN > r_1 + r_2$
(2) Touching externally	$MN = r_1 + r_2$
(3) Intersecting	$r_1 - r_2 < MN < r_1 + r_2$
(4) Touching internally	$MN = r_1 - r_2$
(5) One inside the other	$MN < r_1 - r_2$
(6) Concentric	$MN = \text{zero}$

- (3) The tangent to a circle is perpendicular to the radius drawn from the point of tangency.
- (4) The two tangents which are drawn from the two endpoints of a diameter of a circle are parallel.
- (5) The two tangent-segments drawn to a circle from a point outside it are equal in length.
- (6) If $A = (x_1, y_1)$, $B = (x_2, y_2)$ then the midpoint of $\overline{AB} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$
- (7) The equation of the straight line passing through (x_1, y_1)

and its slope (m) is : $\frac{y - y_1}{x - x_1} = m$

- (8) The length of the perpendicular from the point (x_1, y_1) on the straight line whose equation : $ax + by + c = 0$ equals $\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$

Example 1

Find the general form of the equation of the circle whose centre is $(-2, 3)$ and its radius length is 5 length units.

Solution

The equation of the circle is : $(x + 2)^2 + (y - 3)^2 = (5)^2$

$$\therefore x^2 + y^2 + 4x - 6y - 12 = 0 \quad \text{"After simplify"}$$

Another solution :

$$\therefore L = 2, \quad K = -3, \quad C = L^2 + K^2 - r^2 = (2)^2 + (-3)^2 - (5)^2 = -12$$

$$\therefore \text{The general form of the equation of the circle is } x^2 + y^2 + 4x - 6y - 12 = 0$$

"The same form we obtained before"

Example 2

Find the equation of the circle whose centre is the origin point and its diameter length = $6\sqrt{2}$ length unit, then prove that the circle passing through the point $(\sqrt{2}, -4)$

Solution

The equation of the circle is : $x^2 + y^2 = (3\sqrt{2})^2$

, then $x^2 + y^2 = 18$

by substitute by the point $(\sqrt{2}, -4)$

$\therefore \text{L.H.S.} = (\sqrt{2})^2 + (-4)^2 = 18 = \text{R.H.S.}$

\therefore The point $(\sqrt{2}, -4) \in$ the circle.

Example 3

Find the equation of the circle whose centre $M = (3, -2)$ and passing through the point $A = (-1, 1)$

Solution

$r = MA = \sqrt{(3+1)^2 + (-2-1)^2} = 5$ length unit.

\therefore The equation of the circle is : $(x-3)^2 + (y+2)^2 = 25$

Example 4

Find the equation of the circle whose diameter \overline{AB} where $A = (4, -1)$, $B = (-2, 1)$

Solution

\therefore The centre of the circle M is the midpoint of \overline{AB}

$\therefore M = \left(\frac{4-2}{2}, \frac{-1+1}{2} \right) = (1, 0)$

, $\therefore r = MA = \sqrt{(4-1)^2 + (-1-0)^2} = \sqrt{10}$ length unit.

\therefore The equation of the circle is : $(x-1)^2 + (y-0)^2 = (\sqrt{10})^2$

, then : $(x-1)^2 + y^2 = 10$

Example 5

Find the centre and the length of the radius for each of the following circles :

(1) $x^2 + y^2 - 2x + 4y - 4 = 0$

(2) $x^2 + y^2 - 4y - 9 = 0$

(3) $7x^2 + 7y^2 + 42x - 14y + 28 = 0$

Solution

$$(1) \because x^2 + y^2 - 2x + 4y - 4 = 0 \quad \therefore L = -1, K = 2, C = -4$$

$$\therefore \text{The centre} = (-L, -K) = (1, -2)$$

$$, r = \sqrt{L^2 + K^2 - C} = \sqrt{(-1)^2 + (2)^2 - (-4)} = 3 \text{ length unit.}$$

$$(2) \because x^2 + y^2 - 4y - 9 = 0$$

$$\therefore L = \text{zero}, K = -2, C = -9$$

$$\therefore \text{The centre} = (-L, -K) = (0, 2)$$

$$, r = \sqrt{L^2 + K^2 - C} = \sqrt{(0)^2 + (-2)^2 - (-9)} = \sqrt{13} \text{ length unit.}$$

Notice that

$L = 0$ because the coefficient of $x = 0$

$$(3) \text{ By dividing by 7 to make the coefficient of } x^2 = \text{the coefficient of } y^2 = 1$$

$$\therefore \text{The equation will be : } x^2 + y^2 + 6x - 2y + 4 = 0$$

$$\therefore L = 3, K = -1, C = 4$$

$$\therefore \text{The centre} = (-L, -K) = (-3, 1)$$

$$, r = \sqrt{L^2 + K^2 - C} = \sqrt{(3)^2 + (-1)^2 - 4} = \sqrt{6} \text{ length unit.}$$

Example 6

Find the equation of the circle whose centre $(3, -4)$ and touches x -axis

Solution

$$\because L = -3, K = 4$$

\therefore the circle touches x -axis

$$\therefore r = |K|, C = L^2$$

$$\therefore r = 4 \text{ length unit}, C = 9$$

"We can find C using the relation : $C = L^2 + K^2 - r^2$ "

$$\therefore \text{Equation of the circle is : } x^2 + y^2 - 6x + 8y + 9 = 0$$

Example 7

Find the equation of the circle whose the length of its radius is 5 units and touches y -axis at the point $(0, 3)$

Solution

$$\because \text{The circle touches } y\text{-axis at the point } (0, 3)$$

$$\therefore \text{The centre} = (-L, 3), r = |L| \text{ length unit.} \quad \text{i.e. } |L| = 5$$

$$\therefore L = \pm 5, C = K^2 = 9$$

$$\therefore \text{There are two circles one of them has a centre } (-5, 3) \text{ and its equation :}$$

$$x^2 + y^2 + 10x - 6y + 9 = 0 \text{ and the other has a centre } (5, 3) \text{ and its equation :}$$

$$x^2 + y^2 - 10x - 6y + 9 = 0$$

Example 8

Find the equation of the circle which touches the two coordinate axes, and its centre is the point $(-4, 4)$

Solution

\therefore The circle touches the two coordinate axes

$$\therefore C = L^2 = K^2 = 16$$

$$\therefore \text{The equation is : } x^2 + y^2 + 8x - 8y + 16 = 0$$

Example 9

Determine which of the following equations represent a circle :

(1) $x^2 + 3y^2 - 2x + 4y + 5 = 0$

(2) $2x^2 - xy + 2y^2 + 5x - y - 2 = 0$

(3) $x^2 + y^2 + 7x - y + 8 = 0$

(4) $2x^2 + 2y^2 - 6x + 4y + 9 = 0$

(5) $x^2 + y^2 - 16x + 12y + 100 = 0$

(6) $x^2 + 6x - 8y - 7 = 0$

Solution

(1) \therefore The coefficient of $x^2 \neq$ the coefficient of y^2

\therefore The equation doesn't represent a circle.

(2) \therefore The equation has a term contains xy

\therefore The equation doesn't represent a circle.

(3) The coefficient of $x^2 =$ the coefficient of y^2 and the equation is free of a term contains xy

\therefore The equation may represent a circle.

$$\therefore 2L = 7, 2K = -1 \quad \therefore L = \frac{7}{2}, K = -\frac{1}{2}, C = 8$$

$$\therefore L^2 + K^2 - C = \frac{49}{4} + \frac{1}{4} - 8 = \frac{9}{2} > 0$$

$$\therefore \text{The equation is represent a circle whose centre } \left(-\frac{7}{2}, \frac{1}{2} \right)$$

$$r = \sqrt{\frac{9}{2}} = \frac{3\sqrt{2}}{2} \text{ length unit.}$$

(4) \therefore The coefficient of X^2 = The coefficient of y^2 and the equation is free of a term contains Xy

\therefore The equation may represent a circle.

Multiply by $\frac{1}{2}$ to make the coefficient of X^2 = the coefficient of y^2 = 1

$$\therefore X^2 + y^2 - 3X + 2y + \frac{9}{2} = 0$$

$$\therefore 2L = -3, \quad 2K = 2$$

$$\therefore L = -\frac{3}{2}, \quad K = 1, \quad C = \frac{9}{2}$$

$$\therefore L^2 + K^2 - C = \frac{9}{4} + 1 - \frac{9}{2} = -\frac{5}{4} < 0$$

\therefore The equation doesn't represent a circle.

(5) \therefore The coefficient of X^2 = the coefficient of y^2 , and the equation is free of a term contains Xy

\therefore The equation may represent a circle.

$$\therefore 2L = -16, \quad 2K = 12$$

$$\therefore L = -8, \quad K = 6, \quad C = 100$$

$$\therefore L^2 + K^2 - C = 64 + 36 - 100 = \text{zero}$$

\therefore The equation doesn't represent a circle.

(6) \therefore The equation is free of the term y^2

\therefore The equation doesn't represent a circle.

Example 10

Find the equation of the circle whose radius length = 3 units, and the two equations of the straight lines carrying two diameters are $X + y = 2$, $2X - y = 7$

Solution

The centre of the circle is the point of intersection of the two straight lines :

$$X + y = 2 \quad (1), \quad 2X - y = 7 \quad (2)$$

$$\text{by adding} \quad \therefore 3X = 9 \quad \therefore X = 3$$

$$\text{by substitute} \quad \therefore y = -1$$

\therefore The centre is the point (3, -1)

$$\therefore L = -3, \quad K = 1, \quad C = L^2 + K^2 - r^2 = 9 + 1 - 9 = 1$$

$$\therefore \text{Equation of the circle is : } X^2 + y^2 - 6X + 2y + 1 = 0$$

Example 11

A circle whose centre M (-2, 7), and the length of its radius $r = 5$ units, state which of the following points lies on the circle, inside the circle and outside the circle.

$$A = (-1, 3), \quad B = (0, -5), \quad C = (2, 4)$$

Solution

\therefore Equation of the circle is : $(X + 2)^2 + (y - 7)^2 = 25$

by substitute by the points A , B and C in the L.H.S. of the equation :

$\therefore (-1 + 2)^2 + (3 - 7)^2 = 17 < r^2$ \therefore Point A $(-1, 3)$ lies inside the circle.

$\therefore (0 + 2)^2 + (-5 - 7)^2 = 148 > r^2$ \therefore Point B $(0, -5)$ lies outside the circle.

$\therefore (2 + 2)^2 + (4 - 7)^2 = 25 = r^2$ \therefore Point C $(2, 4)$ lies on the circle.

Example 12

Find the equation of the circle whose centre $M = (2, 3)$ and the straight line $3X + 4y + 2 = 0$ is a tangent at the point A

Solution

$\therefore \overline{MA}$ is a radius , \overleftrightarrow{AB} is tangent to the circle.

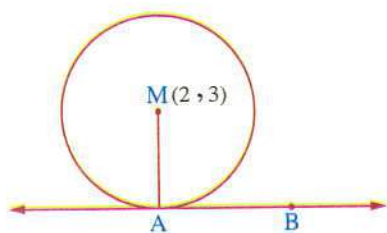
$\therefore \overline{MA} \perp \overleftrightarrow{AB}$

$\therefore MA = \frac{|3 \times 2 + 4 \times 3 + 2|}{\sqrt{3^2 + 4^2}} = 4$ length unit.

$\therefore r = 4$ length unit.

\therefore Equation of the circle is :

$$(X - 2)^2 + (y - 3)^2 = 16$$

**Example 13**

Determine the position of the circle $C_1 : (X - 3)^2 + (y - 2)^2 = 4$ with respect to the circle $C_2 : X^2 + y^2 + 2X + 2y + 1 = 0$

Solution

$$\therefore C_1 : (X - 3)^2 + (y - 2)^2 = 4$$

\therefore The centre $M_1 = (3, 2)$, $r_1 = \sqrt{4} = 2$ length unit.

$$, C_2 : X^2 + y^2 + 2X + 2y + 1 = 0$$

The centre $M_2 = (-1, -1)$, $r_2 = \sqrt{1 + 1} = 1$ length unit.

$\therefore r_1 + r_2 = 2 + 1 = 3$ length unit.

$, M_1 M_2 = \sqrt{(3 + 1)^2 + (2 + 1)^2} = 5$ length unit.

$\therefore M_1 M_2 > r_1 + r_2$ \therefore The two circles are distant.

Example 14

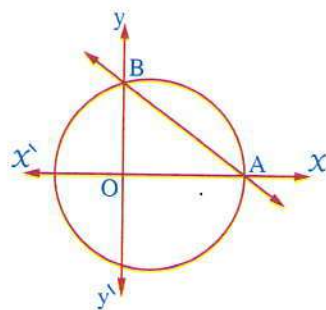
In the opposite figure :

If the equation of \overleftrightarrow{AB}

is : $6x + 8y - 48 = 0$ intersects.

The two coordinate axes at A and B ,

Find the equation of the circle passing through the points A , O and B



Solution

$$\therefore m(\angle AOB) = 90^\circ$$

$\therefore \overline{AB}$ is a diameter of the circle \therefore The equation of \overleftrightarrow{AB} is $6x + 8y = 48$

$$\text{i.e. } \frac{x}{8} + \frac{y}{6} = 1$$

\therefore The straight line cuts x -axis at the point $A = (8, 0)$

, cuts y -axis at the point $B = (0, 6)$

Let M be the centre of the circle.

$$\therefore M \text{ is the midpoint of } \overline{AB} = \left(\frac{8+0}{2}, \frac{0+6}{2} \right) = (4, 3)$$

$$AB = \sqrt{8^2 + 6^2} = 10 \text{ length unit.}$$

$$\therefore r = 5 \text{ length unit.}$$

$$\therefore \text{Equation of the circle is } (x - 4)^2 + (y - 3)^2 = 25$$

Example 15

Find the area of the equilateral triangle whose vertices passing through the circle :

$$x^2 + y^2 + 6x - 2y - 15 = 0$$

"Where each length unit in the plane represents 4 cm."

Solution

$$L = 3, K = -1, C = -15$$

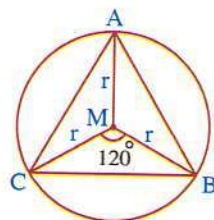
$$\therefore r = \sqrt{L^2 + K^2 - C} = \sqrt{9 + 1 + 15} = 5 \text{ length unit, } M \text{ is the centre of the circumcircle of } \triangle ABC$$

If $\triangle ABC$ is equilateral by drawing

$\overline{MA}, \overline{MB}, \overline{MC}$, then :

$$m(\angle BMC) = \frac{360^\circ}{3} = 120^\circ, \text{ then :}$$

$$\text{Area of } \triangle ABC = 3 \times \text{area of } \triangle MBC$$



$$\begin{aligned}
 &= 3 \times \frac{1}{2} MB \times MC \times \sin (\angle BMC) \\
 &= \frac{3}{2} r^2 \sin 120^\circ \\
 &= \frac{3}{2} \times 25 \sin 60^\circ = \frac{3}{2} \times 25 \times \frac{\sqrt{3}}{2} = \frac{75\sqrt{3}}{4} \text{ square unit.}
 \end{aligned}$$

\therefore Each length unit in the plane represent 4 cm.

\therefore The square unit represent $4^2 = 16 \text{ cm}^2$

$$\therefore \text{Area of } \triangle ABC = \frac{75\sqrt{3}}{4} \times 16 = 300\sqrt{3} \text{ cm}^2.$$

Remark

If the number of sides of a regular polygon = n sides, the length of the radius to the circle passing through its vertices = r , then

$$\text{Area of the regular polygon} = \frac{n}{2} r^2 \sin \left(\frac{360^\circ}{n} \right)$$

For example :

The regular hexagon polygon whose drawn inside a circle of radius length 8 cm., its area equals :

$$\begin{aligned}
 &\frac{6}{2} \times (8)^2 \times \sin \left(\frac{360^\circ}{6} \right) \\
 &= 3 \times 64 \times \sin 60^\circ \\
 &= 3 \times 64 \times \frac{\sqrt{3}}{2} = 96\sqrt{3} \text{ square unit.}
 \end{aligned}$$

Example 16

Find the cartesian equation of the circle passing through the points

$A = (6, 3)$, $B = (2, 3)$ and $C = (4, 1)$, then determine its centre and length of its radius.

Solution

Let the equation is : $x^2 + y^2 + 2Lx + 2Ky + C = 0$

\therefore The points A, B and C lies on the circle.

$$\therefore 36 + 9 + 12L + 6K + C = 0 \quad \text{i.e. } 12L + 6K + C = -45 \quad (1)$$

$$, 4 + 9 + 4L + 6K + C = 0 \quad \text{i.e. } 4L + 6K + C = -13 \quad (2)$$

$$, 16 + 1 + 8L + 2K + C = 0 \quad \text{i.e. } 8L + 2K + C = -17 \quad (3)$$

by subtracting (2) from (1)

$$\therefore 8L = -32 \quad \therefore L = -4$$

and by subtracting (3) from (1)

$$\therefore 4L + 4K = -28$$

$$\therefore L + K = -7$$

$$\therefore -4 + K = -7$$

$$\therefore K = -3$$

by substitute in (3)

$$\therefore -32 - 6 + c = -17$$

$$\therefore C = 21$$

$$\therefore \text{The equation is : } x^2 + y^2 - 8x - 6y + 21 = 0$$

, where the centre = (4, 3)

$$, r = \sqrt{16 + 9 - 21} = \sqrt{4} = 2 \text{ length unit.}$$

Example 17

Find the equation of the circle whose touches X-axis and passing through the points (-1, 2), (-3, 4)

Solution

\therefore The circle touches X-axis

$$\therefore r = |K|, C = L^2$$

Let the equation of the circle is :

$$x^2 + y^2 + 2Lx + 2Ky + L^2 = 0$$

\therefore The circle passing through the point (-1, 2) then :

$$1 + 4 - 2L + 4K + L^2 = 0$$

$$\therefore L^2 - 2L + 4K = -5$$

(1)

\therefore The circle passing through the point (-3, 4) then :

$$9 + 16 - 6L + 8K + L^2 = 0$$

$$\therefore L^2 - 6L + 8K = -25$$

(2)

by multiplying the equation (1) $\times 2$ then subtracting from the equation (2) :

$$\therefore L^2 - 2L = -15$$

$$\therefore L^2 + 2L - 15 = 0$$

$$(L - 3)(L + 5) = 0$$

$$\therefore L = 3 \quad \text{or} \quad L = -5$$

$$\therefore K = -2 \quad \text{or} \quad K = -10$$

\therefore There are two circles in one of them $L = 3$, $K = -2$ and its equation is :

$$x^2 + y^2 + 6x - 4y + 9 = 0$$

and in the other circle : $L = -5$, $K = -10$ and its equation is :

$$x^2 + y^2 - 10x - 20y + 25 = 0$$

SCIENTIFIC SECTION

Mathematics

Applications

By a group of supervisors



FIRST TERM
2
SEC.
2024

EXERCISES



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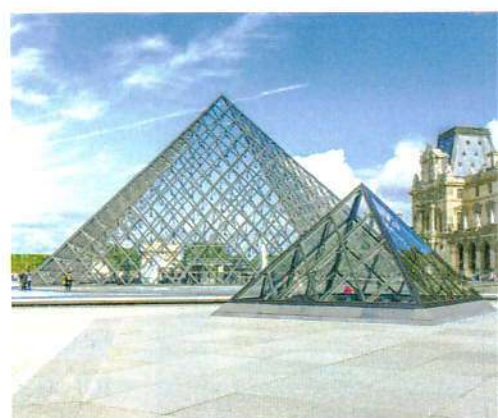
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Unit Two

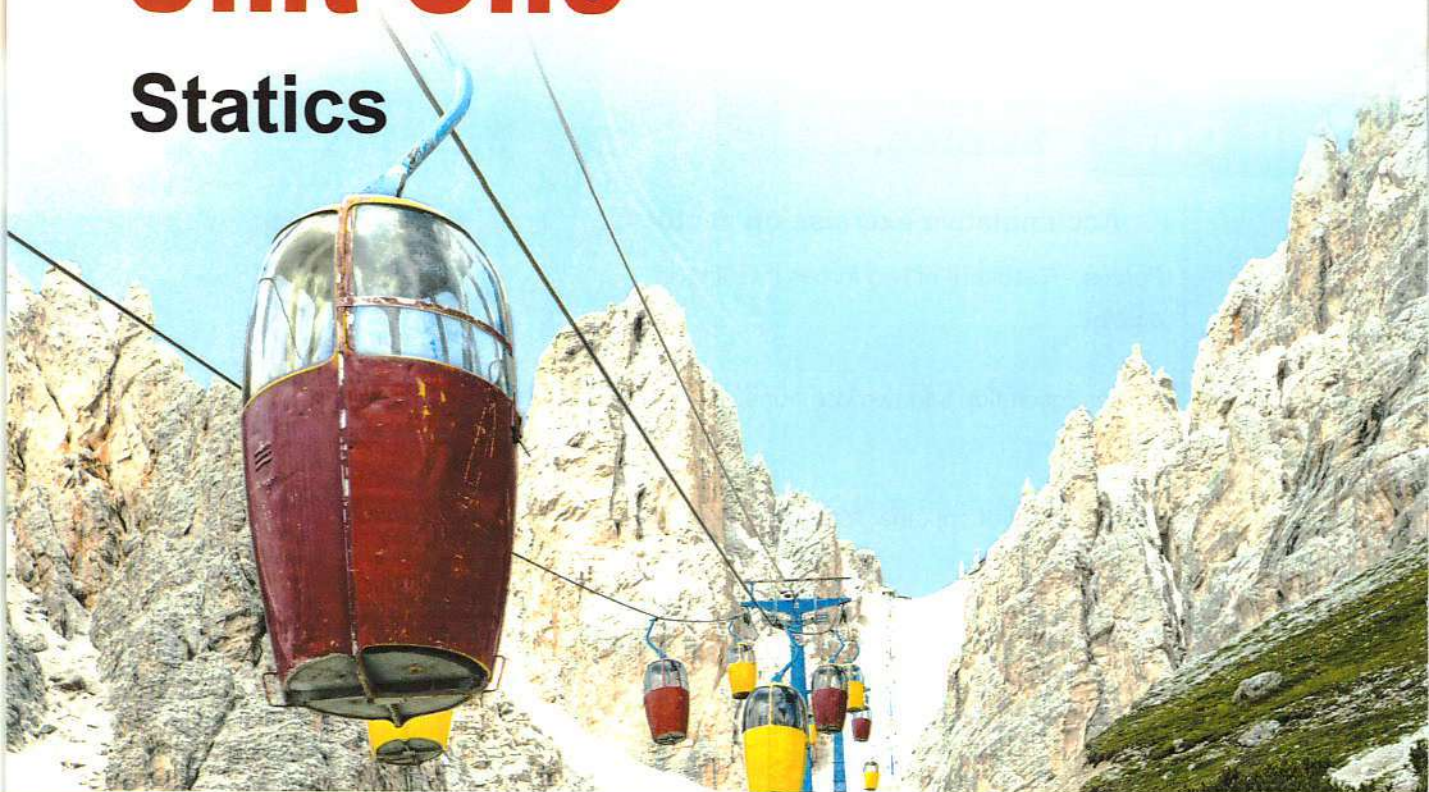
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Unit One

Statics



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1

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- Accumulative exercise on vectors.

Forces - Resultant of two forces meeting at a point.

Forces resolution into two components.

The resultant of coplanar forces meeting at a point.

Equilibrium of a rigid body under the effect of two forces / three forces meeting at a point.

(The triangle of forces rule - Lami's rule).

Follow : The equilibrium

(Meeting lines of action of three equilibrium forces).

Accumulative exercise on vectors



From the school book

Remember Understand Apply Higher Order Thinking Skills

1 Choose the correct answer from the given ones :

- (1) Norm of the vector $\vec{A} = -3\hat{i} + 4\hat{j}$ equals length unit.

(a) 3 (b) 4 (c) 5 (d) 1
- (2) The cartesian form of the vector $\vec{B} = (5\sqrt{2}, 225^\circ)$ is

(a) (5, 5) (b) (-5, -5) (c) (5, -5) (d) (-5, 5)
- (3) Measure of the polar angle of the vector $\vec{B} = -\hat{i} + \sqrt{3}\hat{j}$ equals

(a) 60° (b) 90° (c) 120° (d) 150°
- (4) The polar form of the vector $\vec{A} = \sqrt{2}\hat{i} + \sqrt{2}\hat{j}$ is

(a) (2, 135°) (b) (4, 45°) (c) (2, 45°) (d) (4, 135°)
- (5) The polar form of the vector $\vec{m} = 5\hat{i} + 12\hat{j}$ is

(a) (17, $67^\circ 22' 48''$) (b) (17, $22^\circ 37' 12''$)
(c) (13, $67^\circ 22' 48''$) (d) (13, $22^\circ 37' 12''$)
- (6) The vector that represents a force of magnitude 20 kg.wt. in the direction 30° South of East is written as

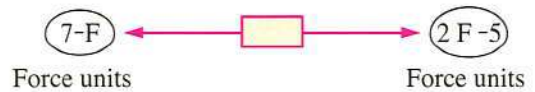
(a) $(10, -10\sqrt{3})$ (b) $(10\sqrt{3}, -10)$ (c) $(-10, 10\sqrt{3})$ (d) $(10\sqrt{3}, 10)$
- (7) If $\vec{F} = k\hat{i} + 2\sqrt{2}\hat{j}$ and $\|\vec{F}\| = 2\sqrt{3}$ newton, then $|k| = \dots\dots\dots$

(a) $6\sqrt{2}$ (b) $2\sqrt{6}$ (c) -2 (d) 2

- (8) If $\vec{F}_1 = (5, -3)$, $\vec{F}_2 = (7, 4)$, then the resultant of the two forces $\vec{R} = \dots\dots\dots$
 (a) $\hat{i} + 12\hat{j}$ (b) $9\hat{i} + 4\hat{j}$ (c) $35\hat{i} - 12\hat{j}$ (d) $12\hat{i} + \hat{j}$
- (9) If $\vec{F}_1 = 5\hat{i}$, $\vec{F}_2 = 7\hat{i} - 5\hat{j}$, then $\|\vec{R}\| = \dots\dots\dots$ force unit.
 (a) 12 (b) 5 (c) 13 (d) $\sqrt[4]{73}$
- (10) If $\vec{F}_1 = 2\hat{i} + 3\hat{j}$, $\vec{F}_2 = \hat{i} + \hat{j}$, then the magnitude of their resultant equals $\dots\dots\dots$ force unit.
 (a) 3 (b) 4 (c) 5 (d) 7
- (11) Two forces of magnitudes 5 newtons and 7 newtons acting in the direction of East , then their resultant equals $\dots\dots\dots$
 (a) 12 newton due East. (b) 2 newton due East.
 (c) 12 newton due West. (d) 2 newton due West.
- (12) If \vec{F}_1 , \vec{F}_2 and \vec{F}_3 are three forces in equilibrium and meeting at one point where :
 $\vec{F}_1 = (2, -5)$, $\vec{F}_2 = (-3, 2)$, then $\vec{F}_3 = \dots\dots\dots$
 (a) (2, 1) (b) (-1, -3) (c) (1, 3) (d) (3, 1)
- (13) If the set of forces $\vec{F}_1 = a\hat{i} + 7\hat{j}$, $\vec{F}_2 = -5\hat{i} - b\hat{j}$, $\vec{F}_3 = \hat{i} + \hat{j}$ are in equilibrium , then (a, b) = $\dots\dots\dots$
 (a) (2, 4) (b) (1, 2) (c) (-4, -8) (d) (4, 8)
- (14) If the set of forces $\vec{F}_1 = 4\hat{i} - 5\hat{j}$, $\vec{F}_2 = a\hat{i} + 3\hat{j}$, $\vec{F}_3 = 7\hat{i} - b\hat{j}$ are in equilibrium , then $a + b = \dots\dots\dots$
 (a) 13 (b) -13 (c) -11 (d) -2
- (15) If the forces $\vec{F}_1 = 4\hat{i} + 5\hat{j}$, $\vec{F}_2 = a\hat{i} - 7\hat{j}$ and $\vec{F}_3 = 3\hat{i} + b\hat{j}$ act at one point and the forces are in equilibrium , then $a + 2b = \dots\dots\dots$
 (a) -5 (b) 5 (c) 7 (d) -3
- (16) If $\vec{F}_1 = 2\hat{i} - 2\hat{j}$, $\vec{F}_2 = 4\hat{i} - 8\hat{j}$, their resultant $\vec{R} = 2a\hat{i} - 3b\hat{j}$, then $a + b = \dots\dots\dots$
 (a) 3 (b) $3\frac{1}{3}$ (c) $6\frac{1}{3}$ (d) 12
- (17) If $\vec{F}_1 = 5\hat{i} + 3\hat{j}$, $\vec{F}_2 = a\hat{i} + 6\hat{j}$ and $\vec{F}_3 = -14\hat{i} + b\hat{j}$ are three forces meeting at one point , $\vec{R} = \left(10\sqrt{2}, \frac{3}{4}\pi\right)$ then : (a, b) = $\dots\dots\dots$
 (a) (-1, 1) (b) (2, 1) (c) (-1, 2) (d) (1, -1)

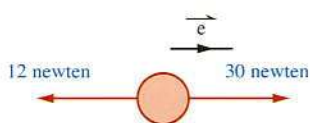
(18) In the opposite figure :

If the system is in equilibrium
 , then $F = \dots\dots\dots$ force units.

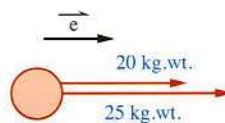


- (a) 4 (b) 7 (c) 2.5 (d) 3.5

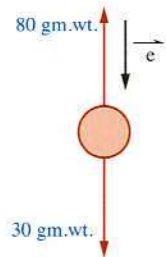
2 Write in terms of the unit vector \vec{e} the resultant of the forces shown in each figure of the following figures :



The resultant is



The resultant is

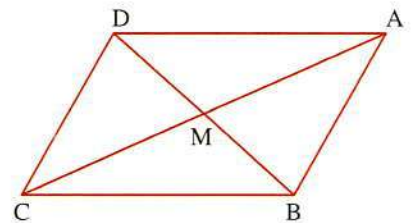


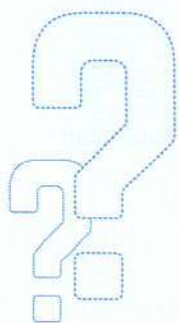
The resultant is

3 In the opposite figure :

ABCD is a parallelogram , M is the point of intersection of its diagonals , then :

- $\overrightarrow{AB} + \overrightarrow{BC} = \dots\dots\dots$
- $\overrightarrow{DA} + \overrightarrow{DC} = \dots\dots\dots$
- $\overrightarrow{AM} + \overrightarrow{CM} = \dots\dots\dots$
- $\overrightarrow{AB} + 2 \overrightarrow{BM} = \dots\dots\dots$
- $\overrightarrow{AB} - \overrightarrow{AM} = \dots\dots\dots$



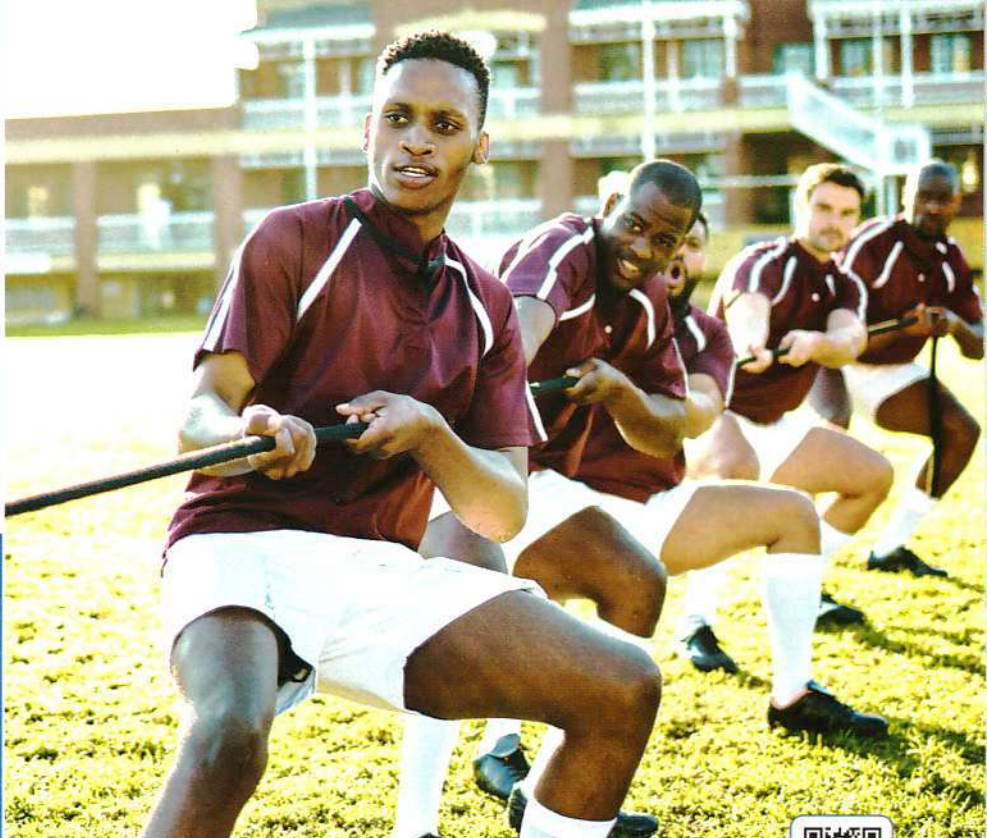


Exercise

1

Forces - Resultant of two forces meeting at a point

From the school book



● Remember ● Understand ● Apply ● Higher Order Thinking Skills



Test yourself

First Multiple choice questions

Choose the correct answer from the given ones :

- (1) The force is defined by
 - (a) its magnitude.
 - (b) its direction.
 - (c) the point of action.
 - (d) all the previous.
- (2) Two forces act at a point. The magnitude of the two forces are 5 , 3 newton and the angle between them 60° , then the magnitude of their resultant = newton.
 - (a) 2
 - (b) 5
 - (c) 7
 - (d) 8
- (3) Two forces act at a point the magnitude of the two forces $8\sqrt{3}$, 8 newton and the measure of the included angle between them 150° , then the magnitude of their resultant = newton.
 - (a) 64
 - (b) 32
 - (c) 16
 - (d) 8
- (4) Two perpendicular forces act at a point. The magnitude of the two forces 12 , 5 newton , then the magnitude of their resultant = newton.
 - (a) 17
 - (b) 7
 - (c) 13
 - (d) 14
- (5) Resultant of two forces 6 newton and 8 newton could be newton.
 - (a) 20
 - (b) 15
 - (c) 12
 - (d) 1

- (6) The magnitude of two forces are 4, 5 N. They act at a point and cosine of their included angle is $\frac{-2}{5}$, then the magnitude of their resultant $R = \dots\dots\dots$ newtons.

(a) 15 (b) 5 (c) 20 (d) 25

- (7) Two forces act at a point. The magnitude of the two forces are 6, 3 newton and their resultant is perpendicular to one of them, then the magnitude of their resultant = $\dots\dots\dots$ newton.

(a) 3 (b) $3\sqrt{3}$ (c) 6 (d) $6\sqrt{3}$

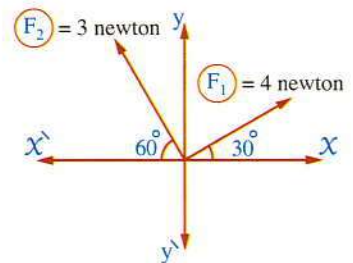
- (8) Two forces enclosing between them an angle of measure θ , then the magnitude of their resultant $\dots\dots\dots$

(a) increase as the value of θ increase.
 (b) doubled as the value of θ doubled.
 (c) increase as the value of θ decrease.
 (d) don't change as change of the value of θ

- (9) In the opposite figure :

The magnitude of the resultant of the two forces in the figure = $\dots\dots\dots$ newton.

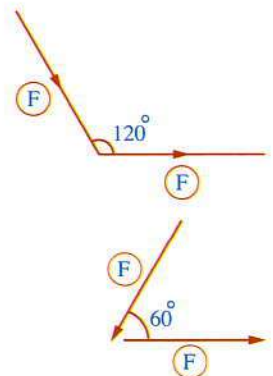
(a) 7 (b) 5
 (c) 1 (d) $\sqrt{7}$



- (10) In the opposite figure :

Magnitude of the resultant of the two forces = $\dots\dots\dots$ newton.

(a) $2F$ (b) F
 (c) $\sqrt{3}F$ (d) zero






- (11) The magnitude of the resultant of the two forces shown in the opposite figure is $\dots\dots\dots$

(a) $\frac{1}{2}F$ (b) F (c) $\sqrt{3}F$ (d) $\sqrt{5}F$

- (12) If the resultant of the two forces F_1, F_2 bisects the angle between them. Which of the following statements is true ?

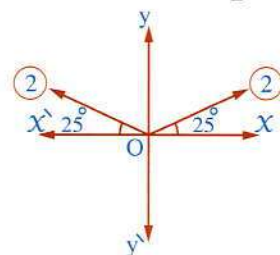
① $F_1 = F_2$ ② $\vec{F}_1 = \vec{F}_2$ ③ $\vec{R} = \vec{F}_1 + \vec{F}_2$
 (a) only ① (b) only ①, ③
 (c) only ②, ③ (d) All the previous.

- (13) Two forces act at a point. The magnitude of the two forces are F , 2 newton and the measure of the angle between them is 60° , if their resultant equal $2\sqrt{3}$ newton, then $F = \dots\dots\dots$ newton.
- (a) 2 (b) 4 (c) 8 (d) 12
- (14) The magnitude of two forces F , 2 newton and the measure of their included angle $= \frac{2\pi}{3}$ and the magnitude of their resultant is F newton, then $F = \dots\dots\dots$ newton.
- (a) 2 (b) 3 (c) 4 (d) $2\sqrt{2}$
- (15)  Two forces of equal magnitudes, enclosing between them an angle of measure $\frac{\pi}{2}$. If the magnitude of their resultant is 8 N, then the value of each force measured in newton is $\dots\dots\dots$
- (a) $2\sqrt{2}$ (b) 4 (c) $4\sqrt{2}$ (d) 8
- (16) Two equal forces in magnitude, the magnitude of their resultant $= 7\sqrt{3}$ newton and the measure of the included angle is $\frac{\pi}{3}$, then the magnitude of each of them $= \dots\dots\dots$ newton.
- (a) 3 (b) $5\sqrt{3}$ (c) 5 (d) 7
- (17) The magnitude of two forces F , F kg.wt., the magnitude of their resultant 24 kg.wt. and inclined to the first force by an angle of measure 30° , then $F = \dots\dots\dots$ kg.wt.
- (a) 8 (b) $8\sqrt{3}$ (c) $8\sqrt{2}$ (d) 12
- (18) Two forces of magnitudes 8 and F gm.wt. The measure of the angle between them is $\alpha \in]0, \pi[$, their resultant bisects the included angle between them, then $F = \dots\dots\dots$ gm.wt.
- (a) 4 (b) 16 (c) $2\sqrt{2}$ (d) 8
- (19)  Two forces of magnitudes 3, F newton and the measure of the angle between them is 120° . If their resultant is perpendicular to the first force, so the value of F in newton is $\dots\dots\dots$
- (a) 1.5 (b) 3 (c) $3\sqrt{3}$ (d) 6
- (20) The magnitude of two perpendicular forces are $(2F - 5)$ and $(F + 2)$ newton and the magnitude of their resultant is $3\sqrt{5}$ newton, then $F = \dots\dots\dots$ newton.
- (a) 7 (b) 4 (c) 6 (d) 3

- (21) Two forces of magnitudes 6 N. and 10 N. , if the magnitude of their resultant is 14 N. , then the measure of the angle between the forces is
- (a) 15° (b) 30° (c) 60° (d) 45°
- (22)  Two equal forces , the magnitude of each of them is 6 N. , the magnitude of their resultant is 6 N. , then the angle between them equals
- (a) 30° (b) 60° (c) 120° (d) 150°
- (23) Two forces of magnitudes 6 N. and 8 N. , if the magnitude of their resultant is 2 N. , then the measure of the angle between the two forces is
- (a) 30° (b) 90° (c) 180° (d) 270°
- (24) Magnitude of resultant of two forces of magnitudes 6 , 2.5 newton is equal to 6.5 newton , then the angle between the two forces is
- (a) an acute angle. (b) an obtuse angle.
(c) a right angle. (d) a straight angle.
- (25) The magnitude of two forces are 2 F , 5 F newton and the measure of their included angle is θ and their resultant is 3 F , then $\theta = \dots\dots\dots$
- (a) zero (b) 60° (c) 90° (d) 180°
- (26) Two forces of magnitudes 3 F and F newton and their resultant is 4 F newton , then the measure of the angle between them =
- (a) 60° (b) 0° (c) 180° (d) 90°
- (27) Two forces of magnitudes F and F act at a particle and their resultant is F , then the measure of the angle between the two forces =
- (a) 120° (b) 60° (c) 45° (d) 90°
- (28) The magnitude of two forces acting at a point F , $\sqrt{3}$ F newton. If the magnitude of their resultant is 2 F newton , then the measure of their included angle equals
- (a) 30° (b) 60° (c) 90° (d) 120°
- (29) If $\vec{R} = \vec{F}_1 + \vec{F}_2$ and $\|\vec{R}\| = \|\vec{F}_1\| - \|\vec{F}_2\|$, then the measure of the angle between \vec{F}_1 , \vec{F}_2 equals
- (a) zero (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{2}$ (d) π
- (30) If the magnitude of the resultant of two forces act at a point is maximum value , then the measure of the angle between the two forces equal
- (a) 180° (b) 120° (c) zero (d) 60°




- (31) The measure of the angle between \vec{F}_1 and the resultant of the two forces $(\vec{F}_1 + \vec{F}_2)$ and $(\vec{F}_1 - \vec{F}_2)$ is
- (a) zero (b) π (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{3}$
- (32) If \vec{R}_1 is the resultant of the two forces (\vec{F}_1, \vec{F}_2) and \vec{R}_2 is the resultant of the two forces $(\vec{F}_1, -\vec{F}_2)$, $\|\vec{F}_1\| = \|\vec{F}_2\|$, then
- (a) $\vec{R}_1 \perp \vec{R}_2$ (b) $\vec{R}_1 = \vec{R}_2$
 (c) $\|\vec{R}_1\| = \|\vec{R}_2\|$ (d) $\vec{R}_1 \parallel \vec{R}_2$
- (33) Two forces of magnitudes 4 and 6 newton. The measure of the angle between them is 90° , then the tangent of the angle between the resultant and the first force equal
- (a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) $2\sqrt{13}$ (d) $\frac{\sqrt{6}}{2}$
- (34) The magnitudes of two perpendicular forces are 6, 8 newton then the measure of the angle between the resultant and the first force is
- (a) $\sin^{-1} \frac{4}{3}$ (b) $\cos^{-1} \frac{4}{3}$ (c) $\tan^{-1} \frac{4}{3}$ (d) $\tan^{-1} \frac{3}{4}$
- (35) Two forces of magnitudes $F, 2F$ newton act at a point, if the resultant of them is perpendicular to one of them, then $R = \dots$
- (a) $\sqrt{5} F$ (b) $\sqrt{3} F$ (c) $3 F$ (d) F
- (36) Two forces of magnitudes $3\sqrt{2}$ and 6 newton and the measure of the angle between them is 135° , then the measure of the angle between their resultant and the second force is
- (a) 30° (b) 45° (c) 60° (d) 90°
- (37) Two forces of magnitudes 12, 15 newton act at a particle and the measure of the enclosing angle between them is θ° , where $\cos \theta = \frac{-4}{5}$, then the measure of the included angle between the resultant and the first force = $^\circ$
- (a) zero (b) 30 (c) 90 (d) $36^\circ 52'$
- (38) The magnitude of two forces acting on a particle are 5, 8 newton, then the smallest value of their resultant = newton.
- (a) 2 (b) 3 (c) 7 (d) 13
- (39) Two forces of magnitudes 9 newton, 1000 dyne, the maximum value of their resultant
- (a) 1009 dyne. (b) 1009 newton. (c) 9.01 dyne. (d) 9.01 newton.

- (40) Two forces of magnitudes 5 , F newton , if the smallest resultant of them is 10 newton , $F > 5$, then $F = \dots\dots\dots$ newton.
 (a) 6 (b) 10 (c) 15 (d) 20
- (41) Two forces act at a point. The magnitude of the two forces are 5 F , 3 F. If the maximum value of their resultant is 40 newton , then the minimum value of their resultant $\dots\dots\dots$ newton.
 (a) 10 (b) 20 (c) 5 (d) zero
- (42) Two forces act at a point. The magnitudes of the two forces are 5 , 3 newton , then the magnitude of their resultant measure by newton $\in \dots\dots\dots$
 (a) $[2, 8]$ (b) $]2, 8[$ (c) $[3, 5]$ (d) $]3, 5[$
- (43) If θ is the angle between two forces of magnitudes 2 newton , 6 newton , $\theta \in]0, \pi]$, then the magnitude of their resultant measured by newton $\in \dots\dots\dots$
 (a) $]4, 8[$ (b) $[4, 8[$ (c) $]4, 8]$ (d) $[4, 8]$
- (44) Two forces of equal magnitude and the magnitude of their resultant equal 16 newton when the measure of the angle between the two forces is $\frac{\pi}{2}$, then the maximum value of their resultant equal $\dots\dots\dots$ newton.
 (a) 32 (b) $8\sqrt{2}$ (c) $16\sqrt{2}$ (d) zero
- (45) Two forces of magnitude F_1, F_2 kg.wt. , where $F_1 > F_2$ and the magnitude of smallest and greatest resultant of them are 3 and 12 gm.wt. respectively , then $F_1^2 - F_2^2 = \dots\dots\dots$
 (a) 12 (b) 3 (c) 9 (d) 36
- (46) The magnitude of two forces are 12 , 17 newton then the difference between the greatest and the smallest value of their resultant = $\dots\dots\dots$ newton.
 (a) 29 (b) 5 (c) 14 (d) 24
- (47) Two forces of magnitude F , $\sqrt{3} F$ newton meeting at a point and the magnitude of their resultant is R_1 when the measure of the angle between the two forces is 90° , and their resultant becomes R_2 when the measure of the angle between the two forces is 150° , then $\dots\dots\dots$
 (a) $R_1 = R_2$ (b) $R_1 = 2 R_2$ (c) $R_1 = \frac{3}{5} R_2$ (d) $R_1 = \frac{1}{2} R_2$
- (48) The direction of the resultant of the forces which represented in the opposite figure is $\dots\dots\dots$
 (a) \overrightarrow{OX} (b) \overrightarrow{OX}
 (c) \overrightarrow{Oy} (d) \overrightarrow{Oy}



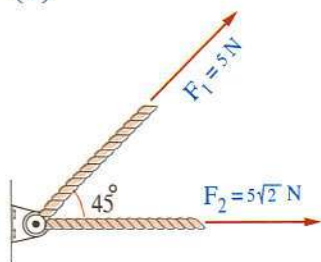
- (49) Two forces act at a point and the magnitude of smallest and greatest resultant of them are 0 and 12 newton respectively, then
- magnitude of one force is three times magnitude of the other.
 - magnitude of one force is twice magnitude of the other.
 - the two forces are equal in magnitude.
 - the two forces are perpendicular.

Second Essay questions

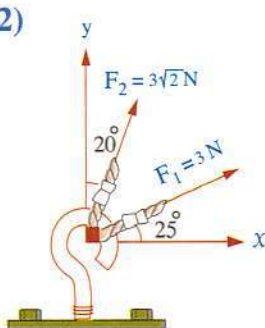
- Find the magnitude and the direction of the resultant of two perpendicular forces of magnitudes 8 and 15 kg.wt. acting at a particle. « 17 kg.wt. , $\theta = 61^\circ 55' 39''$ »
- The magnitude of the resultant of two perpendicular forces is 50 newton. If the resultant makes with the first force an angle of measure 30° , find the magnitude of each of these two forces. « $25\sqrt{3}$, 25 newton »
-  Two forces of magnitudes 30 and 16 newton act at a particle, if the magnitude of their resultant is 26 newton. Find the measure of the angle between these two forces. « 120° »
- Two forces are of magnitudes 9 and 6 kg.wt. act at a particle. The measure of the included angle is α , find α if the magnitude of the resultant is $3\sqrt{7}$ kg.wt., find the measure of the angle between the resultant and the great force. « $\alpha = 120^\circ$, $\theta = 40^\circ 53' 36''$ »
- Two forces acted at a point. If the magnitude of the first is 15 kg.wt. towards East and the second is of magnitude 18 kg.wt. in the direction 30° West of the North. Calculate the magnitude and the direction of the resultant. « $3\sqrt{31}$ kg.wt. , $\theta = 68^\circ 56' 54''$ »
-  Two forces of magnitudes 12, F kg.wt. act on a point. The first force acts in direction of East and the second force acts in direction 60° South of the West. Find the magnitude of F and the magnitude of the resultant if it is known that the line of action of the resultant acts in the direction 30° South of the East. « 6 kg.wt. , $6\sqrt{3}$ kg.wt. »
-  Two forces act at a particle and they include an angle of measure α where $\tan \alpha = \frac{-1}{\sqrt{3}}$
If the resultant is perpendicular to the small force and the magnitude of the great force equals 30 kg.wt. What is the magnitude of each of the small force and the resultant ?
« $15\sqrt{3}$ kg.wt. , 15 kg.wt. »

8 Find the magnitude and the direction of the resultant in each of the following figures :

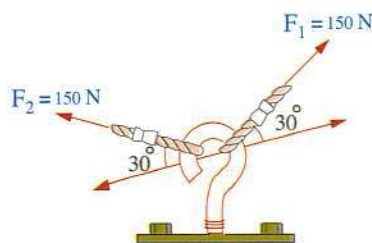
(1)



(2)



(3)



9 Two forces of magnitudes F , 4 newton act on a particle and the measure of the angle between their directions is 120° , the magnitude of their resultant equals $4\sqrt{3}$ newton. Find the magnitude of \vec{F} and the measure of the angle that \vec{R} from with \vec{F} « 8 newton , 30° »

10 Two forces of magnitudes $\sqrt{3} F$ and $2 F$ act at a point. Find the measure of the angle included between them if their resultant is perpendicular to the small force and if $F = 15$ Find the magnitude of the resultant. « 150° , 15 newton »

11 Two forces of magnitudes $2\sqrt{2}$ and F newton act at a particle and the magnitude of their resultant is $\sqrt{2}$ newton. If the resultant is perpendicular to the second force , find F and the measure of the angle between the two forces. « $\sqrt{6}$ newton , 150° »

12 Two forces of magnitudes 16 and F kg.wt. act on a particle and the measure of the angle between them is 120° . If their resultant is inclined to the force 16 kg.wt. by an angle whose measure is 30° , find the magnitude of F and the resultant. « 8 kg.wt. , $8\sqrt{3}$ kg.wt. »

13 Three forces of magnitudes 5 , 10 , $4\sqrt{7}$ N. act on a particle , if the measure of the angle between the first and the second forces equals 60° , find the magnitude of the maximum and the minimum resultant for the three forces. « $9\sqrt{7}$ newton , $\sqrt{7}$ newton »

14 Two forces of magnitudes $2 F$ and $3 F$ newton. The angle between them is of measure θ , find the value of θ if the magnitude of their resultant is :

(1) $3 F$

(2) F


(3) $5 F$

(4) $\sqrt{13} F$

« 109° , 28° , 16° , 180° , zero , 90° »

15 Two forces of magnitudes 2, F newton, the angle between them is of measure 120°

Find F in each of the two cases :

(1)  The direction of the resultant is perpendicular to the second force.

(2) The resultant inclines by 45° to the 2nd force.

« 1, $\sqrt{3} + 1$ newton »

16 F_1 and F_2 newton are magnitudes of two forces intersect at a point and their resultant equals R newton where $R \in [2, 10]$, $F_1 > F_2$, find each of F_1 and F_2 , then find R when the measure of the angle between them is 120°

« 6, 4, $2\sqrt{7}$ newton »


17 Two forces act at a point, the value of one is 3 N. more than the other.

If the magnitude of their resultant is $3\sqrt{3}$ newton and is perpendicular to the smaller force. Find the magnitude of each force and the measure of the angle between them.

« 3, 6 newton, $\alpha = 120^\circ$ »

18 The resultant of two forces F_1 and F_2 is $\sqrt{10}$ newton when $F_1 \perp F_2$ and their resultant becomes $\sqrt{13}$ newton when the angle between F_1 and F_2 becomes 60° , find F_1 and F_2

« 1, 3 newton »

19  Two forces of equal magnitude meeting at a point and the magnitude of their resultant equals 12 kg.wt. if the direction of one of them is reversed then the magnitude of the resultant becomes 6 kg.wt. Find the magnitude of each force.

« $3\sqrt{5}$, $3\sqrt{5}$ kg.wt. »

20 Two forces \vec{F}_1 , \vec{F}_2 meet at a point. Their resultant is R gm.wt. The angle between them is of measure 120° . If the direction of \vec{F}_2 is reversed, the resultant will be $R\sqrt{3}$ gm.wt., prove that $F_1 = F_2$ and the resultant in the first case is perpendicular to the second case.

21 4, F are two forces acting at a point and their resultant is 10 newton and makes an angle of measure 60° with the force 4 newton. Find the value of F.

« $2\sqrt{19}$ newton »

22 The difference between the magnitudes of two forces acting at a point is 15 newton. and their resultant = 35 newton in magnitude when the measure of the angle between the two forces = 120° , find the magnitude of each of the two forces.


« 40, 25 newton »

23 The sum of magnitudes of two forces is 4 newton when the measure of the angle between them is 60° , then the resultant becomes $\sqrt{13}$ newton. Find the magnitude of each of the two forces.


« 1, 3 newton »

24 The sum of magnitudes of two forces acting at a point is 40 kg.wt. the magnitude of their resultant is 20 kg.wt. and it is perpendicular to the smaller force. Find the magnitude of each of the two forces and the cosine of the angle between them.

« 15, 25 kg.wt., $-\frac{3}{5}$ »

- 25**  Two forces of same magnitude F kg.wt. enclose between them an angle of measure 120° . If the two forces are doubled and the measure of the angle between them became 60° , then the magnitude of their resultant increases by 11 kg.wt., than the first case. Find the magnitude of F

« $1 + 2\sqrt{3}$ »

- 26**  $F, 2F$ are two forces act on a particle and enclose between them an angle of measure α . The magnitude of their resultant equals $\sqrt{5}F(m+1)$ and if the measure of the angle between them becomes $(90^\circ - \alpha)$, then the magnitude of the resultant will be $\sqrt{5}F(m-1)$

Prove that : $\tan \alpha = \frac{m-2}{m+2}$

Third Higher skills

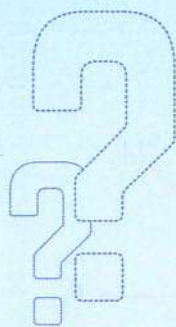
1 Choose the correct answer from those given :

- (1) If the ratio between the maximum and the minimum values of the resultant of two forces is $7:3$, then the ratio between the two forces =
 (a) $7:4$ (b) $7:3$ (c) $5:3$ (d) $5:2$
- (2) If the ratio among magnitudes of two forces and their resultant is $4:3:\sqrt{13}$ respectively, then the measure of the angle between the two forces =
 (a) 30° (b) 60° (c) 90° (d) 120°
- (3) If the resultant of two forces \vec{F}_1, \vec{F}_2 is perpendicular on \vec{F}_1 , then the measure of the angle between the two forces \vec{F}_1, \vec{F}_2 equals
 (a) $\cos^{-1}\left(\frac{F_1}{F_2}\right)$ (b) $\cos^{-1}\left(\frac{-F_1}{F_2}\right)$ (c) $\sin^{-1}\left(\frac{F_1}{F_2}\right)$ (d) $\sin^{-1}\left(\frac{-F_1}{F_2}\right)$
- (4) If the resultant of two perpendicular forces makes an angle of measure θ to the greater force which of the following values could be a value of θ ?
 (a) 90° (b) 70° (c) 45° (d) 10°
- (5) \vec{F}_1, \vec{F}_2 are two forces acting at a point and their resultant is R . If \vec{F}_2 reversed then their resultant rotates with angle of measure 90° , then
 (a) $F_1 = F_2$ (b) $F_1 = 2F_2$
 (c) $F_1 = \frac{1}{2}F_2$ (d) nothing of the previous.
- (6) The magnitudes of two forces acting at a point are $4, F$ newton and the measure of their included angle is 120° , then F which makes the resultant minimum equals newton.
 (a) 1 (b) 2 (c) 3 (d) 4

- (7) If θ_1 is the measure of the angle between the resultant of two forces (\vec{F}_1, \vec{F}_2) and the force \vec{F}_1 and θ_2 is the measure of the angle between the resultant of the two forces $(\vec{F}_1, 2\vec{F}_2)$ and the force \vec{F}_1 , then
- (a) $\theta_1 = \theta_2$ (b) $\theta_1 > \theta_2$ (c) $\theta_1 < \theta_2$ (d) $\theta_1 + \theta_2 = \frac{\pi}{2}$
- (8) The magnitudes of two forces acting at a point are $F, \sqrt{3}F$ newton and the magnitude of their resultant is F newton and θ_1 is the measure of the angle between F, R and θ_2 is the measure between $\sqrt{3}F$ and R , then
- (a) $\theta_1 = \theta_2$ (b) $\theta_1 = \frac{1}{2} \theta_2$ (c) $\theta_1 = 3 \theta$ (d) $\theta_1 = 4 \theta_2$
- (9) The magnitudes of two forces acting at a point are F_1, F_2 where : $3 \leq F_1 \leq 12$, $4 \leq F_2 \leq 16$ and the magnitude of their resultant is R and the measure of their included angle is 90° , then
- (a) $5 \leq R \leq 20$ (b) $7 \leq R \leq 28$ (c) $0 \leq R \leq 18$ (d) $1 \leq R \leq 4$
- (10) Two forces meet at a point, their magnitudes are F_1, F_2 where $1 \leq F_1 \leq 9$, $3 \leq F_2 \leq 7$ and the magnitude of their resultant R , then
- (a) $2 \leq R \leq 16$ (b) $4 \leq R \leq 16$ (c) $6 \leq R \leq 16$ (d) $0 \leq R \leq 16$
- (11) The magnitudes of two forces acting at a point are F_1, F_2 where $5 \leq F_1 \leq 20$, $12 \leq F_2 \leq 21$ and the magnitude of their resultant is R , the measure of the angle between them is θ where $0 \leq \theta \leq \frac{\pi}{2}$ then
- (a) $13 \leq R \leq 29$ (b) $0 \leq R \leq 41$ (c) $13 \leq R \leq 41$ (d) $17 \leq R \leq 29$

2 One of two forces is half the other in magnitude, they have a certain resultant. If the small force increased by 4 kg.wt. and the great force becomes double, then their resultant stays in the same direction of the first case, find the magnitudes of the two forces and the ratio between the magnitudes of the two resultants in the two cases. « 4, 8 kg.wt., 1 : 2 »

3 \vec{F}_1 and \vec{F}_2 are two forces meeting at a point and their resultant is R newton. If the direction of \vec{F}_2 becomes in the opposite direction, then the magnitude of the resultant becomes $R\sqrt{3}$ newton and the resultant becomes perpendicular to the first resultant. Find the measure of the angle between the two forces. « $\alpha = 120^\circ$ »



Exercise

2

Forces resolution into two components



From the school book

Remember Understand Apply Higher Order Thinking Skills



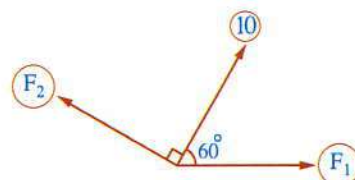
Test yourself

First Multiple choice questions

Choose the correct answer from the given ones :

(1) In the opposite figure :

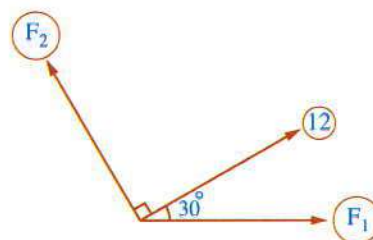
If the force of magnitude 10 N. is resolved into two components \vec{F}_1 and \vec{F}_2 inclined to the force by two angles of measures 60° and 90° respectively , then $F_2 = \dots\dots\dots$ N.



- (a) $5\sqrt{3}$ (b) 10
(c) $10\sqrt{3}$ (d) 20

(2) In the opposite figure :

If the force of magnitude 12 N. is resolved into two components \vec{F}_1 and \vec{F}_2 inclined to the force by two angles of measures 30° and 90° respectively , then $F_2 = \dots\dots\dots$ N.



- (a) 10 (b) $10\sqrt{3}$
(c) $6\sqrt{3}$ (d) $4\sqrt{3}$

(3) In the opposite figure :

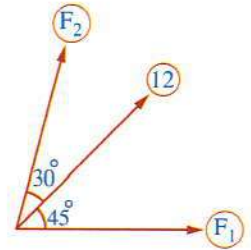
If the force of magnitude 12 N. is resolved into two components \vec{F}_1 and \vec{F}_2 , then $F_1 = \dots\dots\dots$ newton.

(a) $12 \cos 75^\circ$

(b) $12 \cos 45^\circ$

(c) $6 \csc 45^\circ$

(d) $6 \csc 75^\circ$



(4) In the opposite figure :

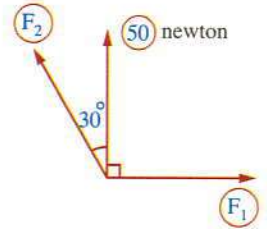
If the force of magnitude 50 newton is resolved into two components \vec{F}_1 and \vec{F}_2 , then $F_1 + F_2 = \dots\dots\dots$ newton.

(a) 50

(b) 25

(c) $50\sqrt{2}$

(d) $50\sqrt{3}$



(5) In the opposite figure :

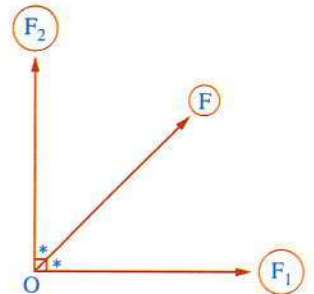
If the force \vec{F} is resolved into the two perpendicular components \vec{F}_1 and \vec{F}_2 , the vector of the force \vec{F} bisects the angle between the directions of \vec{F}_1 and \vec{F}_2 and $\|\vec{F}_1\| = 6\sqrt{2}$ newton, then $\|\vec{F}\| = \dots\dots\dots$ newton.

(a) 6

(b) $6\sqrt{2}$

(c) 12

(d) $12\sqrt{2}$



(6) In the opposite figure :

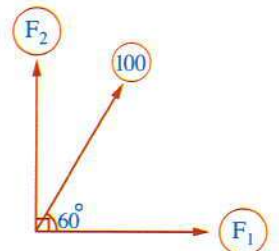
If the force of magnitude 100 newton is resolved into two forces \vec{F}_1 and \vec{F}_2 and the force is measured by newton, then $(F_1, F_2) = \dots\dots\dots$

(a) $(50, 50\sqrt{3})$

(b) $(50\sqrt{3}, 10)$

(c) $(50, 50)$

(d) $(10, 10)$



(7) In the opposite figure :

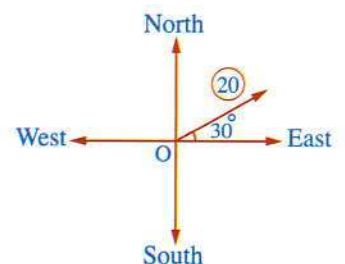
A force of magnitude 20 newton. acts in the direction 30° North of the East is resolved into two perpendicular components, then the magnitude of the component in North direction = $\dots\dots\dots$ newton.

(a) $10\sqrt{3}$

(b) 20

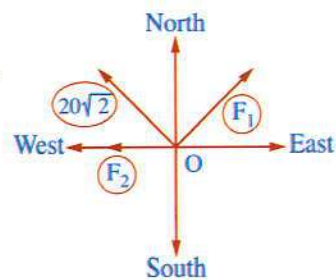
(c) 10

(d) 5



(8) In the opposite figure :

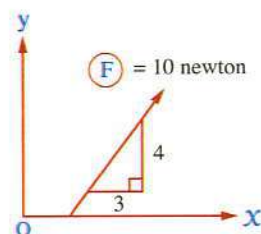
A force of magnitude $20\sqrt{2}$ kg.wt. acts in the Western North direction , is resolved into two component. One of them of magnitude F_1 in the Eastern North direction and the other of magnitude F_2 in the direction of West , then $F_2 = \dots\dots\dots$ kg.wt.



- (a) 30 (b) 40
(c) 50 (d) $40\sqrt{2}$

(9) In the opposite figure :

If a force \vec{F} is resolved into two components in the directions of the coordinate axes , then the magnitude of the component of this force in the direction of \vec{OX} equals $\dots\dots\dots$ newton.



- (a) 10 (b) 6
(c) 8 (d) $\frac{40}{3}$

(10) A force of magnitude $10\sqrt{2}$ gm.wt. acts in the Eastern South direction , is resolved into two perpendicular components , then the magnitude of the component in the South direction = $\dots\dots\dots$ gm.wt.

- (a) 5 (b) 10 (c) $10\sqrt{2}$ (d) $5\sqrt{2}$

(11) A force of magnitude 6 newton acts in direction of North. It is resolved into two perpendicular components , so its component in direction of the East of magnitude $\dots\dots\dots$ newton.

- (a) zero (b) 3 (c) $3\sqrt{2}$ (d) 6

(12) A force of magnitude $4\sqrt{2}$ newton acts in direction of East. It is resolved into two perpendicular components , so its component in the direction of Northern East of magnitude $\dots\dots\dots$ newton.

- (a) zero (b) $4\sqrt{2}$ (c) 4 (d) 6

(13) The magnitude of a force is 6 newton and acts towards the North. It is resolved into two perpendicular components then its component in direction of Eastern North of magnitude $\dots\dots\dots$ newton.

- (a) 6 (b) $3\sqrt{2}$ (c) $2\sqrt{3}$ (d) zero

(14) A force of magnitude $5\sqrt{3}$ newton acts in the direction 30° East of the North, is resolved into two perpendicular components, then the magnitude of its component in the East direction = newton.

- (a) $\frac{5\sqrt{3}}{2}$ (b) $\frac{15}{2}$ (c) $\frac{15\sqrt{3}}{2}$ (d) $15\sqrt{3}$

(15) The magnitude of a force is 8 newton and acts in East direction. It is resolved into two components, the angle between the two components is 120° , then its component in South direction = newton.

- (a) 16 (b) 8 (c) $8\sqrt{3}$ (d) $\frac{8\sqrt{3}}{3}$

(16) A force of magnitude 40 newton acts vertically upwards is resolved into two components one of them is horizontal of magnitude 20 newton, then the magnitude of the other = newton.

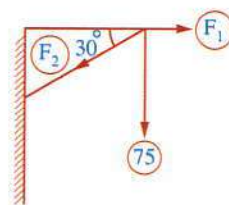
- (a) 20 (b) $20\sqrt{3}$ (c) $20\sqrt{5}$ (d) $10\sqrt{3}$

(17) Force of magnitude F newton is resolved into two components \vec{F}_1 and \vec{F}_2 and they make angles of measure 60° , 90° respectively but on different sides from the line of action of \vec{F} , then $F_1 =$

- (a) $2F_2$ (b) $\frac{\sqrt{3}}{2}F_2$ (c) $\frac{2}{\sqrt{3}}F_2$ (d) $\frac{1}{2}F_2$

(18) In the opposite figure :

A vertical force of magnitude 75 newton is resolved into two components, one of them is horizontal of magnitude F_1 and the other is of magnitude F_2 , then $F_2 =$ newton.

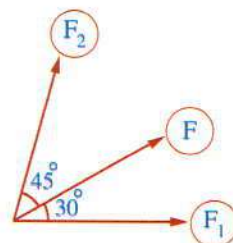


- (a) 75 (b) $75\sqrt{3}$
 (c) 150 (d) $150\sqrt{3}$

(19) In the opposite figure :

The force \vec{F} is the resultant of the two forces \vec{F}_1 , \vec{F}_2 , then $\frac{F_1 + F_2}{F} =$

- (a) $\sin 30^\circ + \sin 45^\circ$ (b) $\frac{\sin 75^\circ + \sin 30^\circ}{\sin 75^\circ}$
 (c) $\frac{\sin 45^\circ + \sin 30^\circ}{\sin 75^\circ}$ (d) $\frac{\sin 75^\circ}{\sin 30^\circ} + \frac{\sin 75^\circ}{\sin 45^\circ}$



- (20) ABCDEF is a regular hexagon. A force of magnitude 20 newton acts in direction of \overrightarrow{AD} , then the magnitudes of the components of the force in direction of \overrightarrow{AC} , \overrightarrow{AF} respectively are

(a) $10\sqrt{3}$, 10 (b) $5\sqrt{3}$, 10 (c) 10, $10\sqrt{3}$ (d) $20\sqrt{3}$, 20

- (21) In the opposite figure :

The force \vec{F} has been resolved into two components

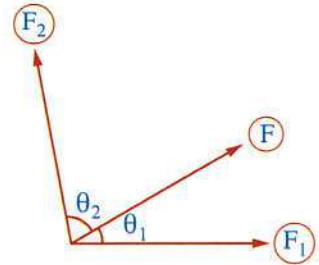
\vec{F}_1 , \vec{F}_2 , then $\frac{F_1}{F_2} = \dots\dots\dots$

(a) $\frac{\sin \theta_2}{\sin \theta_1}$

(b) $\sin \left(\frac{\theta_2}{\theta_1} \right)$

(c) $\sin (\theta_1 + \theta_2)$

(d) $\frac{\sin \theta_1}{\sin \theta_2}$



- (22) In the opposite figure :

ABCDEF is a regular hexagon. Force of magnitude

15 N. acts along \overrightarrow{AC} and it has been resolved into two components \vec{F}_1 and \vec{F}_2 as shown in the figure

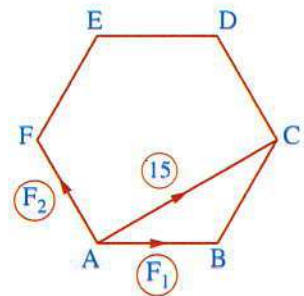
$F_1 : F_2 = \dots\dots\dots$

(a) $\sqrt{3} : 2$

(b) 2 : 1

(c) 1 : 2

(d) $1 : \sqrt{3}$



- (23) In the opposite figure :

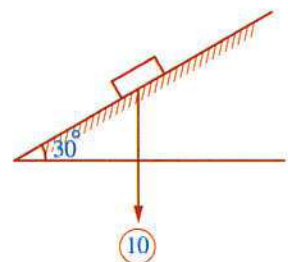
If a body of weight 10 newtons is placed on a smooth plane inclined to the horizontal at an angle of measure 30° , then the component of the weight in direction of line of the greatest slope downward = N.

(a) $5\sqrt{2}$

(b) $5\sqrt{3}$

(c) 5

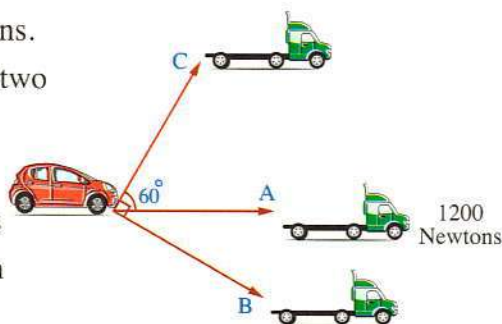
(d) $10\sqrt{3}$



- (24) If a body of weight (W) is placed on a smooth plane inclined to horizontal by angle (θ), so the component of its weight in direction of the plane equals
- (a) W (b) $W \sin \theta$ (c) $W \cos \theta$ (d) $W \tan \theta$
- (25) If a body of weight (W) is placed on an inclined smooth plane makes an angle of measure (θ) with the horizontal, then its weight component in the perpendicular direction of the plane is
- (a) $W \sin \theta$ (b) $W \cos \theta$ (c) $W \tan \theta$ (d) $W \csc \theta$
- (26) If a body of weight (W) is placed on an inclined smooth plane makes an angle of measure (θ) with the vertical, then its weight component in direction of the plane is
- (a) $W \sin \theta$ (b) $W \cos \theta$ (c) W (d) $W \tan \theta$
- (27) A body of weight (W) newton is placed on an inclined plane makes an angle of measure (θ) with the horizontal, then the components of its weight in direction line of greatest slope and its perpendicular are 7, 24 newton respectively, then the magnitude of the weight (W) = newton.
- (a) 7 (b) 24 (c) 25 (d) 31

- (28) A tractor drags a car with a force 1200 newtons.

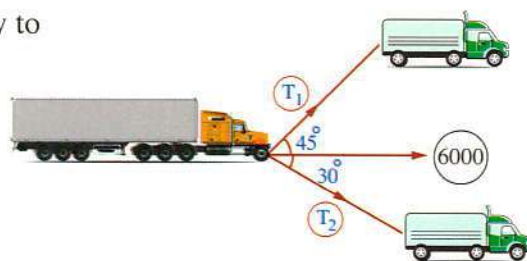
It's required to replace the tractor by another two tractors at B and C attached with two cables to the car and the angle between the two cables is 90° . If one of the two cables inclined to the tractor A at an angle 60° , then the tensions in the two cables B and C are newtons.



- (a) 600, 600 (b) 800, 400
(c) $600\sqrt{3}$, 600 (d) 700, 500

- (29) A truck has broken down traffic officers try to pull the truck by using two draging cars.

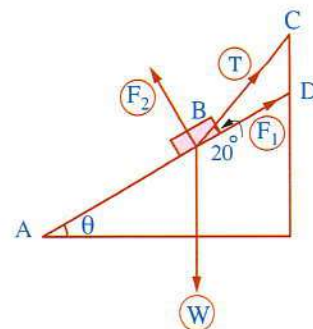
The resultant of their tensions is a horizontal tension of magnitude 6000 newtons as shown in the figure then $T_2 = \dots\dots\dots$ to the nearest newton.



- (a) 3105 (b) 3606
(c) 4392 (d) 4293

(30) In the opposite figure :

A body of weight (W) newtons is placed on a plane inclined to the horizontal at an angle of measure (θ). It is tied by a light string \overline{BC} inclined to the plane at an angle of measure 20° above the plane. F_1 and F_2 are the components of the tension in direction of the plane and perpendicular to the plane then.....



(a) $F_2 = T \cos \theta$

(b) $F_1 = T \sin (20^\circ + \theta)$

(c) $F_1 = T \cos (20^\circ + \theta)$

(d) $T = F_1 \sec 20^\circ$


Second Essay questions

- 1 A force of magnitude 600 kg. wt. acts on a particle. Find its two components in two directions making with the force two angles of measures 30° and 45° « 439.23 , 310.68 gm.wt. »
- 2 A force of magnitude 100 gm.wt. acts in the direction of Western North. Find its components in the North direction and in West direction. « $50\sqrt{2}$, $50\sqrt{2}$ gm. wt. »
- 3 A force of magnitude 12 kg. wt. acting in the direction of Eastern North was resolved into two components. One in the direction of East and the other in the direction of Western North. Find these two components. « $12\sqrt{2}$, 12 kg.wt. »
- 4 Resolve a horizontal force of magnitude 160 gm.wt. in two perpendicular directions. One of them inclined to the horizontal with an angle of measure 30° upwards. « $80\sqrt{3}$, 80 gm.wt. »
- 5 A force of magnitude 300 dyne. acts in the North direction. Find the magnitudes of the two perpendicular components if one of them acts in the direction 30° North of East. « 150 , $150\sqrt{3}$ dyne »
- 6 A force of magnitude 18 newton acts in the direction of South. Find its two components in the two directions 60° East of the South and the other direction towards 30° West of the South. « 9 , $9\sqrt{3}$ newton »
- 7 Resolve a force of magnitude 90 newton into two equal forces in magnitude and the measure of the angle between their lines of action is 60° « $30\sqrt{3}$ newton »

- 8** A body of weight 80 newton is placed on a horizontal plane. Find the two perpendicular components of the weight if one of them inclines to the horizontal with 30° downwards.
- « 40 , $40\sqrt{3}$ newton »

- 9** Two forces act at a point. α is the angle between them and $\tan \alpha = -\frac{1}{\sqrt{3}}$,
If their resultant is perpendicular to the smaller force and the greater force 30 newton. Find the magnitude of the other force and the resultant.
- « $15\sqrt{3}$, 15 newton »

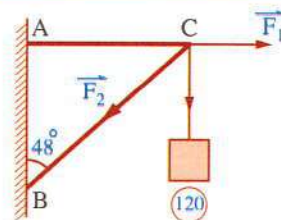
- 10** Resolve a force of magnitude F newton in the North direction into two components , the first in the direction 30° North of East with magnitude 40 newton and the other is in the West direction. Find each of the magnitude of the force F and the magnitude of the other component.
- « 20 , $20\sqrt{3}$ newton »

- 11**  A rigid body of weight 42 newton is placed on a plane inclined to the horizontal with an angle of measure 60° . Find the two components of the weight of the body in the direction of the line of the greatest slope and the direction normal to it. « $21\sqrt{3}$, 21 newton »

- 12** A body of weight 60 newton is placed on an inclined plane , at an angle of measure θ where $\tan \theta = \frac{3}{4}$, find the magnitudes of the two components of the weight in the direction of the line of greatest slope of the plane and the perpendicular to it.
- « 36 , 48 newton »

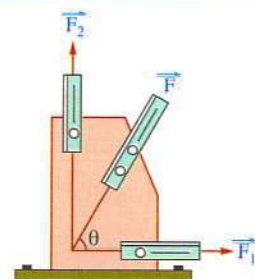
- 13**  **In the opposite figure :**

Resolve the vertical force of magnitude 120 gm.wt. into two components, one of them in the horizontal direction and the other inclined by an angle of measure 48° with the line of action of the force.



« 133.27 , 179.34 gm.wt. »

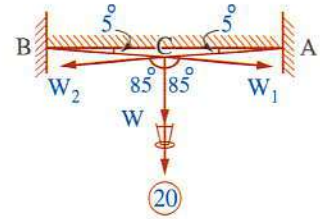
- 14** The opposite figure represents an angle of a bridge , the force \vec{F} of magnitude 30 newton is resolved into two perpendicular components , the magnitude of one of them is $15\sqrt{3}$ newton Find the magnitude of the other component.



« 15 newton »

15  In the opposite figure :

A lamp of weight 20 newton suspended by two metal rods \overline{AC} , \overline{BC} inclined to the horizontal by two equal angles, the measure of each is 5° :



- (1) Resolve the weight of the lamp into two components in the directions \overrightarrow{AC} , \overrightarrow{BC} approximating the result to the nearest newton.
- (2) What happens to the magnitude of the components of the weight in the directions of the two metal rods if the measure of the inclination angle to the horizontal decreased to be smaller than 5° ? And what do you expect to the components when the rods become horizontal ? Justify your answer.

« 114.74 , 114.74 newton »

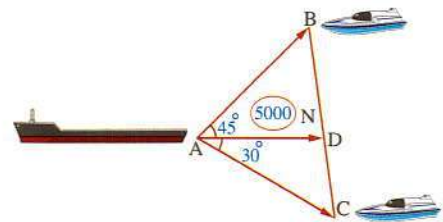
16  An inclined plane of length 130 cm. and height 50 cm. a rigid body of weight

390 gm.wt. is placed on it. Find the two components of the weight in the direction of the line of greatest slope of the plane and the perpendicular to it.

« 150 , 360 gm.wt. »

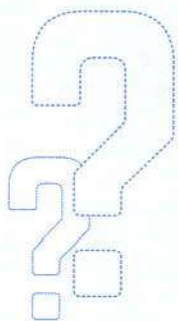
17  In the opposite figure :

A cruiser is pulled by two ships B and C using two strands hanged to a point A on the cruiser, the measure of the angle between the two strands equals 75° , if the measure of the angle between one of the strands and \overrightarrow{AD} equals 45° and the resultant of the forces used to pull the cruiser equals 5000 newton and acts on \overrightarrow{AD}



Find the tension in the two strands.

« 2588.2 , 3660.3 newton »



Exercise

3

The resultant of coplanar forces meeting at a point

From the school book

Remember Understand Apply Higher Order Thinking Skills



Test yourself



First Multiple choice questions

Choose the correct answer from those given :

(where \hat{i} and \hat{j} are the two fundamental unit vectors in two perpendicular directions)

- (1) If $\vec{F}_1 = \hat{i} - \hat{j}$, $\vec{F}_2 = 2\hat{i} - 4\hat{j}$, $\vec{R} = 2a\hat{i} - 3b\hat{j}$, then $a + b = \dots\dots\dots$

(a) 3 (b) $3\frac{1}{3}$ (c) $3\frac{1}{6}$ (d) 12
- (2) If $\vec{F}_1 = 3\hat{i} - 2\hat{j}$, $\vec{F}_2 = a\hat{i} - \hat{j}$, $\vec{F}_3 = 4\hat{i} - b\hat{j}$, $\vec{R} = 6\hat{i} - 4\hat{j}$, then $(a, b) = \dots\dots\dots$

(a) (1, -1) (b) (-1, 1) (c) (-1, -1) (d) (1, 1)
- (3) If $\vec{F}_1 = 4\hat{i}$, $\vec{F}_2 = 8\hat{i} - 5\hat{j}$, then $\|\vec{R}\| = \dots\dots\dots$ force unit.

(a) 12 (b) 5 (c) 13 (d) $\sqrt{73}$
- (4) If $\vec{F}_1 = 3\hat{i} + 2\hat{j}$, $\vec{F}_2 = a\hat{i} + 7\hat{j}$, $\vec{F}_3 = -12\hat{i} + b\hat{j}$ are three coplanar forces meeting at a point and the resultant $\vec{R} = (6\sqrt{2}, \frac{3}{4}\pi)$, then $a - b = \dots\dots\dots$

(a) -3 (b) 3 (c) zero (d) 6
- (5) Three coplanar forces $\vec{F}_1 = 6\hat{i} + 7\hat{j}$, $\vec{F}_2 = a\hat{i} - 9\hat{j}$, $\vec{F}_3 = 5\hat{i} + b\hat{j}$ act at a particle and they are in equilibrium , then $a + 2b = \dots\dots\dots$

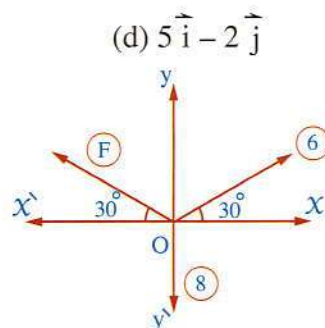
(a) -9 (b) 5 (c) 7 (d) -7

- (6) If \vec{F}_1 , \vec{F}_2 and \vec{F}_3 are three coplanar equilibrium forces meeting at a point, and $\vec{F}_1 = 2\hat{i} - 3\hat{j}$, $\vec{F}_2 = 3\hat{i} + 5\hat{j}$, then $\vec{F}_3 = \dots\dots\dots$

(a) $-5\hat{i} - 2\hat{j}$ (b) $-5\hat{i} + 2\hat{j}$ (c) $5\hat{i} + 2\hat{j}$ (d) $5\hat{i} - 2\hat{j}$

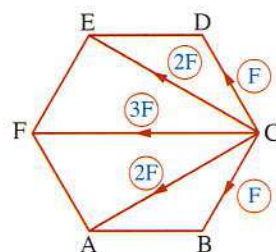
- (7) If the resultant of the forces in the given figure acts in direction of y-axis, then $F = \dots\dots\dots$ force unit.

(a) 2 (b) 6
(c) 8 (d) 14



- (8) The resultant of the forces in the opposite figure acts in direction of $\dots\dots\dots$

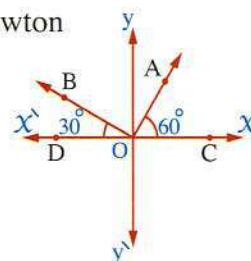
(a) \vec{CD} (b) \vec{CE}
(c) \vec{CF} (d) \vec{CA}



- (9) In the opposite figure :

The magnitude of four coplanar forces are 1, 2, $4\sqrt{3}$, $3\sqrt{3}$ newton act at point O in the direction of \vec{OX} , \vec{OA} , \vec{OB} and \vec{OY} , $m(\angle AOC) = 60^\circ$, $m(\angle BOD) = 30^\circ$, then the magnitude and the direction of the resultant of the forces is $\dots\dots\dots$

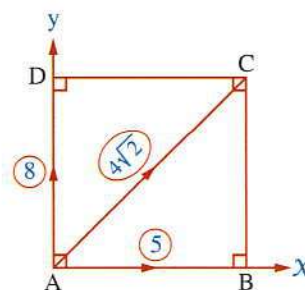
(a) (4, 180°) (b) (4, 0°)
(c) (3, 0°) (d) (5, 90°)



- (10) In the opposite figure :

ABCD is a square, the forces of magnitudes 5, 8, $4\sqrt{2}$ newton act on \vec{AB} , \vec{AD} and \vec{AC} respectively, then the polar form of the resultant is $\dots\dots\dots$

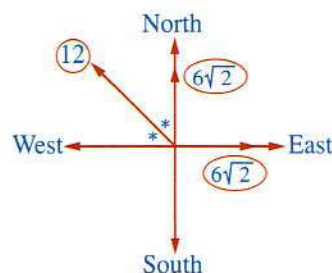
(a) (5, 54°) (b) (15, 60°)
(c) (15, $53^\circ 8'$) (d) (13, 90°)



- (11) In the opposite figure :

The direction of the resultant of the forces is $\dots\dots\dots$

(a) South. (b) East.
(c) West. (d) North.



(12) In the opposite figure :

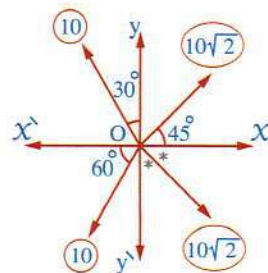
The magnitude of the resultant of the forces (R) = newton.

(a) 20

(b) $10\sqrt{2}$

(c) 10

(d) zero



(13) In the opposite figure :

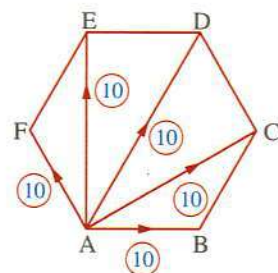
Five equal forces each of magnitude 10 newton act at one vertex of a regular hexagon and in direction of the other vertices of the hexagon, then the magnitude of the resultant of these forces = newton.

(a) 50

(b) 20

(c) $30\sqrt{3}$

(d) $20 + 10\sqrt{3}$



(14) In the opposite figure :

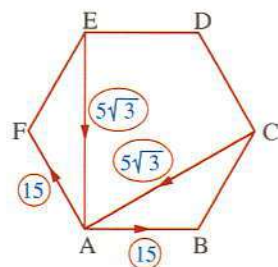
ABCDEF is a regular hexagon, the forces of magnitudes 15, $5\sqrt{3}$, $5\sqrt{3}$, 15 newton act on \overrightarrow{AB} , \overrightarrow{CA} , \overrightarrow{EA} , \overrightarrow{AF} respectively, then the magnitude of their resultant = newton.

(a) 5

(b) 10

(c) 25

(d) zero



(15) In the opposite figure :

ABCDEF is a regular hexagon, forces of magnitudes 2, $4\sqrt{3}$, 8, $2\sqrt{3}$ and 4 kg.wt. act at point A in directions \overrightarrow{AB} , \overrightarrow{AC} , \overrightarrow{AD} , \overrightarrow{AE} and \overrightarrow{AF} respectively.

First : The magnitude of their resultant = kg.wt.

(a) $14 + 6\sqrt{3}$

(b) 20

(c) $20\sqrt{3}$

(d) $20 + \sqrt{3}$

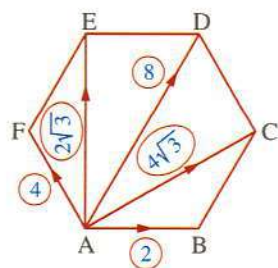
Second : The direction of the resultant inclined by an angle of measure with \overrightarrow{AB}

(a) 30°

(b) 45°

(c) 60°

(d) 90°



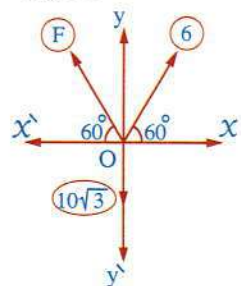
(16) If the resultant of the forces represented in the opposite figure acts in X-axis, then F = newton.

(a) 10

(b) 14

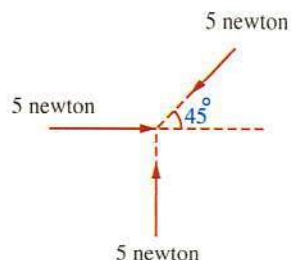
(c) 18

(d) 6



- (17) The opposite figure represents some of forces meeting at a point , then the magnitude of the resultant of these forces = newton.

- (a) $15\sqrt{2}$ (b) 5
(c) $5\sqrt{2} - 5$ (d) zero



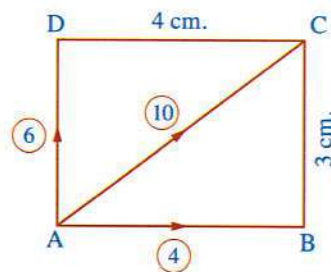
- (18) Three coplanar forces meeting at a point , their magnitudes are 40 , 30 , 40 newton , the first is in direction 60° West of North , the second is towards West and the third in the direction 30° North of East , then the magnitude of their resultant equal newton.

- (a) 30 (b) 110
(c) 60 (d) 50

- (19) In the opposite figure :

ABCD is a rectangle $AB = 4$ cm. , $BC = 3$ cm.
forces 4 N , 10 , 6 N acts along \overrightarrow{AB} , \overrightarrow{AC} , \overrightarrow{AD} respectively. The resultant of these forces makes with \overrightarrow{AB} an angle of measure

- (a) 45° (b) 60°
(c) 30° (d) $\sin^{-1}\left(\frac{3}{5}\right)$



- (20) ABCD is a right trapezium at A and D , in which $AD = CD = 4$ cm. , $AB = 7$ cm. , $M \in \overline{AB}$ where $AM = 4$ cm. , a set of forces their magnitudes 25 , F and $15\sqrt{2}$ gm.wt. act at \overrightarrow{CB} , \overrightarrow{CM} and \overrightarrow{CA} respectively and the norm of the resultant of these forces equals 45 gm.wt. , then the value of F = gm.wt.

- (a) 10 (b) 50
(c) 20 (d) 30

- (21) The forces of magnitudes F , 12 , $8\sqrt{2}$, $10\sqrt{2}$, k newton act on a particle in the directions of East , North , Western North , Western South and South respectively. If the magnitude of the resultant = 4 newton due to North , then $F - K =$ newton

- (a) 24 (b) 27
(c) 12 (d) 6

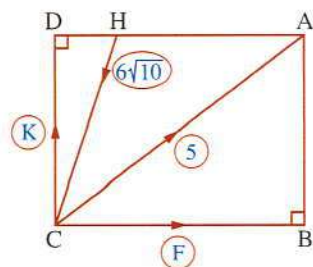
(22) In the opposite figure :

The forces of magnitude F , 5 , K and $6\sqrt{10}$ N act in the rectangle ABCD in the directions \vec{CB} , \vec{CA} , \vec{CD} , \vec{HC}

Such that : $AB = 6$ cm. , $BC = 8$ cm. , $AH = 6$ cm.

If these forces are in equilibrium , then $K = \dots\dots\dots$ newton.

- (a) 12 (b) 15 (c) 18 (d) 20



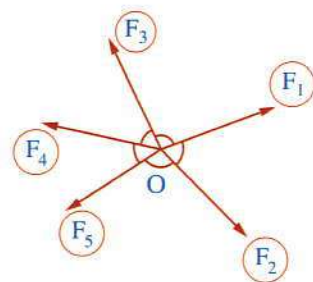
(23) The coplanar forces of magnitudes 5 , 4 , F , 3 , k , 7 kg.wt. act at a particle and the measure of the angle between each two consecutive forces is 60° , if the system is in equilibrium , then $F + 2K = \dots\dots\dots$ kg.wt.

- (a) 21 (b) 6 (c) 9 (d) 15

(24) The opposite figure represents a set of forces meeting at a point (O)

Mohamed took (O) as an origin of coordinate system and the positive direction of X -axis in direction of \vec{F}_1

The magnitude of the resultant was R_1 and made angle of measure (θ_1) with the positive direction of X -axis and Ebrahim took (O) as an origin of coordinate system and the positive direction of X -axis in direction of \vec{F}_2 , the magnitude of the resultant was R_2 and made an angle of measure (θ_2) with the positive direction of X -axis , then $\dots\dots\dots$

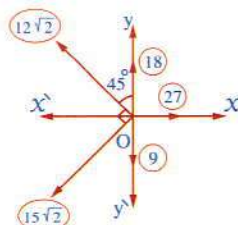


- (a) $R_1 = R_2$, $\theta_1 = \theta_2$ (b) $R_1 = R_2$, $\theta_1 \neq \theta_2$
 (c) $R_1 \neq R_2$, $\theta_1 = \theta_2$ (d) $R_1 \neq R_2$, $\theta_1 \neq \theta_2$

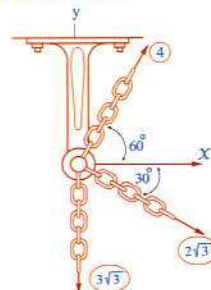
Second Essay questions



1 Find the resultant (magnitude and direction) of the set of forces in each of the following figures (where each force magnitude is in newton) :

(1)



(2)



- 2** Three coplanar forces of magnitudes 1, 2, $\sqrt{3}$ newton act at M, their directions are \overrightarrow{MA} , \overrightarrow{MB} and \overrightarrow{MC} respectively where $m(\angle AMB) = 60^\circ$, $m(\angle BMC) = 30^\circ$, $m(\angle AMC) = 90^\circ$, find the resultant.
« 4 newton, in direction of \overrightarrow{MB} »
- 3** The forces 8, $4\sqrt{3}$, $6\sqrt{3}$ and 14 newton act at a point, the measure of the angle between the first force and the second force is 30° , between the second and the third is 120° and between the third and the fourth is 90° taken in the same cyclic order. Find the magnitude and direction of the resultant of these forces.
« 4 newton, in direction of 4th force »
- 4** The coplanar forces of magnitudes 2, $3\sqrt{2}$, $2\sqrt{3}$ and $\sqrt{3}$ newton act at a point. If the measures between the first force and the second force is 45° , the measure between the second and the third is 105° and the measure between the third and the fourth is 120° taken in the same cyclic order, find the resultant of these forces.
« $\sqrt{13}$ newton, $11^\circ 19'$ with 2nd force »
- 5** Five coplanar forces meeting at a point, their magnitudes are 9, 6, $4\sqrt{2}$, $5\sqrt{2}$ and 5 newton act due to East, North, Western North, Western South and in the direction of South respectively. Prove that the set of forces are in equilibrium.
- 6** Three coplanar forces of magnitudes 60, 88 and 60 gm.wt. act at a point, the 1st is towards North, the second is in the direction 30° South of West and the 3rd in the direction 30° South of East. Find the magnitude of the resultant of these forces and its direction.
« 28 gm.wt., 30° South of West »
- 7**  Four coplanar forces act on a particle the first of magnitude 4 newton acts in the Eastern direction, the second of magnitude 2 newton, acts in direction 60° North of the East, the third of magnitude 5 newton, acts in direction 60° North of the West and the fourth of magnitude $3\sqrt{3}$ newton acts in direction 60° West of the South. Find the magnitude and direction of their resultant.
« 4 N., 120° »
- 8** The forces of magnitudes 2 F, 3 F and 4 F newton act on a particle in the directions parallel to the sides of an equilateral triangle in the same cyclic order. Find the magnitude and the direction of the resultant of these forces.
« $\sqrt{3}$ F newton, perpendicular to the force 3 F »
- 9**  ABC is an equilateral triangle. M is the point of intersection of its medians. the forces of magnitude 15, 20 and 25 newton act on a particle at the point M in the directions of \overrightarrow{MC} , \overrightarrow{MB} , \overrightarrow{MA} . Find the magnitude and the direction of the resultant of these forces.
« $5\sqrt{3}$ newton, 30° with \overrightarrow{MA} »

- 10** ΔABC is an isosceles triangle where $m(\angle BAC) = 120^\circ$, the forces of magnitudes 4, $6\sqrt{3}$, 4 newton act at A in the directions \overrightarrow{AB} , \overrightarrow{CB} , \overrightarrow{CA} respectively.
Find the magnitude and the direction of the resultant of these forces.
« $10\sqrt{3}$ newton in the direction of \overrightarrow{CB} »
- 11** Four coplanar forces of magnitude 2, 1, 4 and $3\sqrt{3}$ N. act at a point A in directions of \overrightarrow{BC} , \overrightarrow{BA} , \overrightarrow{CA} and \overrightarrow{AD} where ABC is an equilateral triangle and D is the midpoint of \overline{BC} .
Find the magnitude and direction of their resultant.
« 1 newton in the direction of \overrightarrow{AC} »
- 12** ABCD is a rectangle where $AB = 4$ cm., $BC = 3$ cm. the forces of magnitudes 2, 5 and 3 kg.wt. act at the point A in the directions \overrightarrow{AB} , \overrightarrow{AC} and \overrightarrow{AD} respectively.
Find the resultant of these forces and the measure of its angle of inclination on \overrightarrow{AB} .
« $6\sqrt{2}$ kg.wt., 45° »
- 13** ABCD is a rectangle in which $AB = 8$ cm., $BC = 6$ cm., $E \in \overline{CD}$ where $ED = 6$ cm., a set of forces their magnitudes 12, 40, $26\sqrt{2}$ and 4 newton act at \overrightarrow{AB} , \overrightarrow{CA} , \overrightarrow{AE} and \overrightarrow{AD} respectively.
Find the magnitude and the direction of the resultant of these forces.
« $6\sqrt{2}$ newton, 45° with \overrightarrow{AB} »
- 14** ABCD is a rectangle in which : $AB = 21$ cm., $BC = 9$ cm. The point $O \in \overline{AB}$ where $AO = 9$ cm. four forces of magnitudes 4, 10, 6 and $12\sqrt{2}$ kg.wt. act at the point O in the directions \overrightarrow{OB} , \overrightarrow{OC} , \overrightarrow{BC} and \overrightarrow{OD} respectively.
Find the magnitude of the resultant of these forces and prove that it is parallel to \overrightarrow{BC} .
« 24 kg.wt. »
- 15** ABCDEF is a regular hexagon, the forces of magnitudes 8, $6\sqrt{3}$, 5, $4\sqrt{3}$ newton act on \overrightarrow{AB} , \overrightarrow{AC} , \overrightarrow{AD} and \overrightarrow{AE} respectively. Find the magnitude and the direction of their resultant.
« $\sqrt{651}$ newton, 40° with \overrightarrow{AB} »
- 16** ABCDHE is a regular hexagon. Forces of magnitudes 2, $4\sqrt{3}$, 8, $2\sqrt{3}$ and 4 kg.wt. act at point A in directions \overrightarrow{AB} , \overrightarrow{AC} , \overrightarrow{AD} , \overrightarrow{AH} , \overrightarrow{AE} respectively.
Find the magnitude and the direction of their resultant.
« 20 kg.wt., 60° with \overrightarrow{AB} »
- 17** ABCDEF is a regular hexagon. M is the point of intersection of its diagonals. the forces of magnitudes 4, 1, 4, 5, 2 and 3 gm.wt. act at M in the directions of \overrightarrow{MA} , \overrightarrow{MB} , \overrightarrow{MC} , \overrightarrow{MD} , \overrightarrow{ME} and \overrightarrow{MF} .
Find the resultant of these forces and prove that it is in the direction of \overrightarrow{MD} .
« 2 gm.wt. »

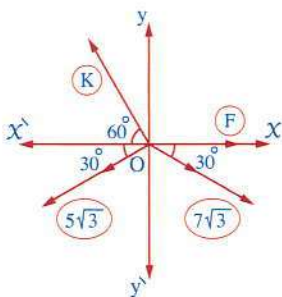
- 18** ABC is a right-angled triangle at B where $AB = 80$ cm. , $BC = 60$ cm. , $D \in \overline{AC}$ where $BD = DC$
The four forces of magnitudes 8 , 12 , 15 and 10 newton act at the point B in the directions \overrightarrow{AB} , \overrightarrow{BC} , \overrightarrow{CA} and \overrightarrow{BD} respectively.
Find the resultant of these forces and prove that it acts in \overrightarrow{BD} « 15 newton »
-
- 19** ABCD is a square of side length is 12 cm. $H \in \overline{BC}$ where $BH = 5$ cm.
forces of magnitudes 2 , 13 , $4\sqrt{2}$, 9 gm.wt. act in directions of \overrightarrow{AB} , \overrightarrow{AH} , \overrightarrow{CA} and \overrightarrow{AD} respectively.
Find the magnitude of the resultant of these forces. « $10\sqrt{2}$ gm.wt. in direction of \overrightarrow{AC} »
-
- 20** ABCD is a square of side length 6 cm. The point E is the midpoint of \overline{BC} and F is the midpoint of \overline{DC} , the five forces of magnitudes 2 , $12\sqrt{5}$, $6\sqrt{2}$, $4\sqrt{5}$ and 4 kg.wt. act at the point A in the directions of \overrightarrow{AB} , \overrightarrow{AE} , \overrightarrow{CA} , \overrightarrow{AF} and \overrightarrow{AD} respectively.
Find the magnitude and the direction of the resultant of these forces. « 30 kg.wt. , $36^\circ 52' 12''$ »
-
- 21** ABCD is a square , $E \in \overline{AD}$, four forces of magnitudes 4 , $4\sqrt{3}$, $10\sqrt{2}$, F kg.wt. act at point B in the directions \overrightarrow{BA} , \overrightarrow{BE} , \overrightarrow{DB} , \overrightarrow{BC} , if these forces are in equilibrium , find $m(\angle ABE)$ and the value of F « 30° , $2(5 - \sqrt{3})$ kg.wt. »
-
- 22** The coplanar forces of magnitudes 5 , 4 , F , 3 , K and 7 kg.wt. act at a particle and the measure of the angle between each two consecutive forces is 60°
Find the magnitude of F and K that makes the system in equilibrium. « 9 , 6 kg.wt. »
-
- 23** The forces of magnitudes F , 6 , $4\sqrt{2}$, $5\sqrt{2}$, K newton act on a particle in the directions of East , North , Western North , Western South and South respectively.
Find the values of F and K if the magnitude of the resultant = 2 newton due to North. « 9 , 3 newton »
-
- 24** Forces of magnitudes F , $4\sqrt{3}$, $12\sqrt{3}$, 36 gm.wt. act at a particle. The last three forces are in the directions of North , 60° West of North , 60° South of East respectively. If the resultant of these four forces = 8 gm.wt. in magnitude in the direction of East.
Determine the value of F and its direction. « 16 gm.wt. , 60° North of East »
-
- 25** The forces of magnitudes F , 8 , K , 5 , $8\sqrt{3}$ newton act at a point in the directions of : East , 30° East of North , North , West and South respectively.
Find the values of F and K if the resultant is 4 newton in magnitude in the direction of 60° North of East. « 3 , $6\sqrt{3}$ newton »

- 26** ABCD is a right trapezium at A and D, in which $AD = CD = 40$ cm., $AB = 70$ cm., $M \in \overline{AB}$ where $AM = 40$ cm., a set of forces their magnitudes 25 , F , $10\sqrt{2}$ and 35 gm.wt. act at \overrightarrow{CB} , \overrightarrow{CM} , \overrightarrow{CA} and \overrightarrow{CD} respectively and the norm of the resultant of these forces equals 50 gm.wt. Find F

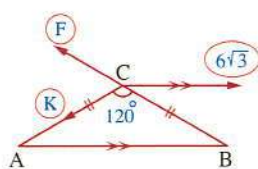
« $F = 10$ gm.wt. »

- 27** In each of the following figures find the magnitudes of F and K in newton that makes the system in equilibrium :

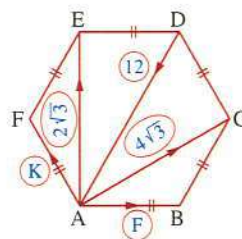
(1)



(2)



(3)



- 28** Coplanar forces of magnitudes F , $3\sqrt{2}$, $2\sqrt{3}$ and $\sqrt{3}$ newton act on a particle. The first force acts in the east direction. The angle between the first and the second force is of measure 45° , the angle between the second and the third force is of measure 105° , the angle between the third and the fourth force is of measure 120° . If the magnitude of their resultant is $3\sqrt{2}$ newton, then find the value of F and measure of the angle between the resultant and the first force.

« 3 newton, 45° »

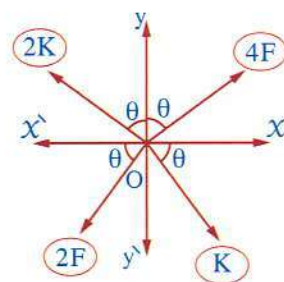
- 29** ABCDEF is a regular hexagon. Forces of magnitudes 4 , $2\sqrt{3}$, F , $2\sqrt{3}$ and K kg.wt. act in the directions of \overrightarrow{AB} , \overrightarrow{AC} , \overrightarrow{AD} , \overrightarrow{AE} and \overrightarrow{AF} respectively. If the resultant of these forces is of magnitude 20 kg.wt. in the direction of \overrightarrow{AD} Find the values of F , K

« 10 , 4 kg.wt. »

- 30** In the opposite figure :

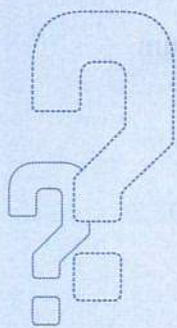
Four coplanar forces act at the point (O) in the directions shown in the figure where $\sin \theta = \frac{4}{5}$ and the resultant of these forces is $8\sqrt{2}$ N. and makes an angle of measure 135° with \overrightarrow{OX} , then find the values of F , K

« 3 , 14 newton »



- 31** If $\vec{F}_1 = 5\vec{i} + 3\vec{j}$, $\vec{F}_2 = a\vec{i} + 6\vec{j}$, $\vec{F}_3 = -14\vec{i} + b\vec{j}$ are three coplanar forces meeting at a point and their resultant is $\vec{R} = (10\sqrt{2}, 135^\circ)$, then find the values of a and b

« $a = -1$, $b = 1$ »



Exercise

4

Equilibrium of a rigid body under the effect of two forces / three forces meeting at a point
(The triangle of forces rule - Lami's rule)



From the school book

Remember Understand Apply Higher Order Thinking Skills



Test yourself

First Multiple choice questions

Choose the correct answer from the given ones :

- (1) If three forces meeting at a point and acting up on a particle are in equilibrium , then the magnitude of each force is proportional to the of the included angle between the other two forces.
(a) cosine (b) sine (c) tangent (d) cotangent
- (2) If a body is in equilibrium under action of two forces \vec{F}_1, \vec{F}_2 , then
(a) $\vec{F}_1 = \vec{F}_2$ (b) $F_1 = F_2$
(c) $\vec{F}_1 + \vec{F}_2 \neq 0$ (d) \vec{F}_1, \vec{F}_2 are not on the same line.
- (3) If a body is kept in equilibrium under action of several forces , then the least number of forces could cause equilibrium equals
(a) 1 (b) 2 (c) 3 (d) 4
- (4) The least number of coplanar unequal in magnitude forces could be in equilibrium is
(a) 1 (b) 2 (c) 3 (d) 14
- (5) If \vec{F}_1, \vec{F}_2 and \vec{F}_3 are three forces meeting at a point and they are in equilibrium , then the magnitude of the resultant of \vec{F}_1 and $\vec{F}_2 =$
(a) F_1 (b) $F_1 + F_2$ (c) F_3 (d) zero

- (6) Three equal forces in magnitude meeting at a point and they are in equilibrium, then the measure of the angle between each two forces =
- (a) 60° (b) 90° (c) 120° (d) 150°
- (7) If \vec{F} is in equilibrium with two perpendicular forces of magnitudes 8 newton and 15 newton, then $F = \dots\dots\dots$ newton.
- (a) 7 (b) 17 (c) 23 (d) $7\sqrt{2}$
- (8) If a force of magnitude (F) is in equilibrium with two forces of magnitudes 5 and 3 newton and the measure of the angle between them is 60° , then $F = \dots\dots\dots$ newton.
- (a) $\sqrt{19}$ (b) $\sqrt{34}$ (c) 7 (d) 6
- (9) Which of the following sets of forces could be in equilibrium?
- ① 8 newton, 8 newton, 8 newton.
 ② 8 newton, 8 newton, 16 newton.
 ③ 8 newton, 8 newton, 20 newton.
- (a) ① only. (b) ② only. (c) ①, ② (d) ②, ③
- (10) Which of the following systems of forces could not be in equilibrium?
- (a) 10 newton, 10 newton, 5 newton (b) 4 newton, 6 newton, 10 newton
 (c) 11 newton, 7 newton, 8 newton (d) 8 newton, 4 newton, 14 newton
- (11) Three coplanar forces not on the same straight line meeting at a point are in equilibrium, the magnitude of two forces of them are 7 and 3 newton, then the magnitude of the third could be
- (a) 10 (b) 4 (c) 5 (d) 3
- (12) Three coplanar forces are in equilibrium act at a particle, the measure of the angle between the first two forces is 60° , and between the second and third forces is 150° then the ratio between forces is
- (a) $1 : 1 : \sqrt{3}$ (b) $1 : 2 : \sqrt{3}$ (c) $\sqrt{2} : \sqrt{3} : 1$ (d) $\sqrt{3} : \sqrt{3} : 1$
- (13) The force which is in equilibrium with two perpendicular forces F , F newton makes with one of the two forces an angle of measure
- (a) 90° (b) 120° (c) 135° (d) 150°
- (14) Three coplanar forces of magnitudes 5, 6, 7 newton act at a particle. If the forces are in equilibrium, then the cosine of the angle between the second and the third force =
- (a) $\frac{7}{5}$ (b) $-\frac{5}{7}$ (c) $\frac{15}{17}$ (d) $\frac{1}{2}$

(15) If a body is in equilibrium under action of three forces as shown in the figure.

Which of the following statements is true ?

① $F_1 + F_2 + F_3 = \text{zero}$ ② $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = \vec{\text{zero}}$

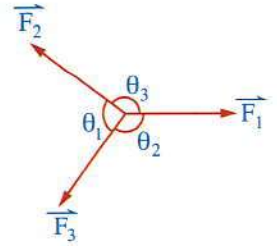
③ $\frac{F_1}{\sin \theta_1} = \frac{F_2}{\sin \theta_2} = \frac{F_3}{\sin \theta_3}$

(a) ① , ② only.

(b) ② , ③ only.

(c) ③ only.

(d) ① , ② , ③



(16) In the opposite figure :

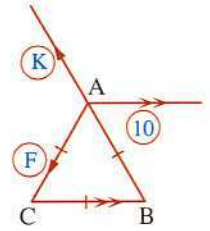
If the three coplanar forces meeting at a point and in equilibrium , then $F = \dots\dots\dots$ N.

(a) 10

(b) 12

(c) 20

(d) 13



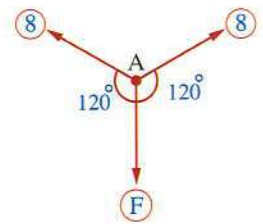
(17) Particle A is balanced under action of three forces as shown in the opposite figure where \vec{F} is balanced with two forces the magnitude of each is 8 newton and makes an angle of measure 120° with each of them , then $F = \dots\dots\dots$ newton.

(a) zero

(b) 8

(c) 16

(d) $8 \sin 120^\circ$



(18) In the opposite figure :

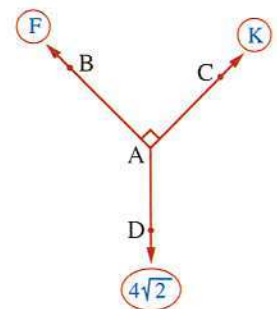
Three equilibrium forces of magnitudes F , k and $4\sqrt{2}$ newton , $m(\angle BAC) = 90^\circ$, $m(\angle BAD) = 135^\circ$, then $(F , K) = \dots\dots\dots$

(a) $(4 , 4)$

(b) $(4 , \sqrt{2})$

(c) $(\sqrt{2} , 4)$

(d) $(2 , 2)$



(19) In the opposite figure :

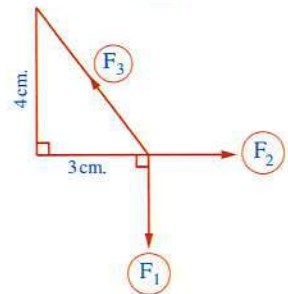
A body is in equilibrium under action of three forces meeting at a point of magnitudes F_1 , F_2 and F_3 newton and the sides of the right-angled triangle are parallel to the lines of action of the forces in the same cyclic order , then $F_1 : F_2 : F_3 = \dots\dots\dots$

(a) $3 : 4 : 5$

(b) $3 : 5 : 4$

(c) $4 : 5 : 3$

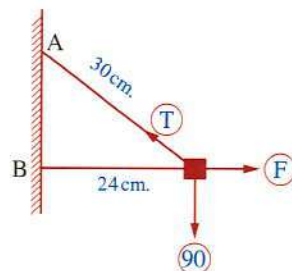
(d) $4 : 3 : 5$



(20) In the opposite figure :

A body of weight 90 gm.wt. is attached to the end of a string of 30 cm. long. The body is pulled by a horizontal force.

It comes to equilibrium when it is 24 cm. apart from the wall \overline{AB} then $T - F = \dots\dots\dots$ gm.wt.



(a) 150

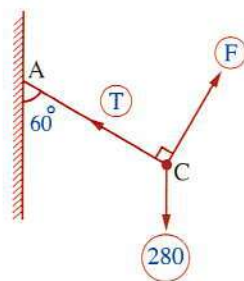
(b) 120

(c) 50

(d) 30

(21) In the opposite figure :

A lamp of weight 280 gm.wt. is attached to the end of a string. It is in equilibrium under the effect of a force perpendicular to the string when it is inclined to the vertical by an angle of measure 60° , then $\frac{F}{T} = \dots\dots\dots$



(a) 2

(b) $\frac{1}{2}$

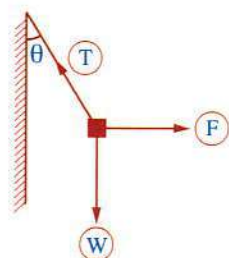
(c) $\frac{1}{\sqrt{3}}$

(d) $\sqrt{3}$

(22) In the opposite figure :

A body of weight (W) newton is suspended from the end of a string. The other end of the string is fixed to a vertical wall.

The body is pulled by a horizontal force of magnitude (F) newton. The body is in equilibrium when the string makes an angle θ to the wall which of the following statements is false in case of equilibrium ?



(a) $F = W \tan \theta$

(b) $\vec{W} + \vec{F} + \vec{T} = \vec{Zero}$

(c) $T^2 = F^2 + W^2$

(d) $T = F + W$

(23) In the opposite figure :

The weight of a body is 20 kg.wt , the body is in

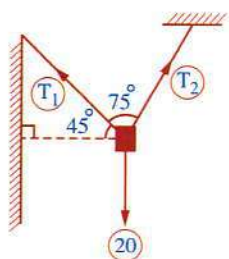
equilibrium then $\frac{T_1}{T_2} = \dots\dots\dots$

(a) $\frac{1}{2}$

(b) $\frac{1}{\sqrt{2}}$

(c) $\frac{2}{3}$

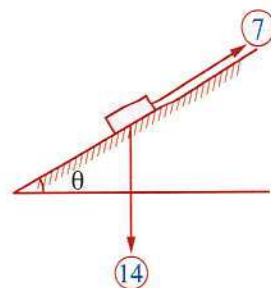
(d) $\frac{\sqrt{3}}{2}$



(24) In the opposite figure :

If the body is in equilibrium when it is placed on an inclined smooth plane , then $m (\angle \theta) = \dots\dots\dots$

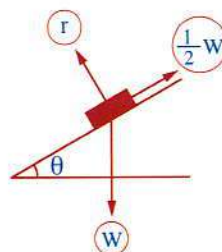
- (a) 60° (b) 45°
(c) 30° (d) 75°



(25) In the opposite figure :

If the body is in equilibrium under action of forces shown , then $m (\angle \theta) = \dots\dots\dots$

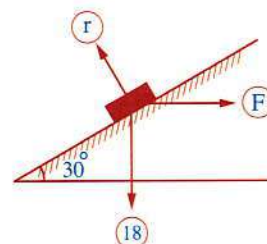
- (a) 30° (b) 60°
(c) 45° (d) 15°



(26) In the opposite figure :

A body of weight 18 newton is placed on a smooth plane inclined to the horizontal at an angle of measure 30° , it is kept in equilibrium by a horizontal force of magnitude F newton , then $F + r = \dots\dots\dots$ newton.

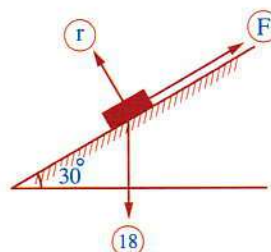
- (a) $6\sqrt{3}$ (b) $12\sqrt{3}$ (c) $18\sqrt{3}$ (d) $24\sqrt{3}$



(27) In the opposite figure :

A body of weight 18 newton is placed on a smooth plane inclined to the horizontal by an angle of measure 30° , it is kept in equilibrium by a force of magnitude F newton in the direction of the plane upward , then $F + R = \dots\dots\dots$ newton.

- (a) $6\sqrt{3}$ (b) $9\sqrt{3}$ (c) $18\sqrt{3}$ (d) $9 + 9\sqrt{3}$



(28) The weight of a body is 6 kg.wt. It is placed on a smooth inclined plane makes an angle 30° to the horizontal and kept in equilibrium by a horizontal force , then the magnitude of this horizontal force = $\dots\dots\dots$ kg.wt.

- (a) $\sqrt{3}$ (b) $2\sqrt{3}$ (c) $4\sqrt{3}$ (d) 6

(29) The weight of a body is 6 newton. It is placed on a smooth inclined plane makes an angle 30° to the horizontal and kept in equilibrium with a force of magnitude 49 newton which makes an angle of measure θ upwards the line of greatest slope of the plane , then $\cos \theta = \dots\dots\dots$

- (a) $\frac{3}{49}$ (b) $\frac{3}{4}$ (c) $\frac{3}{5}$ (d) $\frac{4}{5}$

(30) The weight of a body is 20 kg.wt. It is placed on a smooth inclined plane makes an angle θ to the horizontal, where $\sin \theta = \frac{3}{5}$ and it prevent from sliding by a horizontal force F , then $F = \dots\dots\dots$ kg.wt.

(a) 30

(b) 15

(c) 10

(d) $5\sqrt{3}$

(31) In the opposite figure :

A body of weight (W) is hanged by two strings.

The two strings inclined to the horizontal as shown

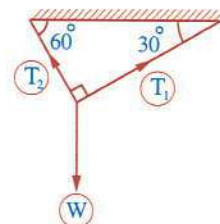
in the figure, then $T_1 = \dots\dots\dots$

(a) $\frac{1}{3} W$

(b) $\frac{1}{2} W$

(c) $\frac{\sqrt{3}}{3} W$

(d) $\frac{\sqrt{3}}{2} W$



(32) In the opposite figure :

A body of weight 150 gm.wt. is in equilibrium by suspending it by two perpendicular strings of lengths 60 cm. and 45 cm., and the other two ends C and B are on a horizontal line, then :

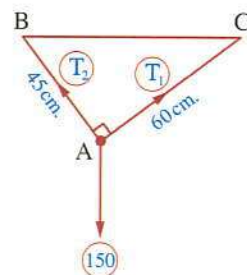
$T_2 - T_1 = \dots\dots\dots$ gm.wt.

(a) 120

(b) 90

(c) 60

(d) 30



(33) A body of weight 28 kg.wt. is suspended by two perpendicular strings, if the measure of the angle between one strings and the line of the weight is 120° , then the magnitude of the tension of this strings equals $\dots\dots\dots$ kg.wt.

(a) 14

(b) 28

(c) $14\sqrt{3}$

(d) $28\sqrt{3}$

(34) In the opposite figure :

A man of weight 70 kg.wt. is walking on a rope.

If the rope lowered 10° from the horizontal when

the man becomes at the middle of the rope, then

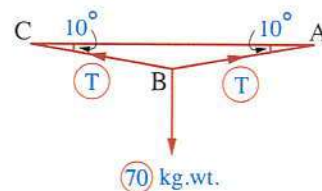
the tension in the rope (T) = $\dots\dots\dots$ kg.wt.

(a) $\frac{70 \sin 20^\circ}{\sin 100^\circ}$

(b) $\frac{70 \sin 100^\circ}{\sin 160^\circ}$

(c) $\frac{70 \cos 100^\circ}{\sin 160^\circ}$

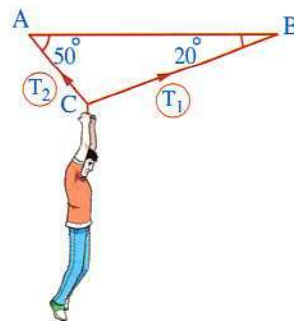
(d) $\frac{\sin 100^\circ}{70 \sin 160^\circ}$



(35) In the opposite figure :

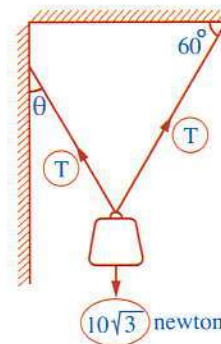
A man of weight (W) suspended vertically at C by two ropes \overline{CB} , \overline{CA} as shown in the figure and $T_2 = 60$ kg.wt. then (W) = kg.wt.

- (a) 87.7 (b) 70.6
(c) 60 (d) 49.8



(36) A body of weight $10\sqrt{3}$ newton is suspended by two strings as shown in the figure, then the value of θ which makes both tensions are equal is

- (a) 15° (b) 30°
(c) 45° (d) 60°



Second Essay questions

- 1 Three forces of magnitudes F_1 , F_2 and 75 newton intersecting at one point they are represented by the line segments \overline{AB} , \overline{BC} and \overline{CA} of ΔABC respectively where : $AB = 3$ cm. , $BC = 4$ cm. and $CA = 5$ cm.

Find the value of each of F_1 and F_2

« 45 , 60 newton »

- 2 Three coplanar forces of magnitudes 60 , F and K newton meeting at a point and in equilibrium. If the angle between the 1st and the 2nd force measures 120° and between the 2nd and the 3rd measures 90°

Find the value of each of F and K

« 30 , $30\sqrt{3}$ newton »

- 3 A body of weight 12 kg.wt. is placed on a smooth plane inclined to the horizontal at an angle of measure 30° , it is kept in equilibrium by a horizontal force.

Find the magnitude of each of the force and the reaction of the plane. « $4\sqrt{3}$, $8\sqrt{3}$ kg.wt. »

- 4 A body of weight (W) newton is placed on a smooth plane inclined with the horizontal at an angle of measure 30° and kept in equilibrium by the effect of force of magnitude 36 newton acts in the direction of the line of greatest slope of the plane upwards. Find the magnitude of the weight W and the magnitude of the reaction of the plane. « 72 , $36\sqrt{3}$ newton »

- 5 The magnitudes of three coplanar concurrent forces are $F_1 = 8$ gm.wt. , $F_2 = 4\sqrt{3}$ gm.wt. and $F_3 = 4$ gm.wt. If these forces are in equilibrium, then find the measures of the angles between these forces.

« 90° , 120° , 150° »

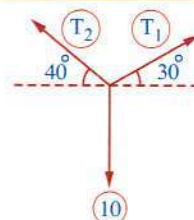
- 6 If M is the point of intersection of the two diagonals of a square ABCD , E is the midpoint of \overline{AB} , F is the midpoint of \overline{BC} and F_1 , F_2 and 42 are the magnitudes of three forces in equilibrium act on \overline{ME} , \overline{MF} , \overline{MD} Calculate the value of F_1 and F_2

« $21\sqrt{2}$, $21\sqrt{2}$ gm.wt. »

- 7  In the opposite figure :

A weight of magnitude 10 newton is suspended by two strings , the first is inclined by an angle of measure 30° to the horizontal and the second is inclined by an angle of measure 40° to the horizontal.

Find T_1 , T_2 in case of equilibrium.



« 8.15 , 9.216 newton »

- 8 A string of length 40 cm. is fixed from its two ends at two points on a horizontal line where the distance between them is 32 cm. A body of weight 180 kg.wt. is suspended at the midpoint of the string. Find the values of the tensions in the two branches of the string.


« 150 , 150 kg.wt. »

- 9 A body of weight 15 kg.wt. is placed on a smooth plane inclines to the horizontal at an angle of measure $\sin^{-1} \frac{1}{2}$, a force inclined to the horizontal at an angle of measure 60° acted on the body to keep it in equilibrium. Find the magnitude of each of the force and the reaction of the plane.

« $5\sqrt{3}$, $5\sqrt{3}$ kg.wt. »

- 10 A body of weight (W) kg.wt. is placed on a smooth plane inclines to the horizontal at an angle of measure $\cos^{-1} \frac{1}{2}$, it is kept in equilibrium by means of a force inclined to the horizontal at an angle of measure 30° upwards. Find the magnitude of each of the force and the reaction of the plane in terms of (W)

« $F = r = W$ »

- 11  A weight of magnitude 200 gm.wt. is suspended by two strings of lengths 60 cm. and 80 cm. , from two points on one horizontal line where the distance between them is 100 cm. Find the magnitude of tension in each string in case of equilibrium.


« 160 , 120 gm.wt. »

- 12 A body of weight 6.5 newton is suspended by two strings of lengths 0.5 and 1.2 m. the two other ends are fixed at two points on a horizontal line such that the strings are perpendicular to each other. Find the tension in each of the two strings in case of equilibrium.

« 6 , 2.5 newton »

- 13 A weight of 50 gm.wt. is suspended by means of two perpendicular strings. If the tensions in the two strings are of magnitudes $25\sqrt{3}$, 25 gm.wt. Find the measures of the angles which the two strings are inclined to the vertical in case of equilibrium.

« 30° , 60° »

- 14** A weight of 200 gm.wt. is suspended at the end of a light string, the other end of which is attached to the ceiling of a room. The weight is pulled by a horizontal force until the string is inclined to the vertical by an angle of measure 30° .
Find the magnitude of each of the horizontal force and the tension in the string.
 $\left\langle \frac{200\sqrt{3}}{3}, \frac{400\sqrt{3}}{3} \text{ gm.wt.} \right\rangle$
-
- 15** A weight of 60 gm.wt. is suspended at the end of a string and the other end is fixed at a point of a vertical wall. A horizontal force of magnitude F acts on the weight in a perpendicular direction to the wall, the weight becomes in equilibrium when the string is inclined to the wall with an angle of measure θ where $\tan \theta = \frac{3}{4}$.
Find the magnitude of each of F and the tension in the string. $\left\langle 45, 75 \text{ gm.wt.} \right\rangle$
-
- 16** A weight of 16 newton is suspended at the end of a light string and the other end is fixed at a point of a vertical wall. A force of magnitude F acts on the weight in a perpendicular direction of the string till it become in equilibrium when the string is inclined to the wall with an angle of measure 30° .
Find the magnitude of the force F and the tension of the string. $\left\langle 8, 8\sqrt{3} \text{ newton} \right\rangle$
-
- 17**  The ball of a pendulum of weight 600 gm.wt. is displaced until the string makes an angle of measure 30° with the vertical under the action of a force perpendicular to the string.
Find the magnitude of each of the force and the tension in the string. $\left\langle 300, 300\sqrt{3} \text{ gm.wt.} \right\rangle$
-
- 18** A light string of length 170 cm., its end A is fixed at a point of a ceiling of a room. From the other end B there is a lamp of weight 34 gm.wt. Find the magnitude of each of the tension and the required force to make the lamp in equilibrium at a distance 80 cm. down the ceiling in each of the following cases :
(1) If the force is horizontal. $\left\langle 72.25, 63.75 \text{ gm.wt.} \right\rangle$
(2) If the force is perpendicular to \overline{AB} $\left\langle 16, 30 \text{ gm.wt.} \right\rangle$
-
- 19** A body of weight 6 N. is placed on a smooth plane inclines to the horizontal by an angle θ . The body is kept in equilibrium by means of a force of magnitude $2\sqrt{3}$ N. inclines to the line of greatest slope of the plane by an angle of measure θ up.
Find the value of θ and the magnitude of the normal reaction of the plane.
 $\left\langle \theta = 30^\circ, r = 2\sqrt{3} \text{ N.} \right\rangle$

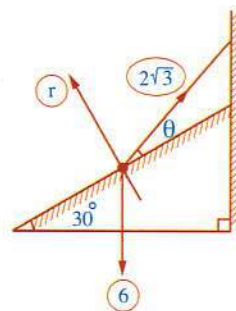
- 20** A body is in equilibrium on a smooth plane inclined to the horizontal at an angle under the action of a force acting in the direction of the plane upwards. Its magnitude equals half the magnitude of weight of the body. Find the measure of the angle of inclination of the plane and the magnitude of the reaction of the plane.

$$\ll 30^\circ, \frac{\sqrt{3}}{2} W \gg$$


- 21**  **In the opposite figure :**

A body of weight 6 kg.wt. is placed on a smooth plane inclined to the horizontal by an angle of measure 30° . The body is kept in equilibrium by a tension force (T) of magnitude $2\sqrt{3}$ kg.wt. The tension force acts along one end of the string of which is fixed by the body and the other end at a point on a vertical wall.

Find the measure of the angle of inclination of the string to the plane and the magnitude of the reaction of the plane on the body.




$$\ll 30^\circ, 2\sqrt{3} \text{ kg.wt.} \gg$$


- 22**  A body of weight 300 gm.wt. is placed on a smooth plane inclined to the horizontal with an angle whose tangent equals $\frac{1}{\sqrt{3}}$. The body is prevented from sliding by a force from with the line of the greatest slope an angle of measure 30° upwards.

Find the magnitude of the force and the reaction of the plane.

$$\ll 100\sqrt{3}, 100\sqrt{3} \text{ gm.wt.} \gg$$

- 23**  A body of weight 800 gm.wt. is placed on a smooth plane inclined to the horizontal by an angle θ , where $\sin \theta = 0.6$ the body is kept in equilibrium by a horizontal force. Find the magnitude of this force and the reaction of the plane on the body.

$$\ll 600, 1000 \text{ gm.wt.} \gg$$

- 24**  A smooth string of length 30 cm. is attached by its end in the two points A, B such that \overline{AB} is horizontally, $AB = 18$ cm. if a smooth ring of weight 150 gm.wt. slides on the string. Prove that in the case of equilibrium the lengths of the two parts of the strings are equal, then find the tension in each part.

$$\ll 93.75 \text{ gm.wt.} \gg$$

- 25** A body of weight 24 newton is suspended at one end of a string of length 130 cm., the other end is fixed at a point of a vertical wall. A horizontal force acts on the body to become in equilibrium. Find the magnitudes of the force and the tension in the string.

(1) When the body is at a distance = 50 cm. from the wall.

$$\ll 10, 26 \text{ newton} \gg$$


(2) When the string is inclined to the wall with an angle of measure 30°

$$\ll 8\sqrt{3}, 16\sqrt{3} \text{ newton} \gg$$

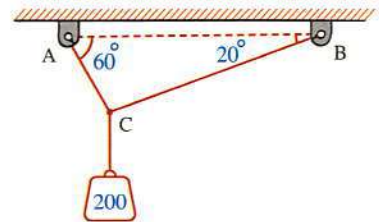
- 26** A body of weight 72 gm.wt. is suspended at one end of a string. The other end of the string is fixed at a point A on a vertical wall. Another string is attached to the first one at a point B 25 cm. a part from A and pulled horizontally until the point B becomes 7 cm. apart the wall.

Find the tension in the horizontal string and in each part of the first string.

« 21 , 75 , 72 gm.wt. »

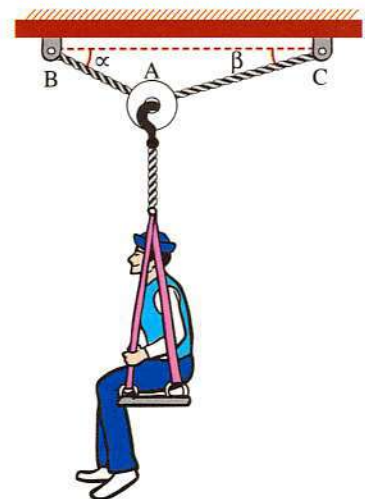
- 27**  A particle of weight 200 gm.wt. is suspended by two light strings. One of them is inclined to the vertical by an angle of measure θ and the other inclined to the vertical by an angle of measure 30° . If the magnitude of the tension in the first string is 100 gm.wt. , then find θ and the magnitude of the tension in the second string. « 60° , $100\sqrt{3}$ gm.wt. »

- 28** The opposite figure represents a weight of magnitude 200 newton hanged vertically at a point C by two strings \overline{BC} and \overline{AC} which make with the horizontal the angles of measures 20° , 60° respectively find in the state of equilibrium the tension in the two strings to the nearest newton.



« 102 , 191 newton »

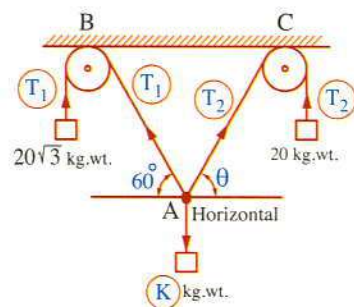
- 29** **Join with navigation :** The operation of saving a nautilus is done by using the captain chair which is hanged in a bully. Two ropes \overline{AB} and \overline{AC} are passing over the bully making two angles α , β with the horizontal whose measures are 25° , 15° respectively. If the tension in the rope \overline{AB} equals 80 newton , find the weight of the nautilus and the chair together and the tension in the rope \overline{AC} in the state of equilibrium.



« 53 , 75 newton »

30 In the opposite figure :

A weight of magnitude K is suspended by an end of a string, the other end is suspended by two strings passing over two smooth pulleys at B , C and carries two weights of magnitudes $20\sqrt{3}$ kg.wt. and 20 kg.wt. Find the value of the weight K and the measure of angle θ in state of equilibrium.



« 40 kg.wt. , 30° »

Third Higher skills

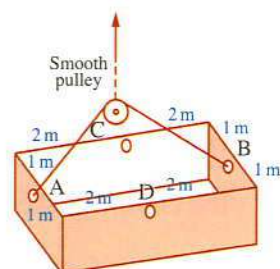
- 1 A body of weight 400 gm.wt. is suspended at point A by a string. From a point B on the string another string is attached and pulled horizontally by a second string \overline{BC} passing over a smooth fixed pulley and carries at its other end a body of weight 300 gm.wt. Find the inclination of \overline{AB} with the vertical and the tension in each of the two strings \overline{AB} , \overline{BC}

« $36^\circ 52'$, 500 , 300 gm.wt. »

- 2 \overline{AB} is a light string, its two ends are fixed at two points on a horizontal line. C and D are two points of the string. Two weights K and 20 gm.wt. are suspended from C and D respectively. If the set of forces are in equilibrium when \overline{CD} is horizontal and the two parts \overline{AC} and \overline{BD} of the string incline to the vertical by angles of measures 30° and 60° respectively. Find the magnitudes of tensions in the three parts of the string and the value of K

« $40\sqrt{3}$, $20\sqrt{3}$, 40 kg.wt. , $K = 60$ gm.wt. »

- 3 A box of weight 20 newton is suspended by a string as in the opposite figure. If the box can be fixed with the string through two methods one of them is A , B and the other is C , D which of these two methods can produce the less tension in the string to become the set in equilibrium ?





Exercise

5

Follow : The equilibrium

(Meeting lines of action of three equilibrium forces)

From the school book



Remember Understand Apply Higher Order Thinking Skills

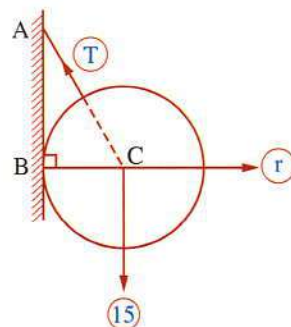
First Multiple choice questions

Choose the correct answer from the given ones :

(1) In the opposite figure :

A solid uniform sphere of weight 15 gm.wt. and radius length 10 cm. is in equilibrium by a string of length 10 cm. attached to a point of its surface and the other end of the string is fixed at a point in the vertical smooth plane above the tangency point , then $(r, T) = \dots\dots\dots$

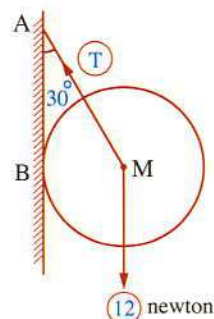
- (a) $(4\sqrt{3}, 8\sqrt{3})$ (b) $(5\sqrt{3}, 10\sqrt{3})$
(c) $(5, 10)$ (d) $(5\sqrt{3}, 8\sqrt{3})$



(2) In the opposite figure :

If the sphere is in equilibrium , then $T - r = \dots\dots\dots$ newton
(Where r is the magnitude of the wall reaction on the sphere)

- (a) $8\sqrt{3}$ (b) $4\sqrt{3}$
(c) 4 (d) 8



- (3) A solid uniform sphere of weight 20 kg.wt. and its radius length 5 cm. If it is in equilibrium by a string of length 5 cm. attached to a point of its surface and the other end of the string is fixed at a point in the vertical smooth plane above the tangency point, then the reaction of the vertical plane $r = \dots\dots\dots$ kg.wt.

(a) $\frac{20}{\sqrt{3}}$

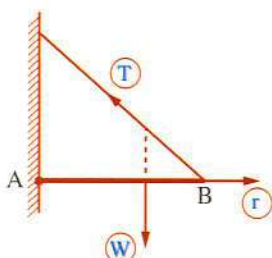
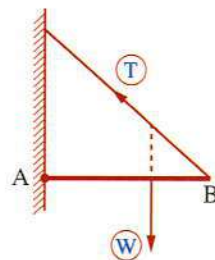
(b) 20

(c) $\frac{20}{\sqrt{5}}$

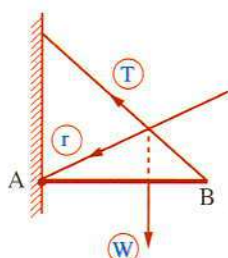
(d) zero

- (4) In the opposite figure :

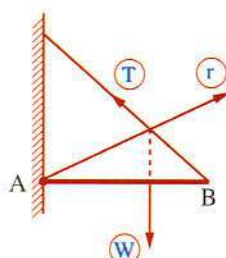
\overline{AB} is a rod. The end A is attached to a hinge fixed on a vertical smooth wall, if the rod is in equilibrium, then which of the following figures represent the correct direction of the reaction of the hinge ?



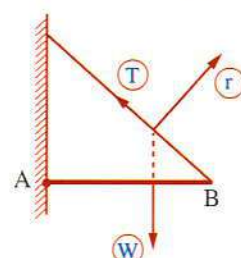
(a)



(b)



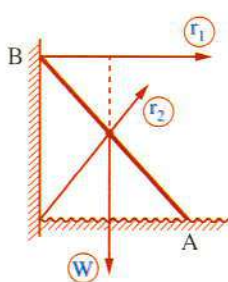
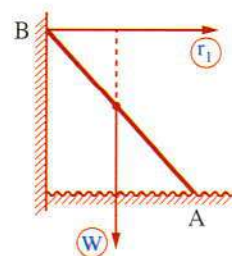
(c)



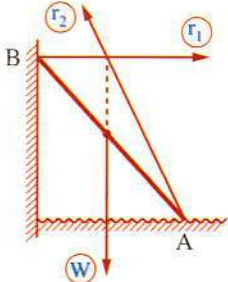
(d)

- (5) In the opposite figure :

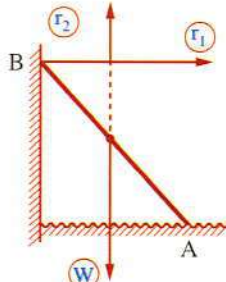
\overline{AB} is a uniform rod and its weight is wres at the end A against a horizontal rough ground, and the end B on a vertical smooth wall. Then which of the following figures represent the correct direction of the reaction of the ground ?



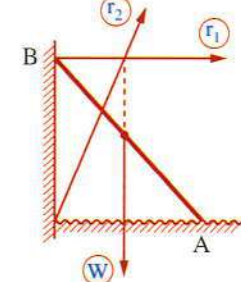
(a)



(b)



(c)

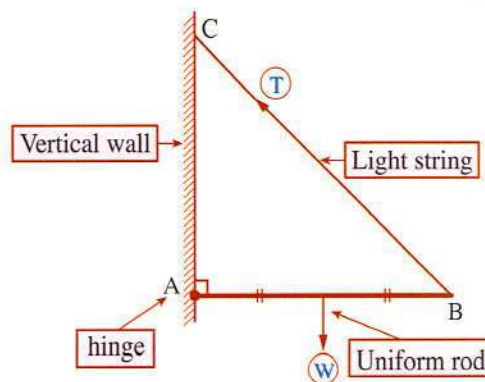


(d)

(6) In the opposite figure :

The direction of the reaction of the hinge on the rod at A

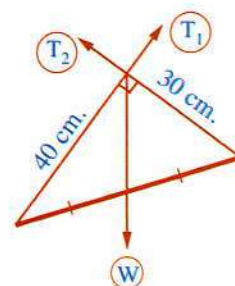
- (a) in the direction of \overrightarrow{AB}
- (b) in the direction of \overrightarrow{AC}
- (c) bisects \overline{BC}
- (d) perpendicular to \overline{BC}



(7) In the opposite figure :

$T_1 : T_2 : W = \dots\dots\dots$

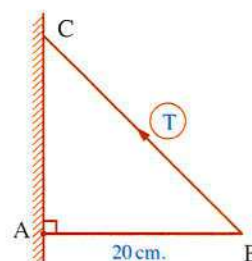
- (a) 5 : 3 : 4
- (b) 8 : 5 : 4
- (c) 4 : 3 : 5
- (d) 5 : 4 : 3



(8) In the opposite figure :

\overline{AB} is uniform rod with length 20 cm. and weight 30 newton is connected to a hinge on the vertical wall at A. If the rod kept in equilibrium horizontally by a light string connected to the rod at B of length $20\sqrt{2}$ cm. fixed at a point C on the wall just above A , then magnitude of the reaction of the hinge = newton.

- (a) $10\sqrt{2}$
- (b) 10
- (c) 15
- (d) $15\sqrt{2}$



(9) In the opposite figure :

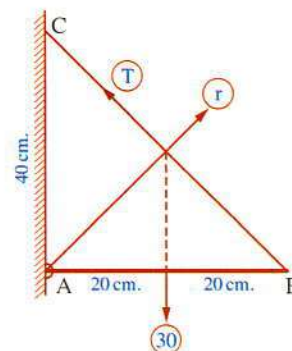
\overline{AB} is a uniform rod with length 40 cm. and weight 30 newton is connected to a hinge at A. If the rod kept in equilibrium horizontally by a light string connected to the rod at B and C where C is located on the wall just above A , $AC = 40$ cm.

First : The reaction of the hinge $r = \dots\dots\dots$ newton.

- (a) 30
- (b) 20
- (c) $40\sqrt{2}$
- (d) $15\sqrt{2}$

Second : The tension of the string $T = \dots\dots\dots$ newton.

- (a) $15\sqrt{2}$
- (b) 30
- (c) 20
- (d) $40\sqrt{2}$



- (10) A uniform rod of weight 20 newton which is movable around a hinge at one end is pulled at the other end by a horizontal force of magnitude 10 newton acting on the other end, then the measure of the angle of inclination of the rod to the vertical when it is in equilibrium =

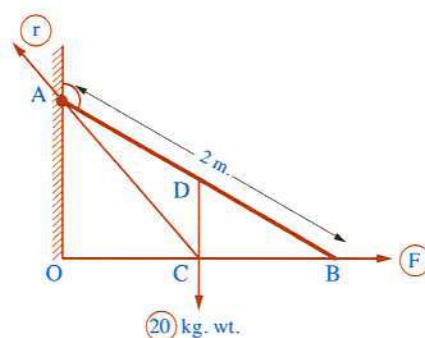
(a) 60° (b) 45° (c) 30° (d) 90°

- (11) A uniform rod of weight 24 newton is placed on two smooth planes inclined at two angles of measures 60° and 30° to the horizontal, then the magnitude of the pressure on each plane newton.

(a) 12, 15 (b) $12, 12\sqrt{3}$
(c) $12\sqrt{3}, 10$ (d) 15, 13

- (12) In the opposite figure :

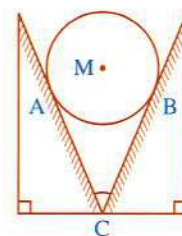
\overline{AB} is a uniform rod of length 2 m. and weight 20 kg.wt. It is connected to a hinge fixed to a vertical wall at A. A horizontal force acts at B. If the rod is kept in equilibrium when it is inclined to the vertical at an angle of measure 60° , then the reaction of the hinge on the rod = kg.wt.



(a) $10\sqrt{3}$ (b) $10\sqrt{5}$ (c) $10\sqrt{7}$ (d) $20\sqrt{2}$

- (13) The opposite figure represents a metal sphere, its weight (W) kg.wt. The sphere is placed such that it touches two smooth planes each makes an angle of measure θ with the vertical.

If the sphere touches the planes at A and B, then the reaction of the plane at A equals kg.wt.



(a) $\frac{1}{2} W$ (b) $W \cos \theta \csc \theta$ (c) $W \sin \theta \csc 2 \theta$ (d) $W \cos \theta \csc 2 \theta$

Second Essay questions

- 1 A smooth sphere of radius length 30 cm. and of weight 200 gm.wt. rests on a vertical smooth wall. It is suspended by a string of length 20 cm., one of its ends is attached to a point on the surface of the sphere and the other end is fixed at a point on the wall above the touch point of the sphere and the wall.

Find the magnitudes of the tension in the string and the reaction of the wall in case of equilibrium.

« 250, 150 gm.wt. »

- 2** A smooth sphere of weight $10\sqrt{3}$ gm.wt. rests against a smooth vertical wall. It is suspended at a point of its surface by means of a string and its other end is fixed to the wall at a point lies directly above the point of tangency of the sphere and the wall. If the string makes with the vertical an angle of measure 30° . Find the tension in the string and the reaction of the wall in case of equilibrium.

« 20 , 10 gm.wt. »

- 3** A smooth sphere of weight 15 newton is on a smooth vertical wall and suspended by a light string from a point on its surface. The other end of the string is attached to a point on the wall above the point of contact between the wall and the sphere. If the length of the string equals the radius length of the sphere. Find the pressure on the wall and the tension in the string in case of equilibrium.

« $5\sqrt{3}$, $10\sqrt{3}$ newton »

- 4** A metallic sphere of weight 15 kg.wt. is put such that it touches two smooth planes , one of them is vertical and the other inclines to the vertical by an angle of measure 30° . Find the reaction of each of the two planes.

« $15\sqrt{3}$, 30 kg. wt. »

- 5** AB is a uniform rod of length 100 cm. and weight 30 kg.wt. is suspended from its two ends A and B by means of two strings , their other ends are fixed at a pin in the ceiling at the point C , if the two strings are perpendicular and $AC = 50$ cm. Find the tension in each of the two strings.


« 15 , $15\sqrt{3}$ kg.wt. »

- 6** A uniform rod of length 130 cm. and weight 26 newton is suspended at its ends by two strings tied at one point. If the length of one of them is 50 cm. and the length of the other one is 120 cm. What is the position in which the rod is in equilibrium and what is the tension in each of the two strings ?

« 24 , 10 newton »

- 7** AB is a uniform rod with length 60 cm. and weight 40 newton is connected to a hinge on the vertical wall at A. If the rod kept in equilibrium horizontally by a light string connected to the rod at B and with point C on the wall just above A and at a distance 60 cm. from A. Find the tension on the string and the reaction on the hinge at A.

« $20\sqrt{2}$, $20\sqrt{2}$ newton »

- 8** \overline{AB} is a uniform rod of length 80 cm. and weight 24 kg.wt. The end A is attached to a hinge fixed on a vertical wall, and the end B is tied by a light string of length $80\sqrt{3}$ cm. fixed at a point C on the wall which lies directly above A and at a distance 80 cm. If the rod is in equilibrium, find the magnitude of the tension and the reaction of the hinge.
« $12\sqrt{3}$, 12 kg.wt. »
-
- 9**  A homogeneous sphere rests on two parallel rods lie on the same horizontal plane. The distance between them equals the radius length of the sphere. Find the pressure on each rod if the weight of the sphere is 60 newton in case of equilibrium. « $20\sqrt{3}$, $20\sqrt{3}$ newton »
-
- 10** A sphere in which M is its centre and its radius length is 12 cm. and its weight is (W) newton rests at B against a smooth vertical wall, from a point C on its surface, it is tied by a string, its other end is fixed at A of the wall lies directly above B. If the tension in the string is 50 newton. Find the length of the string and the weight of the sphere when the reaction of the wall to the sphere equals 25 newton.
« 12 cm., $25\sqrt{3}$ newton »
-
- 11** A uniform rod whose length is 80 cm. and its weight is 12 newton, the rod is freely suspended from its ends by means of two strings, and the other ends are attached to a fixed nail in the ceiling. If the two strings are perpendicular and one of them is of length 48 cm. Find in equilibrium the magnitude of the tension in each of the two strings.
« 7.2, 9.6 newton »
-
- 12** \overline{AB} is a uniform ladder of weight 36 kg.wt. rests at the end A against a vertical smooth wall, and the other end B on a horizontal rough ground. If the ladder is in equilibrium when its end A is at a distance 3 metres from the ground and the end B is at a distance 2.5 metres from the wall. Find the reaction of each of the ground and the wall on the ladder.
« 15, 39 kg.wt. »
-
- 13** \overline{AB} is not a uniform rod of length 60 cm. and weight 16 kg.wt. acts at a point D on the rod where $AD = 20$ cm. The rod is attached to a hinge at A and the hinge is fixed on a vertical wall. The end B of the rod is tied by a light string its other end is fixed at a point C on the wall lying directly above A and at a distance 80 cm. from it, then the rod becomes in equilibrium such that it is perpendicular to the wall. Find the tension in the string and the reaction of the hinge.
« $6\frac{2}{3}$, $\frac{4\sqrt{73}}{3}$ kg.wt. »

- 14** \overline{AB} is a uniform rod of length $2L$ cm. and weight 8 kg.wt. acting at its midpoint. its end A is hinged at a point in a vertical wall where its end B is attached to a light string and the other end of the string is fixed to a point C on the wall situated vertically above A. If $AB = AC = BC$ and the rod is in equilibrium. Find the tension in the string and the reaction of the hinge at A
« 4 kg.wt. , $4\sqrt{3}$ kg.wt. »
- 15** \overline{AB} is a uniform rod of length 60 cm. and weight (W) kg.wt. The end A is attached to a hinge fixed on a vertical wall and the end B is tied by a string of length 80 cm. , its other end is fixed to a point on the wall vertically above A directly and at a distance 100 cm. of it , then the rod became in equilibrium. Find the tension in the string and the reaction of the hinge , also find the measure of the angle of inclination of the reaction of the hinge to the rod.
« $\frac{2}{5} W$, $\frac{\sqrt{13}}{5} W$ kg.wt. , $33^\circ 41'$ »
- 16** \overline{AB} is a uniform rod of length 90 cm. , and weight (W) kg.wt. Its end A is fixed to a vertical wall by a hinge and the rod is kept in equilibrium horizontally by means of a string of length 50 cm. , one of its ends is tied at the point C on the rod at a distance 30 cm. from A , the other end of the string is fixed at a point D on the vertical wall above A directly, calculate the tension in the string and the reaction of the hinge on the rod.
« $\frac{15}{8} W$, $\frac{\sqrt{97}}{8} W$ kg.wt. »
- 17** \overline{AB} is a uniform rod , its end A is attached by a hinge fixed in a vertical wall. A horizontal force acts at the end B to keep the rod in equilibrium while it is inclined to the wall by an angle of measure 45° , if the weight of the rod is 4 kg.wt. acts at its midpoint , then find the magnitude of the force and the reaction of the hinge.
« 2 , $2\sqrt{5}$ kg.wt. »
- 18** A uniform rod which is movable around one of its ends is pulled a side by a horizontal force acting on the other end and equals half the weight of the rod. Find the measure of the angle of inclination of the rod to the vertical when it is in equilibrium and also the reaction at the first end.
« 45° , $\frac{\sqrt{5}}{2}$ of the weight of the rod »
- 19** A uniform rod of weight 4 newton is placed on two smooth planes inclined at 30° and 60° to the horizontal. Find the magnitude of the pressure on each plane and the measure of the angle of inclination of the rod to the horizontal in state of equilibrium.
« $2\sqrt{3}$, 2 newton , 30° »

- 20** A smooth iron sphere of weight (W) kg.wt. rests against a vertical smooth wall and a smooth plane inclines to the horizontal at an angle θ where $\cos \theta = \frac{3}{5}$, if the sphere is in equilibrium. Find the pressure on each of the wall and the inclined plane.

$\ll \frac{4}{3} W, \frac{5}{3} W \text{ kg.wt.} \gg$

- 21** A uniform rod of weight 20 kg.wt. rests at one of its ends against a smooth vertical plane and at the other end on a smooth plane inclined to the vertical at an angle of measure 60° , in the state of equilibrium. Find the magnitude of each of the two reactions of the two planes, also find the measure of the angle at which the rod inclines to the vertical.

$\ll \frac{20\sqrt{3}}{3}, \frac{40\sqrt{3}}{3} \text{ kg.wt.}, 49^\circ 6' \gg$

- 22** A uniform rod \overline{AB} of weight 8 newton acting at its midpoint is placed on two smooth perpendicular planes that are inclined to the horizontal. Such that the vertical plane of the rod and the two lines of greatest slope of the two inclined planes is perpendicular to the intersection line of the two planes. If the magnitude of the pressure on the plane at the end B is 4 newton.

Find the magnitude of the pressure on the other plane and measures of the two inclination angles of the planes to the horizontal, in the state of equilibrium.

$\ll 4\sqrt{3} \text{ newton}, 30^\circ, 60^\circ \gg$

- 23** A uniform hollow sphere of radius length (r) and weight $12\sqrt{3}$ kg.wt. is placed on a smooth plane inclined to the horizontal at an angle of measure 30° , it is prevented from motion on the plane by means of a string fixed at a point on its surface, the length of the string equals the radius length of the sphere. The other end of the string is fixed at a point on the inclined plane. In the state of equilibrium, prove that the string is horizontal, then find the tension in the string and the reaction of the inclined plane upon the sphere.

$\ll 12, 24 \text{ kg.wt.} \gg$

- 24** \overline{AB} is a uniform rod can move in a vertical plane freely around a hinge at A, the other end B is tied to a string passes over a smooth pulley C exactly above A and attached to a weight equals half the weight of the rod. Find the measure of the angle of inclination of the rod to the horizontal in state of equilibrium given that $AC = AB$

$\ll 30^\circ \gg$

- 25** \overline{AB} is a uniform rod which is 40 cm. long and weight 12 N. The rod rests with its end A on a vertical smooth wall. It is kept in equilibrium by means of a light inextensible string , one of its ends is attached to point C where $C \in \overline{AB}$ and $BC = 10$ cm. , and the other end is fixed to a point D on the wall directly vertical above A. If the rod is inclined by an angle whose measure is 60° to the vertical , then find the magnitudes of the tension in the string and the reaction of the wall. « $8\sqrt{3}$, $4\sqrt{3}$ N. »

- 26** A uniform rod \overline{AB} of length 6 metres and weight 8 kg.wt. is attached to a hinge fixed in a vertical wall at its end A. The rod is kept horizontally by attaching it at a point C on the rod (where $AC = 4$ metres) by a string which its other end is fixed at the point D on the wall above A exactly and at a distance 4 metres from it. Calculate the magnitude of the tension in the string and the reaction of the hinge in case of equilibrium. « $6\sqrt{2}$, $2\sqrt{10}$ kg.wt. »

Use the resolution to solve

- 27** A body of weight 100 newton is placed on a smooth plane inclined to the horizontal at an angle of measure θ where $\sin \theta = \frac{3}{5}$, the body is kept in equilibrium by means of a force inclines to the line of the greatest slope at an angle of measure α where $\cos \alpha = \frac{12}{13}$. Find F and the reaction of the plane. « 65 , 55 newton »
- 28** A body is placed on a smooth plane inclined to the horizontal at an angle of measure 30° , it is kept in equilibrium by means of two forces , one of them in the direction of the line of the greatest slope upwards , its magnitude = 50 newton and the second inclines to the line of the greatest slope upwards with an angle of measure 30° and its magnitude is $20\sqrt{3}$ newton. Find each of the weight of the body and the reaction of the plane. « 160 , $70\sqrt{3}$ newton »
- 29** A smooth ring and a string passes through it. The length of the string is 40 cm. its two ends are fixed at the two points A and B on the same horizontal line , the distance between them is 20 cm. A horizontal force \vec{F} acts on the ring to be in equilibrium vertically down B and the string is in tension. Find the value of F and the magnitude of tension in the string given that the weight of the ring = 400 gm.wt. « 200 , 250 gm.wt »

Unit Two

Geometry and Measurement



Exercise

6

The straight lines and the planes in the space.

Exercise

7

The pyramid.

Exercise

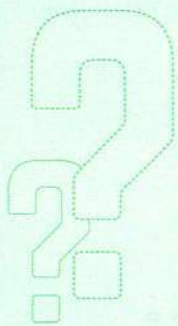
8

The cone.

Exercise

9

The circle.



Exercise

6

The straight lines and the planes in the space



From the school book

Remember Understand Apply Higher Order Thinking Skills





Test yourself

First Multiple choice questions

Choose the correct answer from those given :

- (1) Number of straight lines that are passing through a given point is
(a) 1 (b) 2 (c) 3 (d) an infinite number.
- (2) Number of straight lines that are passing through two given points is
(a) 1 (b) 2
(c) 3 (d) an infinite number.
- (3) Number of planes that are passing through two given points is
(a) 1 (b) 2
(c) 3 (d) an infinite number.
- (4) Number of planes that are passing through three non-collinear points is
(a) 1 (b) 2 (c) 3 (d) an infinite number.
- (5) Number of planes that are passing through three collinear points is
(a) Zero (b) 1 (c) 3 (d) an infinite number.
- (6) All of the following cases determine a plane except
(a) a straight line and a point doesn't belong to it.
(b) two parallel and not coincident straight lines.
(c) two intersecting straight lines.
(d) two skew straight lines.

- (7) All of the following cases determine a plane except
 - (a) two intersecting straight lines.
 - (b) two different parallel straight lines.
 - (c) a straight line and a point belong to it.
 - (d) three non-collinear points.
- (8) Number of planes that are passing through two different parallel straight lines =
 - (a) 1
 - (b) 2
 - (c) 3
 - (d) an infinite number.
- (9) Two skew straight lines are two straight lines which are
 - (a) not intersect.
 - (b) not perpendicular.
 - (c) not parallel.
 - (d) not intersect and not parallel.
- (10) The two straight lines are skew if they are
 - (a) not parallel.
 - (b) not intersecting.
 - (c) not coincident.
 - (d) not contained in the same plane.
- (11)  If the straight line $L \parallel$ the plane X and $A \in X$, then $L \cap X =$
 - (a) \emptyset
 - (b) L
 - (c) X
 - (d) $\{A\}$
- (12)  If the straight line $L \subset$ the plane X and $A \in X$, then $L \cap X =$
 - (a) \emptyset
 - (b) L
 - (c) X
 - (d) $\{A\}$
- (13) If the two straight lines L_1 and L_2 are skew, then $L_1 \cap L_2 =$
 - (a) \emptyset
 - (b) L_1
 - (c) L_2
 - (d) the plane contains L_1 and L_2
- (14) The two not parallel planes are intersecting in
 - (a) a point.
 - (b) a straight line.
 - (c) a plane.
 - (d) ray.
- (15) If X, Y are two planes where $X \cap Y = \emptyset$, then X Y
 - (a) \perp
 - (b) \parallel
 - (c) $=$
 - (d) \subset
- (16) The two planes are coincident if they have in common.
 - (a) only one point
 - (b) two points
 - (c) three collinear points
 - (d) three non-collinear points
- (17) If a straight line and a plane have two point in common, then the straight line
 - (a) is parallel to the plane.
 - (b) intersects the plane in only one point.
 - (c) lies completely inside the plane.
 - (d) intersects the plane in only two points.

- (18) If A , B and C are three points determine a plane , then
 - (a) $AB = BC = AC$ (b) $AB + BC = AC$
 - (c) $AB + BC > AC$ (d) $AB + BC < AC$
- (19) All different vertical straight lines in the space are
 - (a) parallel. (b) skew.
 - (c) contained in the same plane. (d) intersecting.
- (20) Relative position of two straight lines in one plane could be each of the following except
 - (a) parallel. (b) intersecting. (c) skew. (d) coincident.
- (21) If X , Y and Z are planes in the space where $X \cap Y \cap Z = \{A\}$ and $X \cap Y =$ the straight line L , then which of the following is not true ?
 - (a) $A \in L$ (b) $L \cap Z = \{A\}$
 - (c) $L \parallel Z$ (d) $A \in Z$
- (22) If M is a point outside the plane that contains the three points A , B and C , then \overleftrightarrow{MA}
 - (a) lies completely inside the plane. (b) intersects the plane at a point.
 - (c) intersects the plane at two points. (d) is parallel to the plane.
- (23) If $\overleftrightarrow{AB} \subset \text{plane } X$, $\overleftrightarrow{CD} \parallel \text{plane } X$, then \overleftrightarrow{CD} , \overleftrightarrow{AB} are
 - (a) parallel only. (b) skew only.
 - (c) parallel or skew. (d) intersecting.
- (24) X and Y are two parallel planes and straight line $L_1 \subset X$ and straight line $L_2 \subset Y$, then which of the following can not be happen ?
 - (a) $L_1 \parallel L_2$ (b) L_1 and L_2 are skew.
 - (c) $L_1 \parallel Y$ and $L_2 \parallel X$ (d) L_1 and L_2 are intersecting.
- (25) The least number of planes that determine a solid is
 - (a) 1 (b) 2 (c) 3 (d) 4
- (26) ABCD $\hat{=}$ $\hat{=}$ $\hat{=}$ $\hat{=}$ is a cuboid , how many straight lines carry edge from the edges of the cuboid and skew to \overleftrightarrow{AB} ?
 - (a) not exist (b) one (c) two (d) four
- (27) Which of the following statements is not true ?
 - (a) For any two points in the space , there is only one plane is passing through them.
 - (b) Any three non-collinear points in the space determine a plane.
 - (c) Vertices of the triangle determine a plane.
 - (d) For each two intersecting straight lines there is only one plane contains them.

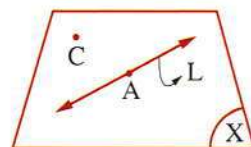
- (28) Which of the following statements is not true ?
- (a) Any two different parallel straight lines determine a plane.
 - (b) For any two different intersecting straight lines there is only one point in common.
 - (c) The two skew straight lines can't be contain in the same plane.
 - (d) For any three non-collinear points there is one plane passing through them at least.

- (29) Which of the following statements is not true ? (where L_1 and L_2 are two straight lines , X and Y are two planes) ?

- (a) If $L_1 \cap L_2 = \emptyset$, then $L_1 \parallel L_2$ or L_1 and L_2 are skew.
- (b) If $L_1 \cap X = \emptyset$, then $L_1 \parallel X$
- (c) If $L_2 \cap X = L_2$, then $L_2 \subset X$
- (d) If $L_2 \subset Y$, then $L_2 \cap Y = \emptyset$

- (30) Using the opposite figure ,
which of the following statements is not true ?

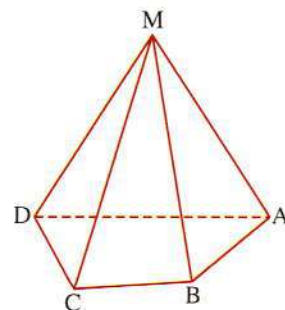
- (a) $L \subset X$
- (b) $A \in L, A \notin X$
- (c) $C \in X, C \notin L$
- (d) $\overline{AC} \cap L = \{A\}$



- (31) In the opposite figure :

The plane $ABD \cap$ The plane $MCD = \dots\dots\dots$

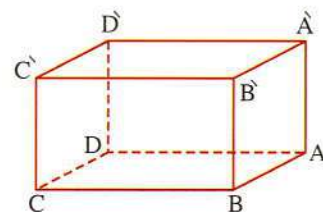
- (a) \overleftrightarrow{AM}
- (b) \overleftrightarrow{CD}
- (c) $\{D\}$
- (d) \overleftrightarrow{MC}



- (32) In the opposite figure :

The plane $AA'B'B \cap$ the plane $ACC'A' = \dots\dots\dots$

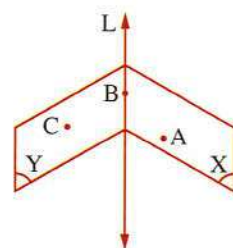
- (a) $\overleftrightarrow{AA'}$
- (b) $\overleftrightarrow{BB'}$
- (c) $\overleftrightarrow{CC'}$
- (d) \overleftrightarrow{AC}



- (33) In the opposite figure :

The plane $X \cap$ the plane $Y = \dots\dots\dots$

- (a) $\{B\}$
- (b) $\{A, B, C\}$
- (c) the straight line L
- (d) \emptyset



(34) In the opposite figure :

First : $L \dots\dots\dots X$

- (a) \in (b) \notin (c) \subset (d) $\not\subset$

Second : $A \dots\dots\dots X$

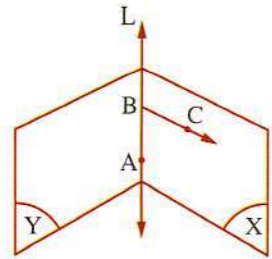
- (a) \in (b) \notin (c) \subset (d) $\not\subset$

Third : $C \dots\dots\dots Y$

- (a) \in (b) \notin (c) \subset (d) $\not\subset$

Fourth : $\overleftrightarrow{BC} \dots\dots\dots Y$

- (a) \in (b) \notin (c) \subset (d) $\not\subset$



(35) In the opposite figure :

First : The plane $ABB'A' \cap$ the plane $BCC'B' = \dots\dots\dots$

- (a) $\overleftrightarrow{BB'}$ (b) \emptyset (c) $\{B'\}$ (d) \overleftrightarrow{AC}

Second : The plane $ABC \cap$ the plane $A'B'C' = \dots\dots\dots$

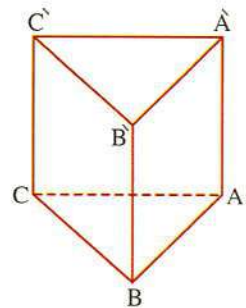
- (a) $\overleftrightarrow{BB'}$ (b) \emptyset (c) $\overleftrightarrow{AA'}$ (d) \overleftrightarrow{AB}

Third : $\overleftrightarrow{AC} \cap \overleftrightarrow{A'C'} = \dots\dots\dots$

- (a) $\{A\}$ (b) $\{C'\}$ (c) $\overleftrightarrow{AA'} \cap \overleftrightarrow{BB'}$ (d) \overleftrightarrow{AC}

Fourth : $\overleftrightarrow{BB'} \cap$ the plane $ABC = \dots\dots\dots$

- (a) $\overleftrightarrow{BB'}$ (b) $\{B'\}$ (c) $\{B\}$ (d) \emptyset



(36) In the opposite figure :

First : The plane $MAB \cap$ the plane $MBC = \dots\dots\dots$

- (a) \overleftrightarrow{AB} (b) \overleftrightarrow{MB} (c) \emptyset (d) $\{M\}$

Second : The plane $MBC \cap$ the plane $ABC = \dots\dots\dots$

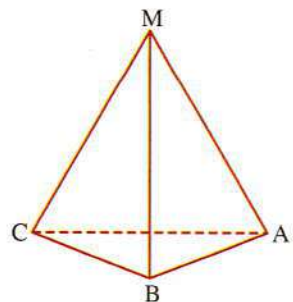
- (a) $\{B\}$ (b) \emptyset (c) \overleftrightarrow{AB} (d) \overleftrightarrow{BC}

Third : $\overleftrightarrow{MB} \cap$ the plane $ABC = \dots\dots\dots$

- (a) \overleftrightarrow{MB} (b) \emptyset (c) $\{B\}$ (d) $\{M\}$

Fourth : The plane $MAB \cap$ the plane $MBC \cap$ the plane $MAC = \dots\dots\dots$

- (a) \overleftrightarrow{MB} (b) \overleftrightarrow{MC}
(c) the solid MABC (d) $\{M\}$



(37) In the opposite figure :

If $A \notin$ the plane BCD , then :

First : $X \cap Y = \dots\dots\dots$

- (a) \overleftrightarrow{AC} (b) \emptyset (c) $\{A\}$ (d) $\{C\}$

Second : $X \cap Z = \dots\dots\dots$

- (a) \emptyset (b) \overleftrightarrow{BC} (c) \overleftrightarrow{AC} (d) $\{C\}$

Third : $Y \cap Z = \dots\dots\dots$

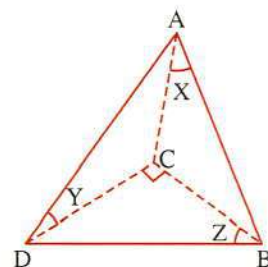
- (a) $\{C\}$ (b) \overleftrightarrow{BC} (c) \overleftrightarrow{CD} (d) \emptyset

Fourth : $\overleftrightarrow{AB} \cap X = \dots\dots\dots$

- (a) \overleftrightarrow{AB} (b) \emptyset (c) \overleftrightarrow{AC} (d) $\{B\}$

Fifth : Let $m(\angle BCD) = 90^\circ$, $BC = 3$ cm. , $CD = 4$ cm. , then $BD = \dots\dots\dots$ cm.

- (a) 6 (b) 5 (c) 4 (d) 7



(38) In the opposite figure :

X and Y are two intersecting planes at the straight line L , $A \in L$, $B \in X$, $B \notin Y$, $C \in Y$, $C \notin X$:

First : The plane $X \cap$ the plane ABC = $\dots\dots\dots$

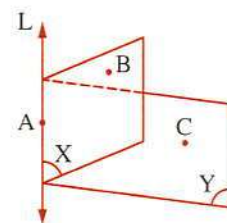
- (a) \overleftrightarrow{AB} (b) \overleftrightarrow{AC} (c) \overleftrightarrow{BC} (d) L

Second : The plane $Y \cap$ the plane ABC = $\dots\dots\dots$

- (a) \overleftrightarrow{AB} (b) $\{A\}$ (c) \overleftrightarrow{CB} (d) \overleftrightarrow{AC}

Third : The plane $X \cap$ the plane $Y \cap$ the plane ABC = $\dots\dots\dots$

- (a) \emptyset (b) L (c) $\{A\}$ (d) $\{B\}$



(39) In the opposite figure :

First : The plane ABCD // the plane $\dots\dots\dots$

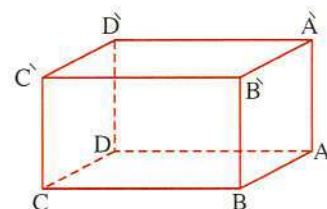
- (a) \overleftrightarrow{ABC} (b) \overleftrightarrow{ABD}
(c) \overleftrightarrow{ABB} (d) \overleftrightarrow{ABC}

Second : The plane $BC\overleftrightarrow{CB}$ // the plane $\dots\dots\dots$

- (a) ABC (b) \overleftrightarrow{ABC} (c) ABD (d) \overleftrightarrow{AAD}

Third : The plane $AB\overleftrightarrow{BA} \cap$ the plane ABCD = $\dots\dots\dots$

- (a) $\{B\}$ (b) $\{A, B\}$ (c) \overleftrightarrow{AB} (d) \overleftrightarrow{AB}



Fourth : The plane $ABB\hat{A} \cap$ the plane $DCC\hat{D} = \dots\dots\dots$

- (a) \overleftrightarrow{BC} (b) \overleftrightarrow{AD} (c) \emptyset (d) $\{C\}$

Fifth : The plane $DCC\hat{D} \cap$ the plane $ABCD \cap$ the plane $A\hat{A}DD = \dots\dots\dots$

- (a) \emptyset (b) \overleftrightarrow{AB} (c) $\{C\}$ (d) $\{D\}$

(40) In the opposite figure :

$ABB\hat{A}$, $BB\hat{C}C$, $ACC\hat{A}$ are three congruent rectangles
 , each pairs are intersecting , D is the midpoint of \overline{CC}
 , if $AB = 5$ cm. , $AA = 10$ cm.

First : The plane $ADA \cap$ the plane $BDB = \dots\dots\dots$

- (a) \overleftrightarrow{CC} (b) \overleftrightarrow{BB} (c) $\{D\}$ (d) \overleftrightarrow{AC}

Second : The plane $ADB \cap$ the plane $ABC = \dots\dots\dots$

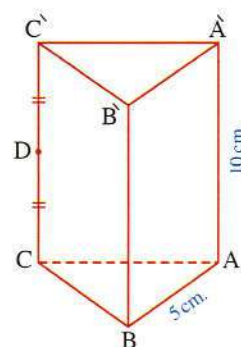
- (a) \emptyset (b) \overleftrightarrow{AB} (c) $\{B\}$ (d) \overleftrightarrow{AC}

Third : The plane $ADB \cap$ the plane $BC\hat{C}B = \dots\dots\dots$

- (a) $\{B\}$ (b) \overleftrightarrow{BC} (c) \overleftrightarrow{BD} (d) \emptyset

Fourth : $m(\angle BDB) = \dots\dots\dots^\circ$

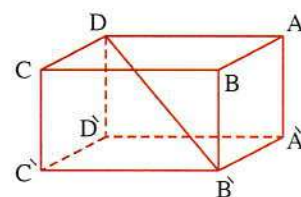
- (a) 60 (b) 120 (c) 90 (d) 100



(41) In the opposite figure :

$ABCD A\hat{B}C\hat{D}$ is a cuboid. How many straight lines
 carrying edges of the cuboid and skew to the straight
 line \overleftrightarrow{DB} ?

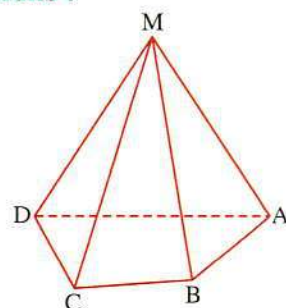
- (a) 2 (b) 4 (c) 6 (d) 8



Second Essay questions

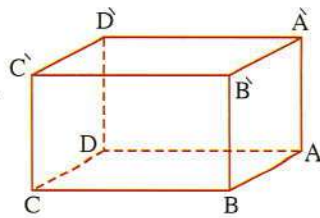
1 **Meditate the opposite figure and answer the following questions :**

- (1) How many lines which carry edges in the figure ?
- (2) State the names of the straight lines which carry edges and passing through point A
- (3) How many planes which carry faces in the figure ?
- (4) State the names of three planes passing through point A



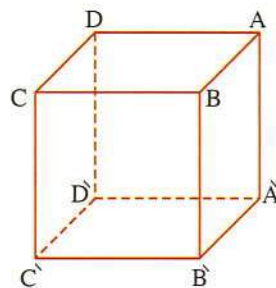
2 Meditate the opposite figure and answer the following questions :

- (1) Write three straight lines passing through point A
- (2) Write the straight lines passing through points A and B together.
- (3) Write three planes passing through point A
- (4) Write three planes passing through points A and B together.



3 The opposite figure represents a classroom , find :

- (1) The lines which carry edges and intersect with \overleftrightarrow{AB}
- (2) The lines which carry edges and parallel to \overleftrightarrow{AB}
- (3) The lines which carry edges and skew to \overleftrightarrow{AB}



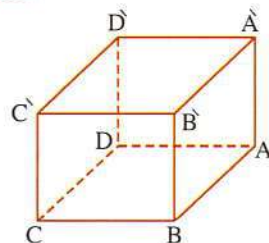
4 Write the number of planes which passing through :

- | | |
|-----------------------------|---------------------------------|
| (1) One given point. | (2) Two different points. |
| (3) Three collinear points. | (4) Three non-collinear points. |

5 In the opposite figure , $ABCD\hat{A}B\hat{C}D$ is a cube of edge length 6 cm.

- (1) Identify the relative positions for each pair of the following straight lines :

- | | |
|---|---|
| ① \overleftrightarrow{AB} , \overleftrightarrow{DD} | ② \overleftrightarrow{AB} , \overleftrightarrow{DC} |
| ③ \overleftrightarrow{AB} , \overleftrightarrow{AD} | ④ \overleftrightarrow{AC} , \overleftrightarrow{AC} |
| ⑤ \overleftrightarrow{AB} , \overleftrightarrow{DC} | ⑥ \overleftrightarrow{AC} , \overleftrightarrow{AA} |



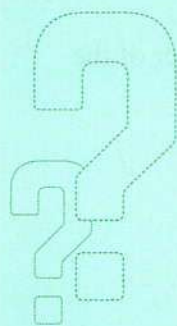
- (2) Identify the relative positions for each pair of the following planes :

- | | | |
|-------------------------------|-------------------------------------|-----------------------------|
| ① $ABB\hat{A}$, $DCC\hat{D}$ | ② $\hat{A}BBA$, $\hat{A}B\hat{C}D$ | ③ ABC , $\hat{D}B\hat{D}$ |
|-------------------------------|-------------------------------------|-----------------------------|

- (3) If $\overleftrightarrow{AB} \perp \overleftrightarrow{AD}$, Find the length of \overline{BD}

6 Draw the figures which represent the plane (X) , the straight line (L) and the point (A) in the following cases :

- | | |
|--|---------------------|
| (1) $A \in L$ | (2) $L \subset X$ |
| (3) $L \cap X = \{A\}$ | (4) $L \parallel X$ |
| (5) $A \in X$, $A \notin L$, $L \subset X$ | |



Exercise

7

The pyramid



From the school book

Remember Understand Apply Higher Order Thinking Skills



Test yourself

First Multiple choice questions

Choose the correct answer from those given :

- (1) The line segment joining the vertex of the pyramid and any vertex of its base vertices is called
 - (a) the height of the pyramid.
 - (b) the slant height of the pyramid.
 - (c) the lateral edge of the pyramid.
 - (d) the side of its base.
- (2) If MABCD is a regular quadrilateral pyramid, then this pyramid must be
 - ① regular faces ② its base is a square ③ right
 - (a) ①, ② (b) ②, ③ (c) ① only (d) ①, ②, ③
- (3) Which of the following statements is true ?
 - (a) The lateral faces of the right pyramid are congruent.
 - (b) The regular pyramid is a right pyramid.
 - (c) The heights of the lateral faces of the right pyramid are equal.
 - (d) The least number of planes that can determine a solid = 3 planes.
- (4) Which of the following statements is not true ?
 - (a) The base of the right pyramid can be a surface of a rhombus.
 - (b) The triangular pyramid has three faces.
 - (c) The pentagonal pyramid has six faces.
 - (d) All lateral faces of the quadrilateral pyramid are surfaces of triangles.

- (5) In the regular pyramid, which of the following is the right ascendingly order of the lengths ?

(a) The length of the lateral edge, the height, the slant height.
 (b) The height, the slant height, the length of the lateral edge.
 (c) The slant height, the height, the length of the lateral edge.
 (d) The length of the lateral edge, the slant height, the height.
- (6) The shape of the base of a regular pyramid must be

(a) parallelogram. (b) rhombus. (c) rectangle. (d) square.
- (7) If MABCD is a regular quadrilateral pyramid, then all lateral edges are

(a) parallel. (b) congruent.
 (c) perpendicular to the base. (d) mutually perpendicular.
- (8) If MABC is a right triangular pyramid, N is the projection of the point M on the plane ABC, E is midpoint of \overline{BC} , then all the following triangles are right except

(a) $\triangle MNC$ (b) $\triangle MNE$ (c) $\triangle MBC$ (d) $\triangle MNA$
- (9) If MABC is a regular faces pyramid, N is the projection of the point M on the plane ABC, E is the midpoint of \overline{AB} , which of the following is an equilateral triangle ?

(a) $\triangle MNE$ (b) $\triangle MBE$ (c) $\triangle ACE$ (d) $\triangle MBC$
- (10) The number of all faces of a regular pentagonal pyramid is

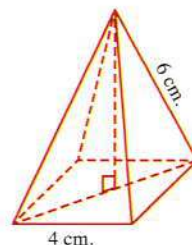
(a) 5 (b) 6 (c) 7 (d) 10
- (11) If the number of the faces of a pyramid = m and the number of its vertices = n, then the number of its edges =

(a) $m + n$ (b) $m + n - 1$ (c) $m + n - 2$ (d) $m + n + 2$
- (12) In the hexagonal pyramid :

number of faces + number of vertices - number of edges =


(a) 1 (b) 2 (c) 3 (d) 4
- (13) The opposite figure represents a regular quadrilateral pyramid of height = cm.

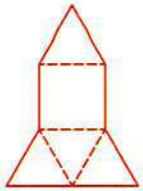
(a) $7\sqrt{2}$ (b) $2\sqrt{7}$
 (c) $4\sqrt{2}$ (d) $2\sqrt{5}$



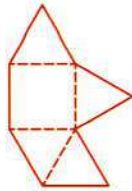
- (14) A regular quadrilateral pyramid, if the length of its base side is 6 cm., the length of its lateral edge is 8 cm., then the length of its height = cm.

(a) $5\sqrt{2}$ (b) $\sqrt{46}$ (c) $\sqrt{85}$ (d) $\sqrt{48}$

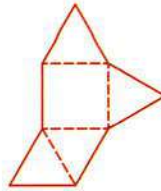
- (15)  Which of the following nets does not make a regular quadrilateral pyramid when it folded ?



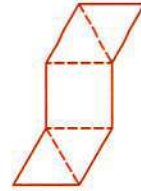
(a)



(b)



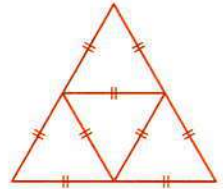
(c)



(d)

- (16) Which solid represents the opposite net ?

- (a) Quadrilateral pyramid.
(b) Regular quadrilateral pyramid.
(c) Triangular regular faces pyramid.
(d) Otherwise.



- (17) The ratio between the edge length of the triangular regular faces pyramid : its height =

(a) $\sqrt{2} : \sqrt{3}$ (b) $\sqrt{3} : 2$ (c) $\sqrt{6} : 2$ (d) $\sqrt{3} : 3$

- (18) The ratio between the length of the edge of the regular faces pyramid : its slant height =


(a) $2\sqrt{2} : \sqrt{3}$ (b) $2\sqrt{3} : 3$ (c) $\sqrt{6} : 2$ (d) $\sqrt{6} : 3$

- (19) If we cut a regular quadrilateral pyramid by a plane parallel to its base, then the resulted section is

- (a) triangle. (b) square. (c) rectangle. (d) circle.

- (20) A right quadrilateral pyramid of height 10 cm., its base is a rhombus whose diagonal lengths are 12 cm. and 8 cm., then its volume = cm^3

(a) 40 (b) 80 (c) 160 (d) 200

- (21)  A regular quadrilateral pyramid whose base perimeter is 36 cm., and its height is 10 cm., then its volume = cm^3

(a) 810 (b) 180 (c) 360 (d) 270

- (22) A regular hexagonal pyramid, the side length of its base = 8 cm. and its height = 10 cm. , then its volume equal cm^3
 (a) $320\sqrt{3}$ (b) $960\sqrt{3}$ (c) $\frac{320\sqrt{3}}{3}$ (d) 160
- (23) A regular pyramid whose volume is 12 cm^3 , and its base area is 4 cm^2 , then its height = cm.
 (a) 3 (b) 6 (c) 9 (d) 2
- (24) A regular quadrilateral pyramid whose volume 64 cm^3 , and its height is 6 cm. , then its base perimeter = cm.
 (a) 8 (b) $8\sqrt{2}$ (c) 16 (d) $16\sqrt{2}$
- (25) A regular quadrilateral pyramid whose volume is 480 cm^3 , and its base length is 12 cm. , then the length of its height = cm.
 (a) 10 (b) 20 (c) 30 (d) 15
- (26) If the volume of a regular hexagonal pyramid equal $8\sqrt{3} \text{ cm}^3$, and its height length equal 4 cm. , then the perimeter of its base = cm.
 (a) 2 (b) 12 (c) 6 (d) $6\sqrt{3}$
- (27) In a regular quadrilateral pyramid , the length of its base is 10 cm. , the length of its slant height is 13 cm. , then its lateral area equal cm^2
 (a) 260 (b) 360 (c) 130 (d) 520
- (28) A regular quadrilateral pyramid , the area of its base = 100 cm^2 and its height is 12 cm. , then its lateral area equal cm^2
 (a) 260 (b) 520 (c) 130 (d) 360
- (29) The total area of a right quadrilateral pyramid , its base is a regular polygon and its diagonal length = $10\sqrt{2} \text{ cm}$. and its height = $5\sqrt{3} \text{ cm}$. equals cm^2
 (a) 40 (b) 100 (c) 200 (d) 300
- (30) A regular quadrilateral pyramid whose lateral area = 30 cm^2 , and its slant height = 5 cm. , then its base perimeter = cm.
 (a) 6 (b) 12 (c) 24 (d) 36
- (31) A triangular regular faces pyramid , its edge length 10 cm. , then its total area equal cm^2
 (a) 40 (b) 100 (c) $100\sqrt{3}$ (d) $25\sqrt{3}$
- (32) If the sum of edge lengths of a triangular regular faces pyramid equals 18 cm. , then its total area = cm^2
 (a) $\frac{27\sqrt{2}}{4}$ (b) $\frac{27\sqrt{3}}{4}$ (c) $9\sqrt{3}$ (d) $\frac{27\sqrt{3}}{2}$

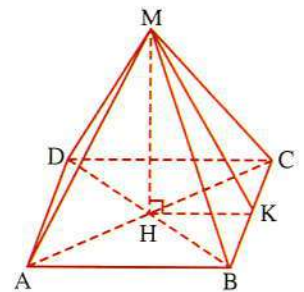
- (33) If the total area of a regular faces pyramid = $36\sqrt{3} \text{ cm}^2$, then the sum of its edges lengths = cm.
 (a) 6 (b) 12 (c) 18 (d) 36
- (34) If the total area of a triangular regular faces is $9\sqrt{3} \text{ cm}^2$, then the length of its edge cm.
 (a) 3 (b) 9 (c) 27 (d) $\sqrt{3}$
- (35) A regular triangular pyramid, its base length is $6 \ell \text{ cm}$. and its height $\ell \text{ cm}$. , then its lateral area = cm^2 .
 (a) $27\sqrt{3} \ell^2$ (b) $18 \ell^2$ (c) $9\sqrt{3} \ell^2$ (d) $36 \ell^2$
- (36) A triangular regular faces pyramid, its edge length 6 cm . , then its volume = cm^3 .
 (a) $27\sqrt{3}$ (b) $36\sqrt{3}$ (c) $54\sqrt{2}$ (d) $18\sqrt{2}$
- (37) A triangular regular faces pyramid, if the sum of the lengths of its edges equal 18 cm . , then its volume = cm^3 .
 (a) $9\sqrt{2}$ (b) $\frac{9\sqrt{2}}{4}$ (c) $\frac{27\sqrt{2}}{5}$ (d) $9\sqrt{3}$
- (38) If the slant height of a triangular regular faces pyramid equals $5\sqrt{3} \text{ cm}$. , then the sum of areas of its faces = cm^2 .
 (a) $\frac{50\sqrt{3}}{3}$ (b) $25\sqrt{3}$ (c) $100\sqrt{3}$ (d) $50\sqrt{3}$
- (39) A right quadrilateral pyramid whose base is a rhombus of side length equals to one of the diagonals of the rhombus equals 6 cm . , if the height of the pyramid = 12 cm . , then its volume = cm^3 .
 (a) $72\sqrt{3}$ (b) $216\sqrt{3}$ (c) 144 (d) 72
- (40) ABCDABCD is a cube of edge length = 6 cm . , then volume of the pyramid BABC = cm^3 .
 (a) 36 (b) 72 (c) $36\sqrt{3}$ (d) $18\sqrt{3}$
- (41) A regular quadrilateral pyramid whose total area = 70 cm^2 and its lateral area = 45 cm^2 , then its height = cm.
 (a) 2.5 (b) 5 (c) $\sqrt{14}$ (d) 4.5
- (42) In a regular quadrilateral pyramid the length of its base side 10 cm . and the area of one of its lateral faces 60 cm^2 , then its total area equal cm^2 .
 (a) 600 (b) 340 (c) 160 (d) 240
- (43) The ratio between the lateral surface area of a triangular pyramid of regular faces to the area of its total surface area =
 (a) 1 : 3 (b) 1 : 4 (c) 3 : 4 (d) 1 : 2

- (44) A quadrilateral regular pyramid, the length of its base side = its slant height, then the ratio between its lateral surface area to its total surface area =
- (a) 2 : 3 (b) 3 : 4 (c) 1 : 2 (d) 3 : 5
- (45) A quadrilateral regular pyramid, the area of any face from its lateral faces equals the area of its base, if the side length of the base of the pyramid is 6 cm., then volume of the pyramid = cm^3
- (a) 36 (b) $6\sqrt{3}$ (c) $36\sqrt{15}$ (d) $216\sqrt{15}$
- (46) If the side length of the base of quadrilateral regular pyramid is doubled but its height remains constant, then its volume
- (a) is doubled. (b) will not change.
(c) become four times its first volume.
(d) become six times its first volume.
- (47) In a regular quadrilateral pyramid, the side length of its base = 18 cm., if its volume = 1296 cm^3 , then its lateral area = cm^2
- (a) 270 (b) 360 (c) 450 (d) 540
- (48) A right pyramid whose base is a square, and all its eight edges are equal in length and each one = a cm., then its lateral area =
- (a) $3a^2$ (b) $4a^2$ (c) $\sqrt{3}a^2$ (d) $4\sqrt{3}a^2$
- (49) In the triangular pyramid MABC, the vertex (M) is at a distance 15 cm. from its base ABC and the sides lengths of its base 5, 6, 7 cm., then its volume = cm^3
- (a) $15\sqrt{3}$ (b) $10\sqrt{6}$ (c) $30\sqrt{6}$ (d) 90

(50) In the opposite figure :

MABCD is a regular quadrilateral pyramid
 , its volume 48 cm^3 , its height 4 cm.
 , $KC = KB$, $\overline{AC} \cap \overline{BD} = \{H\}$,
 $m(\angle MHK) = m(\angle HKB) = m(\angle MKB) = 90^\circ$
 , then its lateral area = cm^2

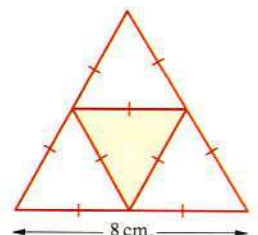
- (a) 18 (b) 24 (c) 36 (d) 60



(51) By using the opposite solid net :

The lateral area of the resulted solid = cm^2

- (a) $8\sqrt{3}$ (b) $12\sqrt{3}$
(c) $16\sqrt{3}$ (d) 24



Second Essay questions

1 In the regular pentagonal pyramid :

- (1) What the number of its lateral faces ?
- (2) What the number of its faces ?
- (3) What the number of its lateral edges ?
- (4) What the number of its edges ?
- (5) The pyramid has one vertex regardless of the vertices of the base. What is the number of all vertices of pentagonal pyramid ? Is your answer prove Euler's rule for any solid , its base is a polygon.

2 MABCD is a regular quadrilateral pyramid , the length of its base side is 10 cm. , and its height is 12 cm. , find its slant height. « 13 cm. »

3 MABCD is a regular quadrilateral pyramid of height 20 cm. and slant height 25 cm. Find the length of its base side. « 30 cm. »

4 MABCD is a regular quadrilateral pyramid , its base as a square ABCD , if its height equals $4\sqrt{3}$ cm. , and its lateral edge length $MA = 4\sqrt{5}$ cm. , find the length of its base side. « 8 cm. »

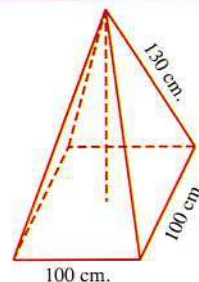
5 MABC is a regular triangular pyramid whose base is the equilateral triangle ABC whose side length 12 cm. , if the height of the pyramid is 6 cm. Find the length of its lateral edge. « $2\sqrt{21}$ cm. »

6 MABC is a regular triangular pyramid whose base ABC as an equilateral triangle of side length 3 cm. , if the length of its lateral edge is $\sqrt{7}$ cm. Find the height of the pyramid. « 2 cm. »

7 MABC is a triangular pyramid with regular faces , the length of its edge is 12 cm. Find its height and its slant height. « $4\sqrt{6}$ cm. , $6\sqrt{3}$ cm. »

8 A regular hexagonal pyramid whose height 8 cm. , its base as a regular hexagon of perimeter $24\sqrt{3}$ cm. Find the length of its lateral edge and its slant height. « $4\sqrt{7}$ cm. , 10 cm. »

9 The opposite figure represents a water tank as a regular quadrilateral pyramid , use the given data to find the height of the lateral face and the height of the tank.



« 120 cm. , $10\sqrt{119}$ cm. »

- 10** Each of the following figures represents a solid net. Describe the solid and find its height :

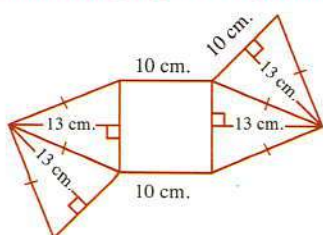


Fig. (1)

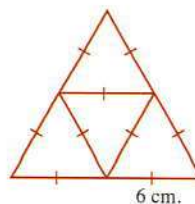


Fig. (2)

« 12 cm. , $2\sqrt{6}$ cm. »

- 11** The great pyramid of Giza (Khopo pyramid) is a regular quadrilateral pyramid the side length of its base is 232 metres and its slant height is 186 metres. Find height of the pyramid.

« 145.4 m. »

- 12** MABC is a right triangular pyramid , the length of its edge MA = 25 cm. , and its base ABC as a right-angled triangle at A. If BA = 16.8 cm. , CA = 12.6 cm. Find the height of the pyramid.

« 24 cm. »

- 13** MABCD is a right quadrilateral pyramid , whose base is the rhombus ABCD where AC = 16 cm. , BD = 12 cm. , N is the point of intersection of its diagonals. If the height of the pyramid MN = 10 cm. Find the lengths of its lateral edges.

« $2\sqrt{34}$ cm. , $2\sqrt{41}$ cm. »

- 14** A regular triangular pyramid whose height 12 cm. , and the side length of its base is 18 cm. Find its volume.

« $324\sqrt{3}$ cm³. »

- 15** MABCD is a regular quadrilateral pyramid , its base ABCD where AB = 10 cm. , and the height of the pyramid = 12 cm.

Find : (1) The length of any slant height.

(2) The volume of the pyramid.

(3) The total area of the pyramid.

« 13 cm. , 400 cm², 360 cm². »

- 16** A regular quadrilateral pyramid the length of its base is 20 cm. , and its height is $10\sqrt{3}$ cm.

Find : (1) Its lateral surface area.

(2) Its volume. « 800 cm² , $\frac{4000}{3}\sqrt{3}$ cm³. »

- 17** A regular quadrilateral pyramid , the length of its base diagonal is $24\sqrt{2}$ cm. , and its slant height = 20 cm. , find its total area and its volume.

« 1536 cm² , 3072 cm³. »

- 18** MABCD is right quadrilateral pyramid , its base is the square ABCD whose side length $8\sqrt{2}$ cm. , and the length of its lateral edge is $4\sqrt{6}$ cm.

Find : (1) The lateral surface area of the pyramid.

(2) The volume of the pyramid.

« $128\sqrt{2}$ cm² , $\frac{512}{3}\sqrt{2}$ cm³. »

- 19** MABCD is a regular quadrilateral pyramid, the side length of its base = 20 cm. and the length of its lateral edge is 26 cm.

Find : (1) The slant height of the pyramid. (2) The height of the pyramid.
(3) The lateral area of the pyramid. (4) The volume of the pyramid.


« 24 cm. , $2\sqrt{119}$ cm. , 960 cm^2 , $\frac{800}{3}\sqrt{119} \text{ cm}^3$ »

- 20** A triangular regular faces pyramid, its edge length = 12 cm. , find its height, volume and total area.


« $4\sqrt{6}$ cm. , $144\sqrt{2} \text{ cm}^3$, $144\sqrt{3} \text{ cm}^2$ »

- 21** MABCD is a right pyramid whose base ABCD as a square of side length = 18 cm. , MA = MB = MC = MD = 15 cm.

Find : (1) The total area. (2) The volume. « 756 cm^2 , $324\sqrt{7} \text{ cm}^3$ »

- 22**  Calculate to the nearest tenth the volume of a regular pentagonal pyramid whose side length of its base = 16 cm. and its height = 12 cm.

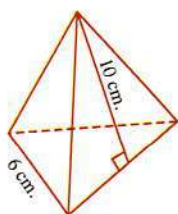
« 1761.8 cm^3 »

- 23**  A regular hexagonal pyramid, the side length of its base = 12 cm. and its slant height = $10\sqrt{3}$ cm. ,

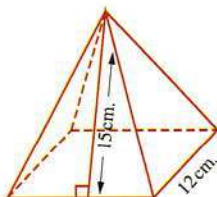
Find : (1) Its lateral area. (2) Its total area. « $360\sqrt{3} \text{ cm}^2$, $576\sqrt{3} \text{ cm}^2$ »

- 24** Find the lateral area and the total area of each regular pyramid of the following :

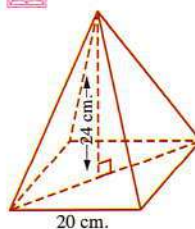
(1) 



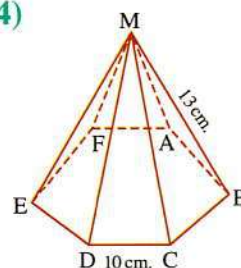
(2) 



(3) 

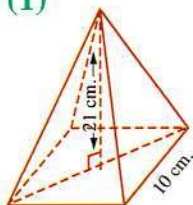


(4)

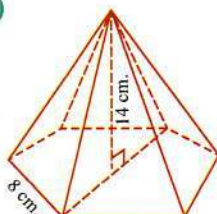


- 25**  Find the volume of each of the following regular pyramids :

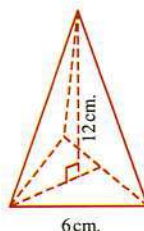
(1)



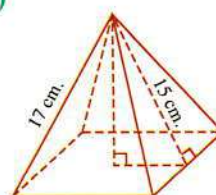
(2)






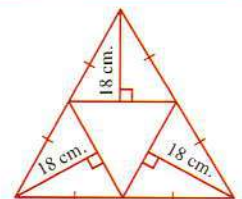
(3)



(4)



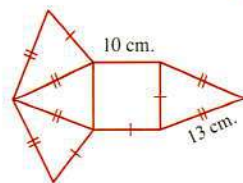
- 26** A quadrilateral pyramid whose height is 12 cm. , and its base as a rhombus of diagonals length 4 cm. and 8 cm. , prove that its volume equals the volume of a cube of edge length 4 cm.
- 27** MABC is a triangular pyramid whose vertex M at distance 15 cm. form its base ABC , and the side lengths of its triangular base are 5 , 6 and 7 cm. Find its volume « $30\sqrt{6} \text{ cm}^3$ »
- 28**  A regular quadrilateral pyramid whose base area is 700 cm^2 and its slant height is 20 cm. Find its volume. « 3500 cm^3 »
- 29** A regular quadrilateral pyramid whose base area is 9 cm^2 and the length of its lateral edge is 5 cm. Find its volume. « 13.6 cm^3 »
- 30** A regular quadrilateral pyramid whose volume is 400 cm^3 and its height is 12 cm. Find its lateral area. « 260 cm^2 »
- 31**  A regular quadrilateral pyramid , the side length of its base is 18 cm. , and its volume is 1296 cm^3
Find its slant height and its lateral surface area. « $15 \text{ cm.} , 540 \text{ cm}^2$ »
- 32** A regular quadrilateral pyramid , the side length of its base = 12 cm. , and its total area = 384 cm^2 Find its volume. « 384 cm^3 »
- 33** A right pyramid whose base is as a square of diagonal length = $10\sqrt{2} \text{ cm}$.
If its lateral area = 260 cm^2 . Find the volume of the pyramid. « 400 cm^3 »
- 34** MABC is a regular triangular pyramid , the side length of its base is 3 cm. , and the length of its lateral edge = $\sqrt{7} \text{ cm}$. Find its volume and lateral area. « $\frac{3\sqrt{3}}{2} \text{ cm}^3 , \frac{9}{4}\sqrt{19} \text{ cm}^2$ »
- 35** A regular hexagonal pyramid whose height is 8 cm. and its base perimeter is $24\sqrt{3} \text{ cm}$. ,
Find its lateral and total area. « $120\sqrt{3} \text{ cm}^2 , 192\sqrt{3} \text{ cm}^2$ »
- 36** Find the volume of the right pyramid whose slant height is 10 cm. , and its base as an equilateral triangle drawn inside a circle of radius length 12 cm. « $288\sqrt{3} \text{ cm}^3$ »
- 37**  Use the opposite net to discribe the solid , then find its total area.



« $432\sqrt{3} \text{ cm}^2$ »

38  **Connecting to industry :**

Products containers of a factory manufactured from cardboard by folding the net of the opposite figure.



(1) Find the area of the used cardboard to produce 1000 containers.

(2) Calculate the costs of the used cardboard if each square metre costs 15 pounds.

« 34 m² , 510 pounds »

39 MABCD is a right quadrilateral pyramid whose base is the square ABCD , if the length of each lateral edge equals $6\sqrt{5}$ cm. and height of the pyramid = $6\sqrt{3}$ cm.

Find : (1) The total area of the pyramid.

(2) The volume of the pyramid.

« 432 cm² , 288 $\sqrt{3}$ cm³ »

40  **Connecting to tourism :**

A model of the great pyramid (regular quadrilateral pyramid) is made of metallic alloy its density is 3.2 gm./cm³. If the length of the model base side 11.5 cm. and its height 7 cm. , then calculate its mass to the nearest one decimal place.

« 987. 5 gm. »

41 France cared about the ancient Egyptian monuments. So it transported some of them to Paris to be shown in their museums. It also set up a pyramid its side faces of transparent glass similar to the great pyramid (regular quadrilateral pyramid) to be a main entrance to the Louvre in Paris. If you know its height 21.6 metres , and the length of its base side 35 metre , then find the area of the glass used in its building to the nearest square cm.

« 1946 m² »

Third Higher skills

1 A regular hexagonal pyramid , the side length of its base = 2ℓ , and its height = 3ℓ ,

Prove that : The lateral area of the pyramid equals twice of its base area.

2 MABCD is a regular quadrilateral pyramid , if the length of its lateral edge = length of the diagonal of its base = ℓ , prove that : the total area of the pyramid = $\frac{\ell^2}{2} (1 + \sqrt{7})$

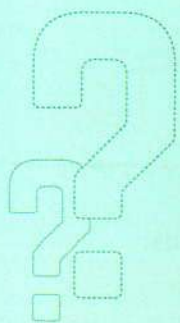
3 A hollow circular cylinder , put inside it a triangular pyramid MABC whose base ABC is an equilateral triangle whose vertices lies on perimeter of the lower base of the cylinder , M (vertex of the pyramid) is the centre of the upper base of the cylinder.

Find the ratio between volume of the pyramid and volume of the cylinder.

« $\frac{\sqrt{3}}{4\pi}$ »

4 A right pyramid whose base as a square , and all its eight edges are equal in length , if its total area = $(\sqrt{3} + 1) A$, find the length of its edge in terms of (A)

« \sqrt{A} »



Exercise

8

The cone



From the school book

Remember Understand Apply Higher Order Thinking Skills

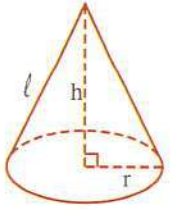


Test yourself

First Multiple choice questions

Choose the correct answer from the given ones :

- (1) The right circular cone is generated by folding a paper in the shape of
 - (a) an equilateral triangle.
 - (b) a right-angled triangle.
 - (c) a circular segment.
 - (d) a circular sector.
- (2) The measure of the smallest rotation angle of an isosceles triangle around its axis of symmetry to form a right circular cone is
 - (a) 90°
 - (b) 180°
 - (c) 270°
 - (d) 60°
- (3) The right circular cone is formed from rotation of a right-angled triangle a complete rotation about
 - (a) its hypotenuse.
 - (b) one of its right sides.
 - (c) any straight line in the plane of the triangle.
 - (d) any straight line passes through one of its vertices and parallel to the opposite side of this vertex.
- (4) If a right circular cone intersected by a plane parallel to its base , then the resulted sector is
 - (a) an isosceles triangle.
 - (b) an equilateral triangle.
 - (c) a circle.
 - (d) a trapezoid.



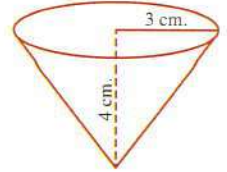
(5) The total area for the opposite right cone equals

- (a) $\pi r l$ (b) $\frac{\pi}{3} \pi^2 h$
(c) $\pi r (r + l)$ (d) $\frac{\pi}{3} r (r h + 3 l)$

(6) In the opposite figure :

The length of the drawer = cm.

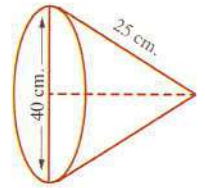
- (a) 2 (b) 3
(c) 4 (d) 5



(7) In the opposite figure :

The height of the cone = cm.

- (a) 15 (b) 20
(c) 25 (d) 40



(8) In a right circular cone , if the length of its height 15 cm. , and the length of its drawer 17 cm. , then its radius length equal cm.

- (a) 10 (b) 8 (c) 7 (d) 9

(9) In a right circular cone , the radius length of its base = 15 cm. and its height = 20 cm. , then its lateral area = cm^2

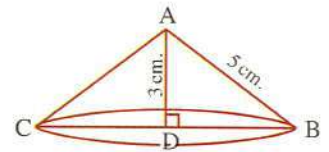
- (a) 375π (b) 600π (c) 1500π (d) 1875π

(10) In the opposite figure :

If AD = 3 cm. , AB = 5 cm.

, then the total area of the cone = cm^2

- (a) 8π (b) 24π (c) 48π (d) 36π



(11) If the length of the diameter of the base of a right circular cone is 12 cm. and its height 8 cm. , then its lateral area equal cm^2




- (a) 60π (b) 28π (c) 10π (d) 48π

(12) The height of a right circular cone is 6 cm. and the circumference of its base is 16π cm. , then its lateral area = cm^2

- (a) 144π (b) 64π (c) 60π (d) 80π

(13) A right circular cone , the length of its drawer equal the length of the diameter of its base , then its total area = cm^2

- (a) $3 \pi r^2$ (b) $3 \pi r^3$ (c) $4 \pi r^2$ (d) $4 \pi r^3$

- (14)  A right circular cone, its height 24 cm., and the length of its drawer 26 cm., then the area of its base cm^2
 (a) 25π (b) 100π (c) 20π (d) 50π
- (15)  The radius length of the base of a right circular cone where its total area = $616\pi \text{ cm}^2$, and the length of its drawer is 30 cm. is cm.
 (a) 44 (b) 14 (c) 30 (d) 34
- (16)  Lamp cover is in the form of a right circular cone, the circumference of its base circle = 88 cm., its height = 20 cm., then its lateral area \approx cm^2 ($\pi = \frac{22}{7}$)
 (a) 88 (b) 596 (c) 1074 (d) 1047
- (17) A right circular cone, the radius length of its base = 6 cm. and the length of its drawer = 10 cm., then its volume = cm^3
 (a) 32π (b) 64π (c) 96π (d) 228π
- (18) A right circular cone where its height 4 cm., the length of its drawer 5 cm., then its volume cm^3
 (a) 36π (b) 15π (c) 24π (d) 12π
- (19) ABC is an equilateral triangle. Its side length (ℓ) it turned around \overline{BC} as a rotation axis a complete turn, then the volume of the generated solid in terms of π and ℓ is
 (a) $\frac{\pi \ell}{4}$ (b) $\frac{\pi}{4} \ell^2$ (c) $\frac{\pi \ell^3}{4}$ (d) $\sqrt{3} \pi \ell^2$
- (20) A right circular cone where its volume $27\pi \text{ cm}^3$, and the circumference of its base $6\pi \text{ cm}$, then its height equal cm.
 (a) 27 (b) 18 (c) 9 (d) 6
- (21) A right circular cone, the radius length of its base 5 cm. and its total area = $90\pi \text{ cm}^2$, then its volume = cm^3
 (a) 105π (b) 95π (c) 100π (d) 120π
- (22) The volume of a right circular cone, if the length of its drawer = 15 cm. and its total area = $216\pi \text{ cm}^2$ equal cm^3
 (a) 205π (b) 220π (c) 280π (d) 324π
- (23) A right circular cone where the length of its drawer 25 cm. and its lateral area 550 cm^2 , then its volume = cm^3 where ($\pi = \frac{22}{7}$)
 (a) 1223 (b) 1232 (c) 1322 (d) 3122
- (24) If the volume of a right circular cone is $9\pi \text{ cm}^3$ and the length of its base radius equal the length of its height, then its base area = cm^2
 (a) 9π (b) 3π (c) 27π (d) 12π

(25) The volume of a right circular cone 100 cm^3 , then its volume when the radius length of its base is doubled = cm^3 .

- (a) 100 (b) 200 (c) 300 (d) 400

(26) In a right circular cone, if the length of its radius base increased to its double, and the length of the height is decreased to its half, then its volume

- (a) don't change. (b) increased to its double.
(c) decreased to its half. (d) increased to its four times.

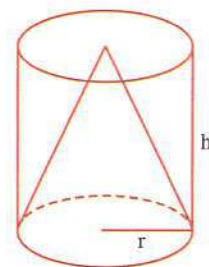
(27) The radius of the base of a right circular cone = twice its height = 6 cm. and uniform quadrilateral pyramid its base length = its height = 6 cm., then the ratio between the volume of the cone : the volume of the pyramid = :

- (a) $3 : \pi$ (b) $\pi : 3$ (c) $\pi : 2$ (d) $2 : \pi$

(28) In the opposite figure :

$\frac{\text{the volume of the cone}}{\text{the volume of the cylinder}} = \dots\dots\dots$

- (a) $\frac{2}{3}$ (b) $\frac{1}{3}$
(c) $\frac{1}{4}$ (d) $\frac{3}{1}$



(29) In the opposite figure :

A right regular pyramid and right circular cone have common vertex and the circle of base of the cone touches sides of the base of the pyramid internally :

First : The lateral area of the right circular cone = cm^2 .

- (a) 60 (b) 60π (c) 48 (d) 48π

Second : The total area of the regular pyramid equals cm^2 .

- (a) 360 (b) 240 (c) 384 (d) 432

Third : The volume of the pyramid equals cm^3 .

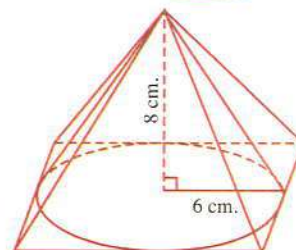
- (a) 64 (b) 96 (c) 480 (d) 384

Fourth : The ratio between the volume of the pyramid and the volume of the cone equals

- (a) $\pi : 3$ (b) $4 : \pi$ (c) $\pi : 4$ (d) $3 : \pi$

Fifth : The ratio between the lateral area of the pyramid and the lateral area of the cone equals

- (a) $\pi : 3$ (b) $4 : \pi$ (c) $\pi : 4$ (d) $3 : \pi$



- (30) The opposite net describes a solid
its volume = cm^3
- (a) 25π (b) 50π
(c) 75π (d) 100π

- (31) The opposite net describes a solid its
volume = $96\pi \text{ cm}^3$
, then its total area = cm^2

- (a) 16π (b) 32π
(c) 48π (d) 96π

- (32) The opposite figure represents the net of a solid
, $MB = 3\pi \text{ cm}$, $m(\angle AMB) = 120^\circ$
, then the volume of the solid = cm^3

- (a) $2\sqrt{2}\pi^2$ (b) $\frac{2\sqrt{2}}{3}\pi^4$
(c) $2\sqrt{2}\pi$ (d) $\frac{2\sqrt{2}}{3}\pi^3$

- (33) In the opposite figure a circle is divided
into two circular sectors such that they
form two right cone nets
, then $\frac{\text{the lateral area of the smallest cone}}{\text{the lateral area of the greatest cone}} = \dots\dots\dots$

- (a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{8}$

- (34) In the opposite figure :

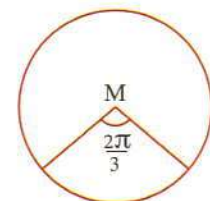
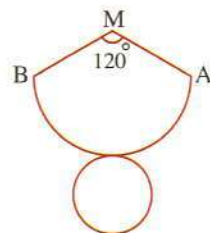
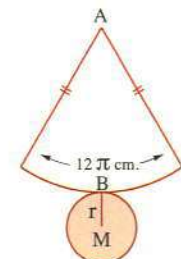
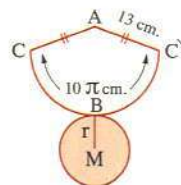
If we fold the shown net it becomes a cone
its base radius length is cm .

- (a) 10 (b) 8
(c) 5 (d) 2.5

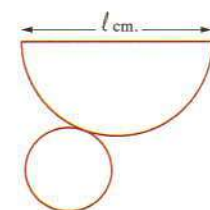
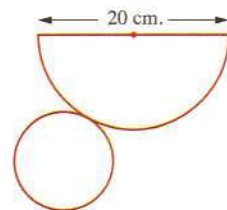
- (35) In the opposite figure :

If we folded this net it becomes a cone
its base radius is cm .

- (a) $\frac{l}{2}$ (b) $\frac{l}{3}$
(c) $\frac{l}{4}$ (d) $\frac{l}{5}$

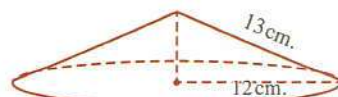


- (d) $\frac{1}{16}$



- (36) The central angle of the sector if be folded it becomes the opposite cone is

(a) acute. (b) obtuse. (c) straight. (d) reflex.



- (37) If we folded the circular sector it becomes a right circular cone , its drawer length 10 cm. and the radius length of its base 5 cm. , then the central angle of this sector is

(a) acute. (b) obtuse. (c) straight. (d) reflex.

- (38) If we have a quarter circle , its radius length 16 cm. , then the radius length of the base of the cone which can be formed from the arc of the quarter circle = cm.

(a) 16 (b) 8 (c) 4 (d) 2

- (39) The area of a circular sector : the total area of the circular solid cone which can be formed from folding this sector

(a) > 1 (b) < 1 (c) $= 1$ (d) ≥ 1

- (40) The ratio between the volume of a regular quadrilateral pyramid and the volume of the smallest circular cone contains the pyramid equals

(a) $2 : \pi$ (b) $4 : \pi$ (c) $6 : \pi$ (d) $8 : \pi$

- (41) In the opposite figure :

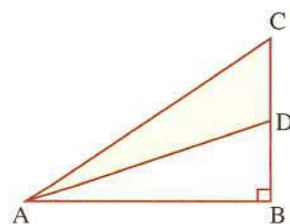
If $AB = 3$ cm. , $BD = CD = 1$ cm. ,

$m(\angle ABC) = 90^\circ$, then the volume of

the solid generated by turning the shaded

part around \overleftrightarrow{AB} as a rotation axis a complete turn =

(a) π (b) 2π (c) 3π (d) 4π



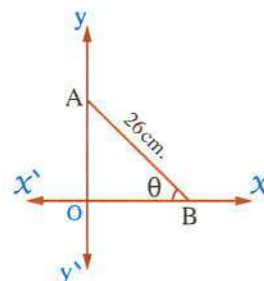
- (42) In the opposite figure :

If $\tan \theta = \frac{5}{12}$, $AB = 26$ cm. , then the lateral area

of the solid generated by rotating the triangle ABO

a complete revolution about the X -axis = π cm.²

(a) 360 (b) 260π
(c) 260 (d) 360π



(43) In the opposite figure :

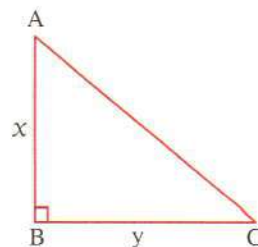
If v_1 is the volume of the cone produced by rotating the triangle ABC about \overleftrightarrow{AB} a complete revolution
 v_2 is the volume of the cone produced by rotating the triangle ABC about \overleftrightarrow{BC} a complete revolution
 , then $\frac{v_1}{v_2} = \dots\dots\dots$

(a) $\frac{1}{2}$

(b) $\frac{\pi}{2}$

(c) $\frac{x}{4}$

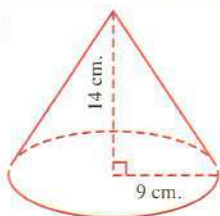
(d) $\frac{y}{x}$



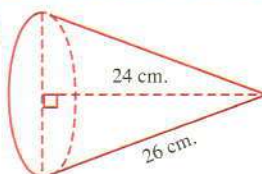
Second Essay questions

1 Find the volume of the right circular cone shown in each figure using the given data :

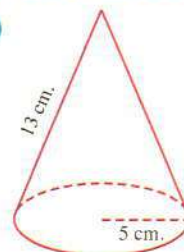
(1)



(2)

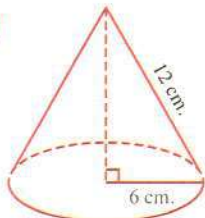


(3)

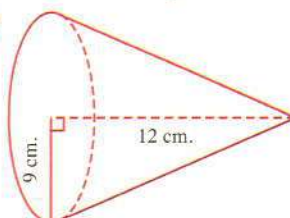


2 Find the lateral and the total areas of each right circular cone due to the given data :

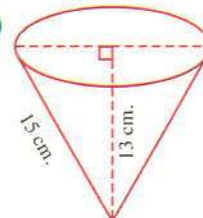
(1)



(2)



(3)



3 A right circular cone , its drawer length = 17 cm. its height = 15 cm. Find :

(1) Its lateral area.

(2) Its total area.

(3) Its volume.

« $136\pi \text{ cm}^2$, $200\pi \text{ cm}^2$, $320\pi \text{ cm}^3$ »

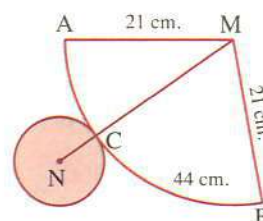
4 Find in terms of π the circumference and the area of the base of a right circular cone whose height is 24 cm. , and the length of its drawer is 26 cm.

« $20\pi \text{ cm}$, $100\pi \text{ cm}^2$ »

5 The opposite figure shows a net of a right cone

, use the given data to find its height ($\pi = \frac{22}{7}$)

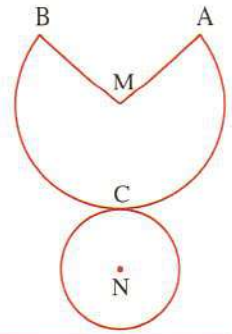
« $14\sqrt{2} \text{ cm}$. »



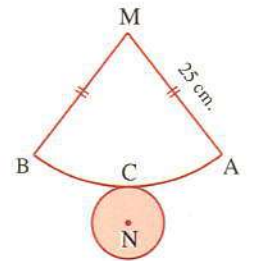
► Exercise 8 ?

- 6 The opposite figure represents a right cone net form from a circular sector whose area is $20\pi \text{ cm}^2$, the length of its arc $\widehat{ACB} = 8\pi \text{ cm}$. Find the height of the solid.

« 3 cm. »

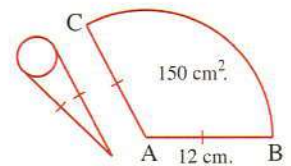


- 7 The opposite figure represents a solid net. Describe the resulting solid of the folding process and find its height if $MA = MB = 25 \text{ cm}$, area of the circle $N = 49\pi \text{ cm}^2$



« 24 cm. »

- 8 The frozen milk is encapsulated (kept) on a right circular cone by folding a piece of healthy - insulated paper in the form of circular sector the length of its radius is 12 cm. and its area is 150 cm^2 , where the two radii of the circle \overline{AB} , \overline{AC} become in contact. Find the height of the cone to the nearest one decimal.



« 11.3 cm. »

- 9 Find to the nearest tenth, the total area of the right circular cone in which the diameter length of its base is 10 cm. and its height is 12 cm.

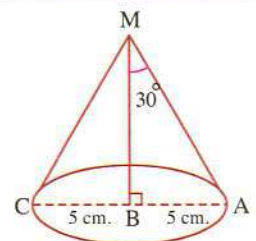
« 282.7 cm^2 . »

- 10 Find the volume of the right circular cone where the circumference of its base is 44 cm. and its height is 25 cm.

« 1283.8 cm^3 . »

- 11 In the opposite figure :

A right circular cone in which $m(\angle AMB) = 30^\circ$, the radius length of the base = 5 cm. Calculate its lateral area and also the total area.



« $50\pi \text{ cm}^2$, $75\pi \text{ cm}^2$. »

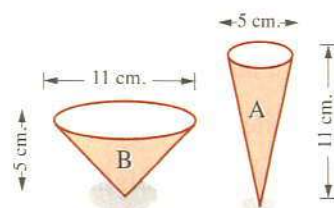
- 12 A right circular cone, the radius length of its base is 8 cm. and its lateral area = $96\pi \text{ cm}^2$. Find to the nearest one decimal the volume of this cone.

« 599.5 cm^3 . »

13 In the opposite figure :

A, B are two cups for drinking which of them has greater capacity ?

Find the difference between their capacities.

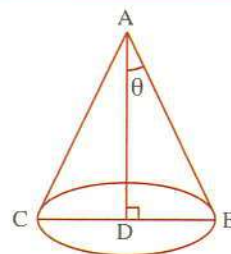


« The capacity of B is the greater, $\frac{55}{2} \pi \text{ cm}^3$. »

14 In the opposite figure :

If $\sin \theta = \frac{3}{5}$, and the height of the cone = 12 cm. ,

Find the total area of the cone.

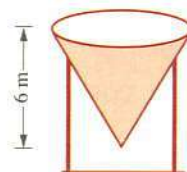


« $216 \pi \text{ cm}^2$ »

15 Civil engineering :

The opposite figure shows a water tank in the shape of a right circular cone , its volume = $32 \pi \text{ m}^3$ and its height = 6 m.

Find the radius length of its base and its total area.



« 4 m. , $(16 + 8\sqrt{13}) \pi \text{ m}^2$ »

16 Which is greater in volume ?

A right circular cone in which the radius length of its base is 15 cm. and its height is 20 cm. or a regular quadrilateral pyramid whose height is 40 cm. and its base perimeter = 48 cm.

17 A right circular cone , its height = h and its volume = πh^3 . Prove that its lateral area equals the lateral area of a right circular cylinder which is common with the cone in the base and the height.

18 Connecting to physics :

A cylindrical shaped vessel contain water , a metal body in the form of a right cone , its height is 12 cm. and the length of its base radius is 2 cm. and is completely immersed in it raising the surface of the water in the vessel with the value 1 cm.

Find the length of base diameter of the vessel.

« 8 cm. »


19 A cube made of wax , its edge length = 20 cm. it is melted and converted to a right circular cone of height 21 cm. Find the radius length of the base of the cone given that 12% from wax had been lost during melting and reforming. ($\pi = \frac{22}{7}$)

« $8\sqrt{5} \text{ cm}$. »

20 A container in the shape of a right cone of capacity 2.2 litre and its height = 21 cm.


Find the radius length of its base. ($\pi = \frac{22}{7}$)

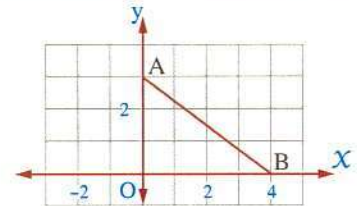
« 10 cm. »

- 21  A circular sector MAB, the radius length of its circle is 18 cm. and the measure of its central angle = 60° , it is folded and their radii are connected to form greatest lateral area of a right circular cone. Find the volume of this cone. « 167.3 cm^3 . »

- 22 AMB is a quadrant of a circle of centre M and its radius length = 20 cm. It is converted to the surface of a right circular cone where M is its vertex such that \overline{MA} coincide \overline{MB}
Find the radius length of the base of the cone also find its volume in π
« $5 \text{ cm.}, \frac{125\sqrt{15}}{3} \pi \text{ cm}^3$. »

- 23 ABC is a right-angled triangle at B in which AB = 6 cm. , BC = 8 cm.
Find the volume of the solid generated by turning ΔABC a complete turn around :
(1) \overrightarrow{BC} (2) \overrightarrow{AC} « $96 \pi \text{ cm}^3$, $76.8 \pi \text{ cm}^3$. »

- 24  The opposite figure shows a coordinate perpendicular plane. Calculate in terms of π the volume of solid generated when revolving triangle ABO one complete revolution around :

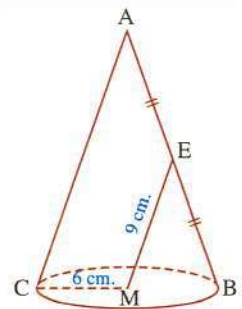


- (1) The X-axis (2) The y-axis « 12π cubic units , 16π cubic units »

- 25 ABC is an isosceles triangle in which AB = AC = 10 cm. and BC = 12 cm. It turned around the base \overline{BC} a complete turn. Calculate the volume of the generated solid. « $256 \pi \text{ cm}^3$. »

- 26 **In the opposite figure :**

Find the lateral area and the total area and the volume of the right circular cone.

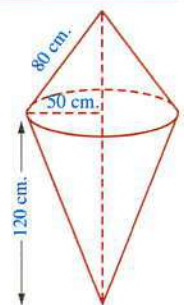


« $108 \pi \text{ cm}^2$, $144 \pi \text{ cm}^2$, $144\sqrt{2} \pi \text{ cm}^3$. »

- 27  **Marine navigation :**

The opposite figure shows a guide sign (Shamandora) (Buoy) to determine the waterway , and it is in the form of two right cones have a common base.

Find the costs of its painting with a material which resists erosion factor , note that each square metre of its costs 300 pound.



« 990 pounds »

28 Connecting with industry :

A regular pentagon pyramid made of copper , the side length of its base = 10 cm. and its height = 42 cm. it is melted and converted to a right circular cone the radius length of its base = 15 cm. given that 10% of copper has been lost during melting and converting it. Find the height of the cone to the nearest one decimal.

« 9.2 cm. »

29 Critical thinking :

A right circular cone of volume 100 cm^3 . Find its volume when :

- (1) Its height is doubled. (2) The length of its radius is doubled.
- (3) Its height is doubled and the length of its radius is doubled.

What you conclude ? Explain your answer.

« 200 cm^3 , 400 cm^3 , 800 cm^3 . »

Third Higher skills

1 Choose the correct answer from those given :

- (1) If the volume of hemisphere with raduis (r) equals the volume of cone with base raduis length (r) and height (h) , then

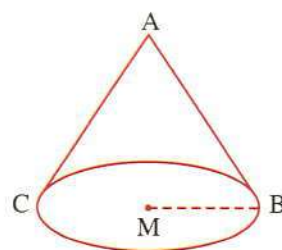
(a) $h = \frac{2}{3} r$ (b) $h = 2 r$ (c) $h = 2 r^2$ (d) $h = 4 r$

- (2) In the opposite figure :

The volume of a right circular cone is $96 \pi \text{ cm}^3$

and $\frac{MB}{AB} = \frac{3}{5}$, then its total surface area = cm^2

(a) 24π (b) 48π (c) 96π (d) 192π



- (3) The arc length of a circular sector that if it is folded it becomes a right circular cone whose volume is $49 \pi \text{ cm}^3$ and height 3 cm. equals cm.

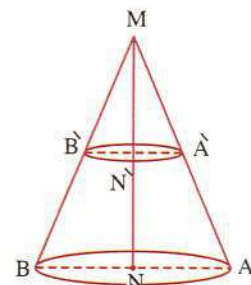
(a) 2π (b) 4π (c) 8π (d) 14π

- (4) In the opposite figure :

If a plane is drawn perpendicular to the cone axis and intersects it at midpoint of \overline{MN} , then

First : $\frac{\text{The volume of the smaller cone}}{\text{The volume of the greater cone}} = \dots\dots\dots$

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$
 (c) $\frac{1}{8}$ (d) $\frac{1}{16}$



Second : $\frac{\text{The lateral area of the smaller cone}}{\text{The lateral area of the greater cone}} = \dots\dots\dots$

- (a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{8}$ (d) $\frac{1}{16}$

- (5) The ratio between the volume of a regular triangular pyramid and the volume of the greatest right circular cone can fit inside of the pyramid equals

- (a) $\frac{3\sqrt{3}}{\pi}$ (b) $\frac{3\sqrt{3}}{2\pi}$ (c) $\frac{\sqrt{3}}{\pi}$ (d) $\frac{3\sqrt{3}}{4\pi}$

- (6) The ratio between the volume of a regular triangular pyramid and the volume of the smallest right circular cone can contain it equals

- (a) $\frac{3\sqrt{3}}{\pi}$ (b) $\frac{3\sqrt{3}}{2\pi}$ (c) $\frac{\sqrt{3}}{\pi}$ (d) $\frac{3\sqrt{3}}{4\pi}$

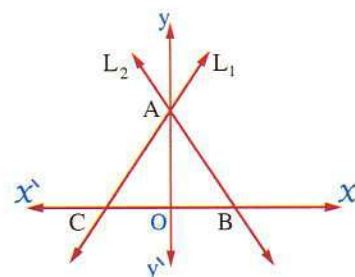
- (7) The volume of a right circular cone is (v). If its base radius length is increased 50 % and its height is increased 50 % and its volume after increase is (\hat{v}) , then

- (a) $\hat{v} = 150 \% v$ (b) $\hat{v} = 225 \% v$ (c) $\hat{v} = 337.5 \% v$ (d) $\hat{v} = 450 \% v$

2 In the opposite figure :

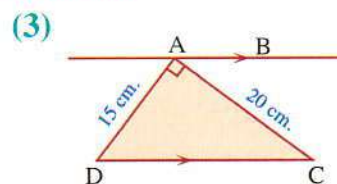
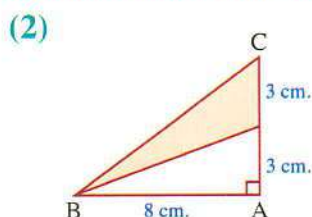
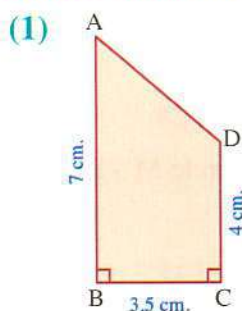
The equation of the straight line L_1 is $3x - \sqrt{3}y + 6 = 0$ and the equation of the straight line L_2 is $\sqrt{3}x + y - 2\sqrt{3} = 0$

Find the volume of the body generated from turning $\triangle ABC$ a complete turn around X-axis.

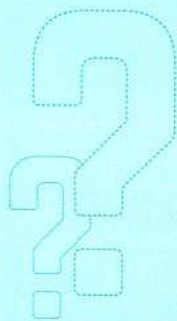


« 16π cube units »

3 Find the volume of the solid generated by turning the shaded part a complete turn around \overleftrightarrow{AB} as an axis of rotation in each of the following figures :



« $192.4, 226.2, 7539.8 \text{ cm}^3$ »



Exercise

9

The circle



From the school book

Remember Understand Apply Higher Order Thinking Skills







Test yourself

First Multiple choice questions

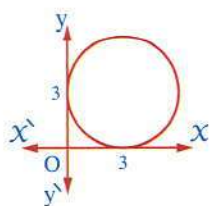
Choose the correct answer from the given ones :

- (1) The centre of the circle in which its diameter is \overline{AB} where $A = (-1, 3)$, $B = (5, -3)$ is
(a) $(4, 0)$ (b) $(2, 0)$ (c) $(-6, -6)$ (d) $(0, 4)$
- (2) The radius length of the circle whose equation $x^2 + y^2 - 4x + 2y - 4 = 0$ is length unit.
(a) 2 (b) 4 (c) 3 (d) 9
- (3) The radius length of the circle whose equation $(x + 2)^2 + y^2 + 2y = 0$ is length unit.
(a) zero (b) 1 (c) 2 (d) 4
- (4) The diameter length of the circle : $4x^2 + 4y^2 + 16x - 8y - 16 = 0$ equals length unit.
(a) 3 (b) 6 (c) 12 (d) 24
- (5) If the two straight lines $y = -6$, $y = 8$ are two tangents to the circle M, then its radius length = length unit.
(a) 1 (b) 2 (c) 7 (d) 14
- (6) If the straight line $y = 2$ touches the circle M whose center is $(6, 9)$, then its diameter length = length unit.
(a) 6 (b) 7 (c) 14 (d) 15

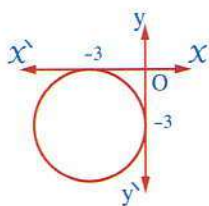
- (7) The radius length of the circle $(n+3)x^2 + y^2 - 4y + (m-2)xy + (m-n)x - 8 = 0$ is length unit.
 (a) 2 (b) 4 (c) 6 (d) $2\sqrt{2}$
- (8) The area of the circle whose equation is $(x-5)^2 + (y+4)^2 = 7$ equals square unit.
 (a) 3.5π (b) 7π (c) 12.25π (d) 49π
- (9) If the equation $2x^2 + ay^2 + bxy - 5 = 0$ represents a circle , then its area = square unit.
 (a) 5π (b) $\sqrt{5}\pi$ (c) $\frac{5}{2}\pi$ (d) $5\sqrt{2}\pi$
- (10) The circumference of the circle whose equation $x^2 + y^2 + 2x - 2y - 2 = 0$ is length unit.
 (a) π (b) 2π (c) 4π (d) 8π
- (11)  The circumference of the circle whose equation is $x^2 + y^2 = 8$ is length units.
 (a) 8π (b) 64π (c) $2\sqrt{2}\pi$ (d) $4\sqrt{2}\pi$
- (12) If the two straight lines : $x = -3$, $x = 4$ touch the circle M , then its circumference = length units. where $(\pi = \frac{22}{7})$
 (a) 22 (b) 44 (c) 12 (d) 14
- (13) If $(x \ y \ 8) \begin{pmatrix} x \\ y \\ -2 \end{pmatrix} = \square$, then the obtained equation represents a circle with diameter length = length unit. (Where \square is the zero matrix)
 (a) 2 (b) 4 (c) 6 (d) 8
- (14) The equation : $\begin{vmatrix} x & y & i \\ y & i & x \end{vmatrix} - 49 = 0$ represents the equation of a circle with radius length length unit.
 (a) 49 (b) 14 (c) 9 (d) 7
- (15) Which of the following equations represent a circle ?
 (a) $x^2 - y^2 + x - y = 6$ (b) $2x^2 + y^2 - x + y = 5$
 (c) $x^2 + y^2 - x = 6$ (d) $x^2 + y^2 - xy = 6$
- (16) If the equation : $2x^2 + (a-1)y^2 + 5x - 3y = 7$ represent a circle , then $a =$
 (a) 1 (b) 2 (c) 3 (d) 4

- (17) If $X^2 + y^2 + 2(\cos \theta)X - 2(\sin \theta)y - 8 = 0$ represent an equation of a circle , then $r = \dots\dots\dots$ length unit.
 (a) $\sqrt{2}$ (b) $2\sqrt{2}$ (c) 3 (d) 8
- (18) The center of the circle whose equation $(X - 2)^2 + (y + 3)^2 = 16$ is
 (a) (2 , 3) (b) (2 , - 3) (c) (13 , 16) (d) (4 , 9)
- (19) The centre of the circle whose equation $X^2 + y^2 - 6X + 8y = 0$ is the point
 (a) (3 , - 4) (b) (4 , - 3) (c) (- 3 , 4) (d) (- 4 , 3)
- (20) The centre of the circle whose equation $2X^2 + 2y^2 + 12X - 16y = 0$ is
 (a) (3 , - 4) (b) (- 6 , 8) (c) (- 3 , 4) (d) (6 , - 8)
- (21)  The circle $(X + 2)^2 + y^2 + 2y = 0$ its centre is the point
 (a) (2 , 2) (b) (- 2 , - 1) (c) (2 , - 1) (d) (- 2 , 0)
- (22) Centre of the circle passes through the origin and the two points A (- 6 , 0) , B (0 , 8) is
 (a) (4 , - 3) (b) (- 5 , 5) (c) (5 , 5) (d) (- 3 , 4)
- (23) If any circle touches the two coordinate axes and it is drawn in the first quadrant , then its centre lies on the straight line
 (a) $y = X$ (b) $y = -X$ (c) $y = X + 1$ (d) $y = X - 1$
- (24) How many circles whose centre (3 , - 5) and touches one of the two axes ?
 (a) 1 (b) 2 (c) 3 (d) 4
- (25) The point (2 , 2) lies the circle whose equation $X^2 + y^2 = 9$
 (a) on (b) inside (c) outside (d) in the centre of
- (26)  The point (2 , 0) lies on
 (a) X-axis. (b) y-axis.
 (c) the straight line $y = 2X$ (d) the circle $X^2 + y^2 = 9$
- (27)  The point which lies on the circle : $(X - 2)^2 + y^2 = 13$ is
 (a) (2 , 3) (b) (3 , - 2) (c) (2 , 5) (d) (4 , 3)
- (28) The circle whose equation : $(X - 1)^2 + (y + 2)^2 = 5$ passes through the point
 (a) (0 , 0) (b) (3 , - 1) (c) (2 , - 4) (d) All the previous.

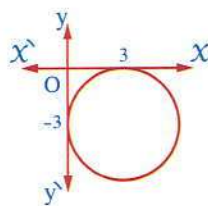
- (29) The circle $C : (X + 3)^2 + (y - 3)^2 = 9$ is represented by the figure



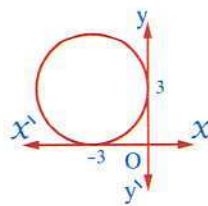
(a)



(b)



(c)



(d)

- (30) The general form of the equation of a circle if its centre is $(2, -1)$ and its radius length = 3 cm. is

(a) $X^2 + y^2 - 4X + 2y - 9 = 0$

(b) $X^2 + y^2 - 4X + 2y - 4 = 0$

(c) $X^2 + y^2 + 2X - y + 3 = 0$

(d) $X^2 + y^2 + 2X - y + 9 = 0$

- (31) The equation of the circle whose centre $(4, 3)$ and touches X -axis is

(a) $(X - 3)^2 + (y - 4)^2 = 16$

(b) $(X - 4)^2 + (y - 3)^2 = 9$

(c) $(X + 3)^2 + (y + 4)^2 = 9$

(d) $(X + 3)^2 + (y - 4)^2 = 16$


- (32) The equation of the circle whose centre $(-4, 4)$ and touches the two coordinate axes is

(a) $X^2 + y^2 + 8X - 8y + 16 = 0$

(b) $X^2 + y^2 = 16$

(c) $X^2 + y^2 - 8X + 8y + 16 = 0$

(d) $X^2 + y^2 = 8$

- (33)  The equation of the circle which is the image of the circle $X^2 + y^2 - 12X + 6y + 20 = 0$ by translation $(X + 2, y - 2)$ is

(a) $(X + 8)^2 + (y + 5)^2 = 25$

(b) $(X - 8)^2 + (y + 5)^2 = 25$

(c) $(X - 8)^2 + (y - 5)^2 = 25$

(d) $(X + 5)^2 + (y - 8)^2 = 25$

- (34) The equation of the circle whose centre $(-4, 3)$ and passes through the origin point is

(a) $(X + 4)^2 + (y - 3)^2 = 5$

(b) $(X - 4)^2 + (y + 3)^2 = 25$

(c) $(X + 4)^2 + (y - 3)^2 = 625$

(d) $(X + 4)^2 + (y - 3)^2 = 25$

- (35) The equation of the circle whose centre $(1, 2)$ and touches the line : $3X + 4y + 9 = 0$ is

(a) $X^2 + y^2 - 2X - 4y = 16$

(b) $X^2 + y^2 + 2X + 4y - 11 = 0$

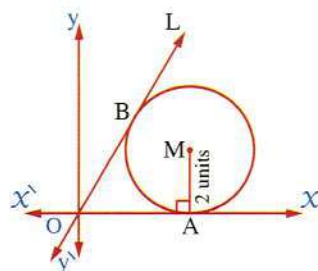
(c) $X^2 + y^2 + 2X + 4y - 16 = 0$

(d) $X^2 + y^2 - 2X - 4y = 11$

- (36) The circle equation whose centre lies on the straight line $y = \frac{1}{2}x$ and touches x -axis could be
- (a) $(x-2)^2 + (y-1)^2 = 4$ (b) $(x-4)^2 + (y-2)^2 = 16$
 (c) $(x-2)^2 + (y-4)^2 = 16$ (d) $(x-4)^2 + (y-2)^2 = 4$
- (37) The equation of the circle which concentric with the circle whose equation $x^2 + y^2 - 6x + 2y - 6 = 0$ and passes through the point $(-3, 4)$ is
- (a) $(x+3)^2 + y^2 = 16$ (b) $(x-3)^2 + (y+1)^2 = 25$
 (c) $(x-3)^2 + (y+1)^2 = 16$ (d) $(x-3)^2 + (y+1)^2 = 61$
- (38) In the following equations : The circle whose centre lies on the y -axis and does not intersect the x -axis is
- (a) $x^2 + (y-1)^2 = 4$ (b) $x^2 + (y-5)^2 = 25$
 (c) $x^2 + (y+5)^2 = 9$ (d) $(x+5)^2 + y^2 = 16$
- (39) The equation of circle whose centre $(-4, -3)$ and its surface area is $25\pi \text{ cm}^2$ is
- (a) $x^2 + y^2 - 8x + 6y - 25 = 0$ (b) $x^2 + y^2 + 8x + 6y = 0$
 (c) $x^2 + y^2 + 4x + 3y + 25 = 0$ (d) $x^2 + y^2 + 8x - 6y = 0$
- (40) ABCD is a rectangle in which $A = (-1, 4)$, $B = (7, 8)$, $C = (9, 4)$, $D = (1, 0)$, then the equation of the circumcircle of the rectangle is
- (a) $(x-4)^2 + (y-4)^2 = 25$ (b) $(x-4)^2 + (y-4)^2 = 16$
 (c) $(x+4)^2 + (y+4)^2 = 25$ (d) $(x-4)^2 + (y+4)^2 = 16$
- (41) The geometrical centre of square ABCD is the origin and its side length is $2\sqrt{3}$, then the equation of the circle that touches its sides is
- (a) $x^2 + y^2 = 3$ (b) $x^2 + y^2 = 12$
 (c) $x^2 + y^2 = 6$ (d) $(x-\sqrt{3})^2 + (y-\sqrt{3})^2 = 3$
- (42) The equation of the circle passes through the vertices of a regular hexagon that has area $6\sqrt{3} \text{ cm}^2$ and the centre of the circle is the origin is
- (a) $x^2 + y^2 = 2$ (b) $x^2 + y^2 = 4$ (c) $x^2 + y^2 = 9$ (d) $x^2 + y^2 = 16$
- (43) The circle whose equation is $(x-a)^2 + (y-b)^2 = a^2$ where $(a \neq b)$
- (a) touches x -axis. (b) touches y -axis.
 (c) touches the two coordinates axes. (d) does not touch any of the two axes.

- (44) If y-axis is a tangent to the circle $X^2 + y^2 + 4X + mY + 4 = 0$, then $m = \dots\dots\dots$
 (a) 4 (b) -4 (c) 0 (d) ± 4
- (45) If the circle whose equation $X^2 + y^2 - 6X + 8Y + c = 0$ touches X-axis, then $c = \dots\dots\dots$
 (a) -9 (b) 9 (c) 6 (d) -6
- (46) If X-axis touches the circle $X^2 + y^2 + mX + 4Y + 7 - 3m = 0$, then $m = \dots\dots\dots$
 (a) 2 or 14 (b) -2 or -14 (c) 2 or -14 (d) -2 or 14
- (47) If the straight line $3X - 4Y - 12 = 0$ touches the circle $(X + 3)^2 + (Y - 1)^2 = r^2$, then the circumference of the circle = $\dots\dots\dots$ length unit (in terms of π)
 (a) 5π (b) 10π (c) 15π (d) 20π
- (48) If the straight line $y = mX$ touches the circle $(X - 2)^2 + (Y - 6)^2 = 4$, then $m = \dots\dots\dots$
 (a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{-4}{3}$ (d) $\frac{4}{3}$
- (49) The straight line $y = 5 - 2X$ the circle whose equation :
 $X^2 + y^2 - 8X - 4Y + 15 = 0$
 (a) touch (b) intersect (c) outside (d) passes the centre
- (50) The two circles $C_1 : (X + 2)^2 + (Y - 1)^2 = 4$, $C_2 : (X - 5)^2 + (Y - 3)^2 = 9$
 (a) distant. (b) touching externally.
 (c) touching internally. (d) intersecting.
- (51) The two circles $C_1 : (X + 2)^2 = 1 - y^2$, $C_2 : X^2 + y^2 - 2X - 8Y - 19 = 0$ are
 (a) intersecting. (b) touching internally.
 (c) distant. (d) touching externally.
- (52) If the straight line $L : 3X + 4Y + 9 = 0$ touches the circle $M : X^2 + y^2 - 22X - 4Y - c = 0$, then $c = \dots\dots\dots$
 (a) 15 (b) -20 (c) 25 (d) -25
- (53) The length of the tangent segment to the circle : $X^2 + y^2 = 9$ from the point $(5, 0)$ equals $\dots\dots\dots$ length unit.
 (a) 14 (b) 3 (c) 5 (d) 4
- (54) If \overrightarrow{AB} is a tangent to the circle $X^2 + y^2 + 6X - 8Y + 15 = 0$ at the point $A(-2, 1)$, then the equation of \overrightarrow{AB} is
 (a) $X - 3Y + 5 = 0$ (b) $X - 3Y = 5$ (c) $3X - Y - 5 = 0$ (d) $3Y - X + 5 = 0$

- (55) If X -axis intersects the circle whose equation $X^2 + y^2 = 49$ at the two points A and B, then $AB = \dots\dots\dots$ length unit.
- (a) 49 (b) 7 (c) 2 (d) 14
- (56) The intersection point of the circle $(X - 2)^2 + y^2 = 16$ with the X -axis is $\dots\dots\dots$
- (a) (6, 0), (-2, 0) (b) (-6, 0), (2, 0)
(c) (4, 0), (-4, 0) (d) (2, 0), (-2, 0)
- (57) If the line $y = 2$ intersects the circle whose equation $(X - 3)^2 + (y - 2)^2 = 25$ at the two points A and B, then $AB = \dots\dots\dots$ length unit.
- (a) $\sqrt{13}$ (b) 7 (c) 8 (d) 10
- (58) If the straight line : $y - 2X + 5 = 0$ cuts the circle $X^2 + y^2 - 4X - 8y = 0$ at the two points A and B, then the distance between the centre and the chord $\overline{AB} = \dots\dots\dots$
- (a) 3 (b) 4 (c) 5 (d) $\sqrt{5}$
- (59) A circle, its centre $M = (5, 4)$ and its radius length = 5 length units and it intersects X -axis at the two points A and B, then the area of $\Delta MAB = \dots\dots\dots$ square units.
- (a) 6 (b) 9 (c) 12 (d) 18
- (60) If the straight line \overleftrightarrow{AB} is the axis of symmetry of the circle whose equation : $X^2 + y^2 = k^2$, and $A, B \in$ the circle where $A = (-2, 5)$, then $B = \dots\dots\dots$
- (a) (2, -5) (b) (2, 5) (c) (0, 0) (d) (5, -2)
- (61) Area of the square whose vertices lie on the circle : $X^2 + y^2 - 4X + 6y + 4 = 0$ is $\dots\dots\dots$ square units.
- (a) 6 (b) 9 (c) 12 (d) 18
- (62) If a circle with radius length 4 cm. and it passes through the vertices of a regular hexagon, then the area of the hexagon = $\dots\dots\dots$ cm²
- (a) $8\sqrt{3}$ (b) $16\sqrt{3}$ (c) 16 (d) $24\sqrt{3}$
- (63) In the opposite figure :
- If $OB = 5$ length unit
- , then the equation of the circle M is $\dots\dots\dots$
- (a) $(X - 2)^2 + (y - 5)^2 = 25$
(b) $(X - 2)^2 + (y - 5)^2 = 4$
(c) $(X - 5)^2 + (y - 2)^2 = 25$
(d) $(X - 5)^2 + (y - 2)^2 = 4$



- (64) The equation of the circle that touches the straight lines $X = 4$, $X = -2$, $Y = 0$ could be

(a) $(x + 2)^2 + (y - 4)^2 = 36$

$$(b) (x-1)^2 + (y-3)^2 = 36$$

(c) $(x-1)^2 + (y+3)^2 = 9$

(d) $(x + 1)^2 + (y + 3)^2 = 9$

- (65) In the opposite figure :**

If the equation of the circle is

$$(x-2)^2 + (y-3)^2 = 25$$

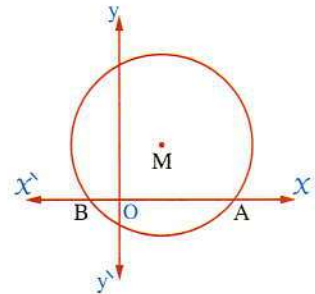
, then $AB = \dots\dots\dots$ length unit.

(a) 8

(b) 4

(c) 6

(d) 5



- (66) In the opposite figure :**

If the equation of the circle M is

$$(x-3)^2 + (y+2)^2 = 25,$$

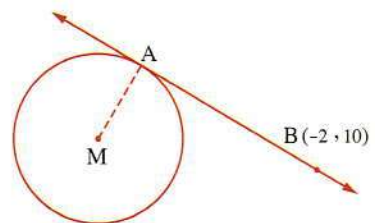
\overleftrightarrow{AB} is a tangent to the circle M at A where

B = (-2, 10), then AB = length unit.

(a) 13

$$(b) \sqrt{194}$$

(c) 12



(d) 5

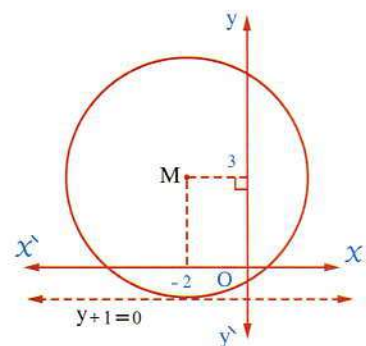
- (67) Which of the following circle equations does represent the circle in the opposite figure ?

$$(a) (x-3)^2 + (y+2)^2 = 16$$

(b) $(x + 2)^2 + (y - 3)^2 = 16$

(c) $(x + 2)^2 + (y - 3)^2 = 4$

(d) $(x + 2)^2 + (y - 3)^2 = 9$



- (68) If M is a circle where its circumference = 10π length unit, intersects X-axis at the two points A (2, 0), B (8, 0), then the equation of the circle M could be

(a) $(x + 5)^2 + (y + 4)^2 = 25$

(b) $(x-5)^2 + (y-4)^2 = 25$

(c) $(x-5)^2 + (y-4)^2 = 9$

(d) $(x-5)^2 + (y-4)^2 = 36$

(69) In the opposite figure :

If the equation of the circle M

$$\text{is } x^2 + y^2 - 6x + 4y - 12 = 0$$

, $\overline{MB} \perp$ the straight line L where

the equation of L is

$$3x - 4y + 23 = 0, A \in \overline{MB}$$

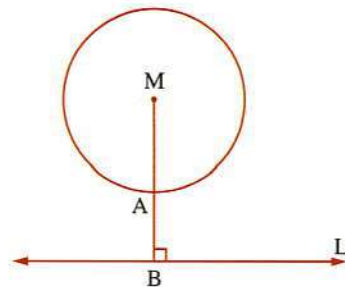
, then the length of $\overline{AB} = \dots\dots\dots$ length unit.

(a) 3

(b) 4

(c) 5

(d) 2.5



(70) The opposite figure represents a disc of a machine.

It is required to make another one like it, if the cost of one

square unit from the surface of the disc is 5 pounds and the

equation of the smaller disc is $x^2 + y^2 = 4$ the length of the

diameter of the greater circle is 10 length units, then the cost of

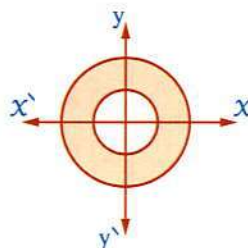
the disc $\approx \dots\dots\dots$ pounds.

(a) 440

(b) 660

(c) 220

(d) 330



(71) The opposite figure represents two gears in a machine

, their centres are M and N, $\overline{MN} \parallel$ the y-axis.

If the radius of the smaller gear = $\frac{1}{3}$ the radius

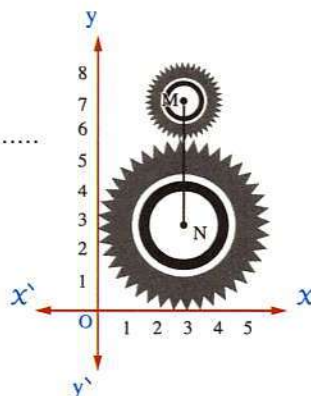
of the greater gear, then the equation of the smaller gear is $\dots\dots\dots$

(a) $(x - 3)^2 + (y - 1)^2 = 9$

(b) $(x - 3)^2 + (y - 7)^2 = 1$

(c) $x^2 + y^2 - 6x - 14y + 58 = 0$

(d) $(x - 1)^2 + (y - 1)^2 = 1$



Second Essay questions

1 Find the equation of the circle whose centre is (M) and its radius length (r) unit in each of the following cases :

(1) $M = (2, 3)$, $r = 5$

(2) $M = (0, 0)$, $r = 3$

(3) $M = (0, -1)$, $r = 2\sqrt{3}$

(4) $M = (-4, -3)$, $r = \frac{3}{2}$

2 Write the general form of the equation of the circle if :

- (1) Its centre M $(-2, 3)$ and its diameter length equals 8 length unit.
- (2) Its centre M $(5, -12)$ and it passes through the origin point.
- (3) Its centre M $(7, -5)$ and passes through the point A $(3, 2)$
- (4) \overline{AB} is a diameter in the circle where A $(6, -4)$ and B $(0, 2)$
- (5) Its centre is the point $(-3, -2)$ and touches X-axis.
- (6) Its centre is the point $(3, 0)$ and touches y-axis.
- (7) Its centre is the point $(5, -5)$ and touches the two coordinate axes.
- (8) It passes through the two points A $(6, 2)$, B $(0, -1)$ and the two tangents to the circle at A and B are parallel.
- (9) Its centre lies on X-axis and it passes through the two points $(2, 0)$, $(8, 0)$
- (10) Its radius length = 6 length units and it touches the two axes given that the circle lies in the fourth quadrant.

3 Find the coordinates of the centre , also find the radius length for each of the following circles :

- | | |
|------------------------------------|----------------------------------|
| (1) $x^2 + y^2 - 8 = 0$ | (2) $(x + 3)^2 + (y - 5)^2 = 49$ |
| (3) $(x + 4)^2 + y^2 = 9$ | (4) $x^2 + (y + 7)^2 = 24$ |
| (5) $x^2 + y^2 - 4x + 6y - 12 = 0$ | (6) $x^2 + y^2 - 8x = 12$ |

4 Show which two circles in the following are congruent and why ?

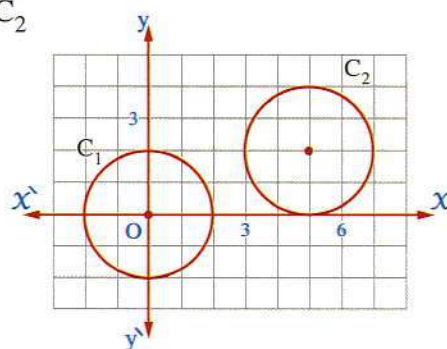
- | | |
|-----------------------------------|------------------------------|
| (1) $x^2 + y^2 - 4x + 8y = 0$ | , $x^2 + y^2 + 12y + 16 = 0$ |
| (2) $x^2 + y^2 + 14y = 1$ | , $x^2 + y^2 + 10x - 25 = 0$ |
| (3) $x^2 + y^2 - 2x + 4y - 3 = 0$ | , $x^2 + y^2 + 6x - 11 = 0$ |

5 The opposite figure shows the two circles C_1 and C_2

Prove that the two circles are congruent
, then find the equation of each of them.

If the circle C_3 is the image of the circle C_1
by translation $(-4, 3)$

Find the equation of the circle C_3



6 Show with reasons which of the following equations represents a circle and which of them does not represent a circle :

(1) $x^2 + xy + y^2 = 25$

(3) $x^2 + 2y^2 + 6x - 5y = 0$

(5) $(x + y)^2 - 3x + 6y - 4 = 0$

(7) $x^2 + y^2 + 2x - 4y + 5 = 0$

(9) $x^2 + x = y^2 + y + 7$

(2) $x^2 + y^2 + 8x - 16y - 1 = 0$

(4) $2x^2 + 2y^2 + 3y - 8 = 0$

(6) $x^2 + y^2 + x + 2y + 7 = 0$

(8) $\frac{1}{4}x^2 + \frac{1}{4}y^2 + x - 8 = 0$

7 M_1 and M_2 are the two centers of two circles where $M_1 = (2, -1)$, $M_2 = (-1, 3)$. Find the equation of each circle given that each of them passes through the centre of the other.

8 Prove that the two circles : $x^2 + y^2 - 2x + 6y + 1 = 0$, $4x^2 + 4y^2 - 8x + 24y + 15 = 0$ are concentric and find the radius length of each of them. « 3 , 2.5 length unit »

9 Show which of the following points belongs to the circle C whose equation : $(x - 6)^2 + (y + 1)^2 = 25$, then determine the position of each of the other points with respect to the circle C where : A (9, 3), B (7, 5), C (3, 3), D (2, -3)

10 A circle of centre (2, -1) passes through the point A = (-1, 3). Show the positions of the following points with respect to the circle M : B = (2, 4), C = (-3, 1), D = (1, 2)

11 Determine the position of the straight line with respect to the circle $(x + 3)^2 + (y - 4)^2 = 9$
If the equation of the straight line is :

(1) $L_1 : 3x - 4y + 5 = 0$ (2) $L_2 : 6x - 8y + 23 = 0$ (3) $L_3 : 3x - 4y + 10 = 0$

12 Determine the position of the circle $C_1 : (x - 5)^2 + (y + 2)^2 = 4$ with respect to the circle $C_2 : (x + 7)^2 + (y - 3)^2 = 1$

13 Are the two circles $C_1 : x^2 + y^2 - 10x - 8y + 16 = 0$ and $C_2 : x^2 + y^2 + 14x + 10y - 26 = 0$ touching externally ? Explain your answer.

14 If the two circles $C_1 : (x + 2)^2 + (y + 11)^2 = k$, $C_2 : (x - 3)^2 + (y - 1)^2 = 16$ are touching. Find the value of k « 81 or 289 »

15 Prove that the two circles : $x^2 + y^2 - 6x - 4y + 12 = 0$, $x^2 + y^2 + 2x - 4y - 4 = 0$ touch each other and find the coordinates of the point of tangency, then find the circle equation whose centre is the point of tangency and passes through the center of the second circle.

16 Write the equation of the unit circle and if the point $(2a \cos \theta, 2a \sin \theta)$ belongs to this circle. Find the real values of a (i.e. $a \in \mathbb{R}$)

17 Find the value of $h \in \mathbb{R}$ which makes each of the following represents an equation of circle :

(1) $x^2 + y^2 - 2x - 4y - h + 2 = 0$

(2) $x^2 + y^2 + 4x - 6y - h^2 + 4 = 0$

(3) $x^2 + y^2 - 4hx - 2hy + 10(h-1) = 0$

(4) $x^2 + y^2 + 6x + 8y + h^2 - 3h + 15 = 0$

(5) $x^2 + y^2 + 2hx - 6hy - 2h^2 + 12h - 3 = 0$

18 Find the value of a in the equation :

$x^2 + y^2 - 2x + 4y + 2a - 3 = 0$ in each of the following cases :

(1) The equation represents a circle.

(2) The equation represents a circle passing through the origin point.


(3) The equation represents a circle touching x -axis.

(4) The equation represents a circle touching y -axis.


(5) The equation represents a circle touching the straight line : $3x + 4y + 15 = 0$

(6) The equation represents a circle of diameter length 14 length unit.

19 Write the general form of the equation of a circle if :

(1)  Its centre $M(5, 4)$ and touches the straight line $x = 2$

(2) Its centre $M(5, 3)$ and touches the straight line passing through the two points $(3, 7), (-1, 3)$

(3)  Its centre M lies in the first quadrant and its radius length = 3 length unit and the two straight lines $x = 1, y = 2$ are tangents to it.

(4) Its radius length = 5 length unit and touches x -axis at the point $(4, 0)$

(5) Its radius length = $3\frac{1}{2}$ unit and touches y -axis at the point $(0, -4)$

(6) Touches the two coordinate axes and passes through the point $(-2, -4)$

(7) Touches x -axis at the point $(-3, 0)$ and touches also y -axis

(8) Touches x -axis at the point $(-2, 0)$ and intercepts from the positive part of y -axis a chord of length $4\sqrt{3}$ length unit.

(9) Touches y -axis at the point $(0, -1)$ and intercepts from the negative part of x -axis a chord of length $4\sqrt{6}$ length unit.

- (10) Touches the X -axis and passes through the two points $(2, 1)$, $(-5, 2)$
- (11) Touches y -axis and passes through the two points $(-4, 2)$, $(-1, 2)$
- (12) Its centre lies on X -axis and passes through the two points $A(1, 3)$, $B(2, -4)$
- (13) Passes through the origin point and intercepts from the two positive parts of the X -axis and y -axis two parts of lengths 12, 16 length units respectively.
- (14) Its centre lies on the straight line : $y - X = 1$ and passes through the two points $A(-2, 4)$, $B(6, 8)$
- (15) Its radius length $= \sqrt{85}$ length unit and passes through the two points $A(-1, 2)$, $B(3, 4)$
- (16) Its diameter \overline{AB} where A and B are the points of intersection between the circle $X^2 + y^2 + 2X + 4y = 0$ and X -axis.

- 20 Find the area of the equilateral triangle which its circumcircle is :

$$X^2 + y^2 + X - 4y - 2 = 0$$

$$\ll \frac{75\sqrt{3}}{16} \text{ square units} \gg$$

- 21 Find to the nearest cm^2 the surface area of a regular pentagon. If the circle :

$X^2 + y^2 + 6X - 12y + 5 = 0$ passes through its vertices knowing that each unit in the coordinate plane represents 5 cm.

$$\ll 2378 \text{ cm}^2 \gg$$

- 22 Find the surface area of the regular hexagon which its circumcircle is :

$$X^2 + y^2 - 10X + 6y + 25 = 0$$

$$\ll \frac{27\sqrt{3}}{2} \text{ square units} \gg$$

- 23 Find the surface area of a regular polygon of 12 sides and the circle :

$$X^2 + y^2 - 16 = 0 \text{ passes through its vertices.}$$

$$\ll 48 \text{ square units} \gg$$

- 24 Find the equation of the circle whose radius length $= 5$ length unit and the equations of two straight lines carrying two diameters in it are : $3X + y + 2 = 0$, $4X - y - 16 = 0$, then prove that the point $(5, -4)$ belongs to the circle.

- 25 Find the equation of the circle whose radius length equals the radius length of the circle :

$$X^2 + y^2 - 2X \cos \theta - 2y \sin \theta - 8 = 0$$

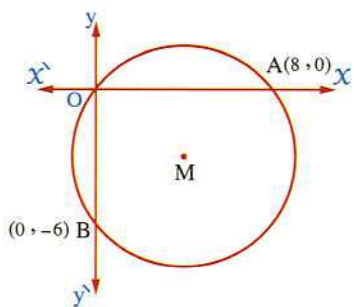
and the equations of two straight lines carrying two diameters in it are

$$X + y = 0, \quad \vec{r} = (1, 5) + k(1, 2)$$

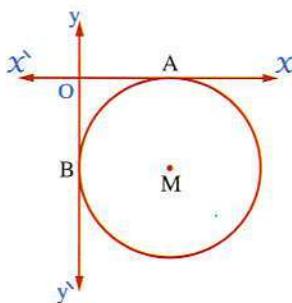
- 26** Find the equation of the circle which passes through the two points of intersection of the two circles $x^2 + y^2 - 10x = 0$ and $x^2 + y^2 + 2x - 12 = 0$ and whose center
- (1) The origin (2) The point $(2, 0)$
- 27** Prove that the points : $A = (0, -1)$, $B = (-1, 0)$, $C = (-9, 0)$ lie on circle whose centre is $M = (-5, -5)$, then find the equation of this circle.
- 28** If the points : $A = (3, -2)$, $B = (3, 8)$, $C = (-1, 0)$ belong to one circle, prove that \overline{AB} is a diameter in it , then write the general form of its equation.
- 29** Prove that the triangle whose vertices are $A(8, 0)$, $B(0, 6)$, $C(0, 0)$ is right-angled , then find the equation of the circle which passes through its vertices.
- 30** Prove that the points : $A = (-2, 0)$, $B = (4, 0)$, $C = (1, 3\sqrt{3})$ are the vertices of the equilateral triangle ABC , then find the equation of the circumcircle of $\triangle ABC$
- 31** Find the equation of the circle which passes through the points : $A = (2, -1)$, $B = (-2, 0)$, $C = (0, -9)$ and determine its centre and its radius length.
- 32** If $A = (3, 0)$, $B = (0, 9)$, $C = (0, 1)$, $D = (-1, 2)$
Prove that the quadrilateral $ABCD$ is cyclic.

33 Find the general form of the equation of the circle M in each of the following figures :

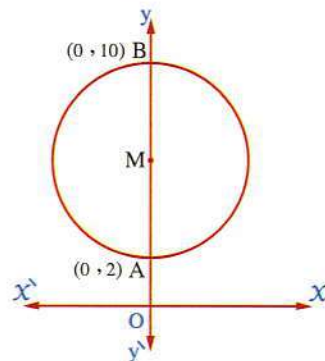
(1) The circle passes through the origin point and passes through the two points A and B



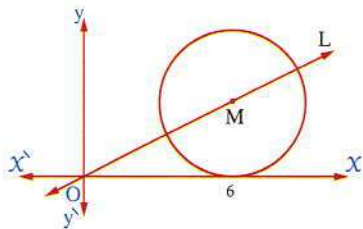
(2) The circle touches the two coordinate axes at A and B and the length of $\overline{MO} = 2\sqrt{2}$



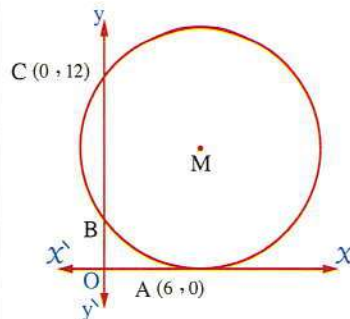
(3) The centre of the circle lies on y -axis and the circle intersects y -axis at A and B



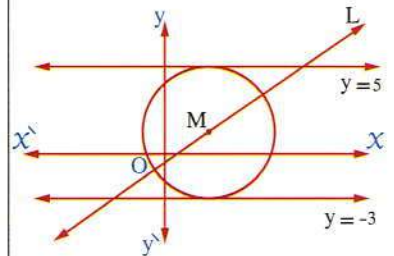
- (4) The straight line whose equation is $x - 3y = 0$ passes through the centre of the circle and the origin point



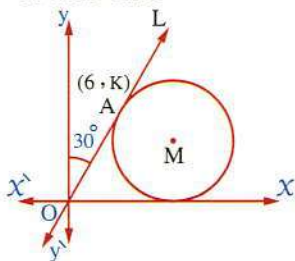
- (5) The circle touches x -axis at A and intersects y -axis at B and C



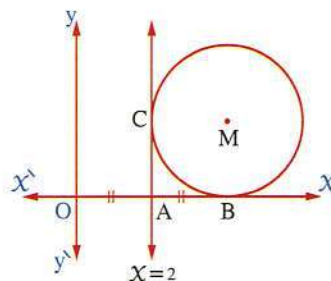
- (6) The straight line $L : 2x - 3y = 1$ passes through the centre of the circle and the two straight lines $y = 5$, $y = -3$ are tangents to the circle.



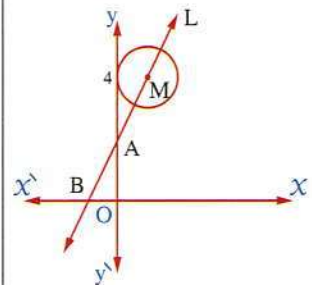
- (7) The straight line L touches the circle at $A(6, k)$ and makes an angle of measure 30° with the positive direction of y -axis and the circle touches x -axis also



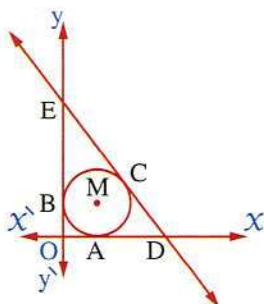
- (8) The circle touches x -axis at B and touches the straight line $x = 2$ at C



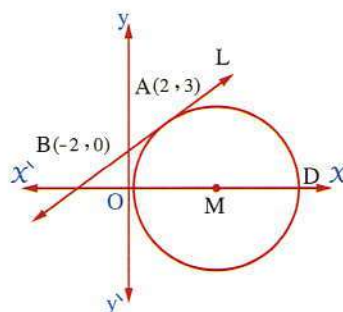
- (9) The circle touches y -axis at the point $(0, 4)$ and the straight line L passes through the centre of the circle and the two points $A(0, 2)$ and $B(-1, 0)$



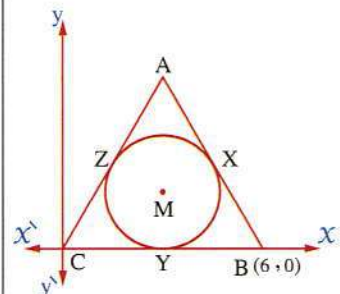
- (10) The two coordinate axes touch the circle M at A and B. If the straight line $4x + 3y - 12 = 0$ is a tangent to the circle M at C



- (11) The straight line L touches the circle at $A(2, 3)$ and intersects x -axis at $B(-2, 0)$



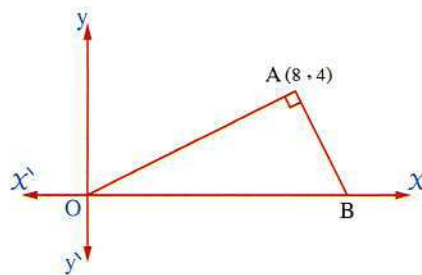
- (12) ABC is an equilateral triangle its sides touch the circle M , $B(6, 0)$



34 In the opposite figure :

If $\overline{OA} \perp \overline{AB}$, $A(8, 4)$

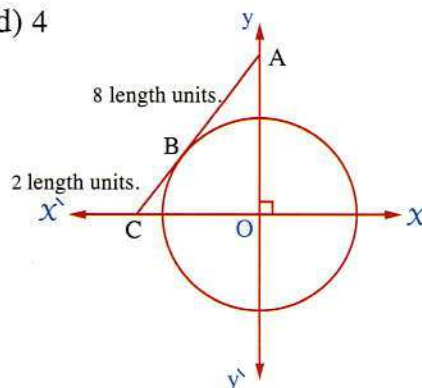
Find the equation of
the circle which passes through
the points A, B and O



Third Higher skills

1 Choose the correct answer from those given :

- (1) The equation : $(k - 2) x^2 + (2 - k) y^2 - k x + 3 k y - 25 = 0$
 - (a) represents a circle when $k = 2$
 - (b) represents a circle when $k \neq 2$
 - (c) represents a circle when $k \in \mathbb{R}$
 - (d) does not represent a circle whatever the value of k .
- (2) The height of a right circular cone is 6 length units and the equation of its circular base is $x^2 + y^2 = 64$ in the xy - plane , then the volume of the cone = cubic units.
 - (a) 96π
 - (b) $\frac{640}{3} \pi$
 - (c) 128π
 - (d) $\frac{128}{3} \pi$
- (3) The least distance between the y -axis and a point on the circle whose equation : $(x - 7)^2 + (y - 5)^2 = 16$ is length units.
 - (a) 11
 - (b) 3
 - (c) 5
 - (d) 7
- (4) Number of circles touch the coordinate axes and their centres lie on the circle : $x^2 + y^2 = 25$ equals
 - (a) zero
 - (b) 1
 - (c) 2
 - (d) 4
- (5) **In the opposite figure :**
The equation of the circle is
 - (a) $x^2 + y^2 = 4$
 - (b) $x^2 + y^2 = 16$
 - (c) $x^2 + y^2 = 64$
 - (d) $x^2 + y^2 = 100$



(6) In the opposite figure :

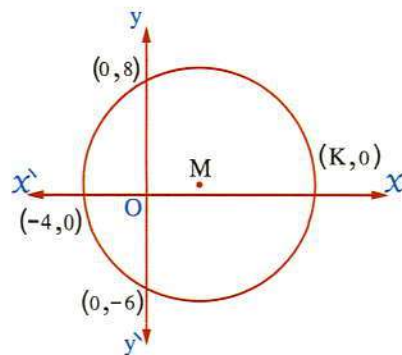
The equation of the circle is

(a) $(x + 4)^2 + (y + 1)^2 = 65$

(b) $(x - 6)^2 + (y - 2)^2 = 64$

(c) $(x - 4)^2 + (y - 1)^2 = 65$

(d) $(x - 4)^2 + (y - 2)^2 = 64$



(7) If O is the origin, \overrightarrow{OA} and \overrightarrow{OB} are two tangents to the circle $x^2 + y^2 - 10x + 4y + 6 = 0$, then the centre of the circumcircle of $\triangle AOB$ is

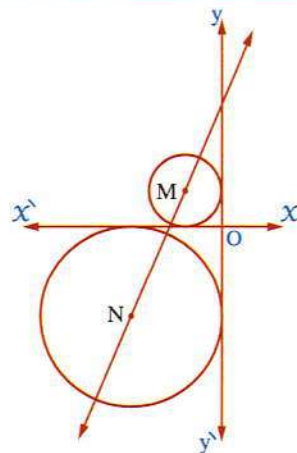
(a) $\left(\frac{7}{4}, 2\right)$ (b) $\left(\frac{5}{2}, -1\right)$ (c) $\left(\frac{7}{4}, -1\right)$ (d) $\left(\frac{5}{2}, 2\right)$

(8) The length of the common chord of the two circles $x^2 + y^2 - 10x - 10y = 0$ and $x^2 + y^2 + 6x + 2y - 40 = 0$ equals length unit.

(a) $5\sqrt{2}$ (b) 10 (c) 12 (d) $10\sqrt{2}$

2 In the opposite figure :

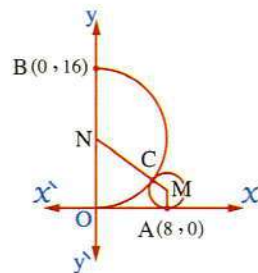
If each of the two circles M and N touches the two coordinate axes and the equation of the line of centres \overleftrightarrow{MN} is : $y = 2x + 1$
Find the equation of each of the two circles M and N



3 In the opposite figure :

A semicircle, its centre N lies on y-axis and touches a circle M at C and the circle M touches x-axis at A where $A = (8, 0)$ If $B = (0, 16)$

Find the general form of the equation of the circle M



Life applications

1 Town designing :

In the drawing for one of the cities in a perpendicular coordinate axes plane, where each unit in it represents 5 metres. It is found that the circle : $x^2 + y^2 - 6x + 8y + 11 = 0$ represents one of its squares. Find to the nearest squared metre the area of the square ($\pi = \frac{22}{7}$)

« 1100 m² »

2 Marine navigation :

A radar is located in the position A (7, -9) and cover a circular region. The length of its radius equals 30 length unit. Write the equation of the circle that determine the radar range in the coordinates plane. Can the radar observe a ship in the position B (25, -30) ? Explain your answer.

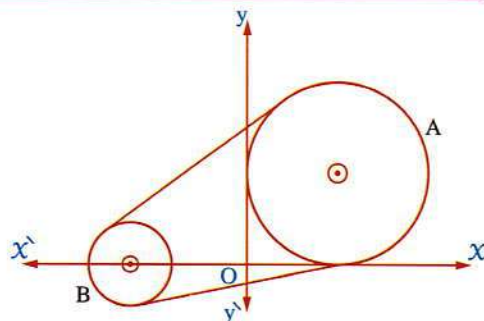
3 Architectural design :

An architect designs a building in the form of a regular octagon. Its vertices passes by a circle $x^2 + y^2 - 4x + 12y - 60 = 0$ Calculate the area of the building to the nearest squared unit.

« $200\sqrt{2}$ square units »

4 Industry :

The opposite figure shows a pulley A in a machine touching the two coordinate axes, it rotates by a wire passing on a small pulley B which the equation of its circle is : $x^2 + y^2 + 14x + 45 = 0$



Find :

(1) The equation of the circle of the pulley A given that its radius length = 5 units.

(2) The distance between the two centres of the two pullies if the plane unit represents 6 cm.

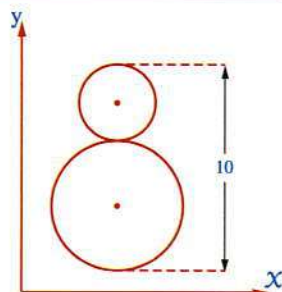
« 78 cm. »

5 Industry :

The opposite figure shows two gears in a machine such that their centres lie on a straight line parallel to y-axis and the maximum distance between their edges is 10 units.

Find the equation of the circle of the small gear given that the equation of the great gear is :

$$x^2 + y^2 - 10x - 8y + 32 = 0$$



SCIENTIFIC SECTION

Mathematics

Applications

By a group of supervisors

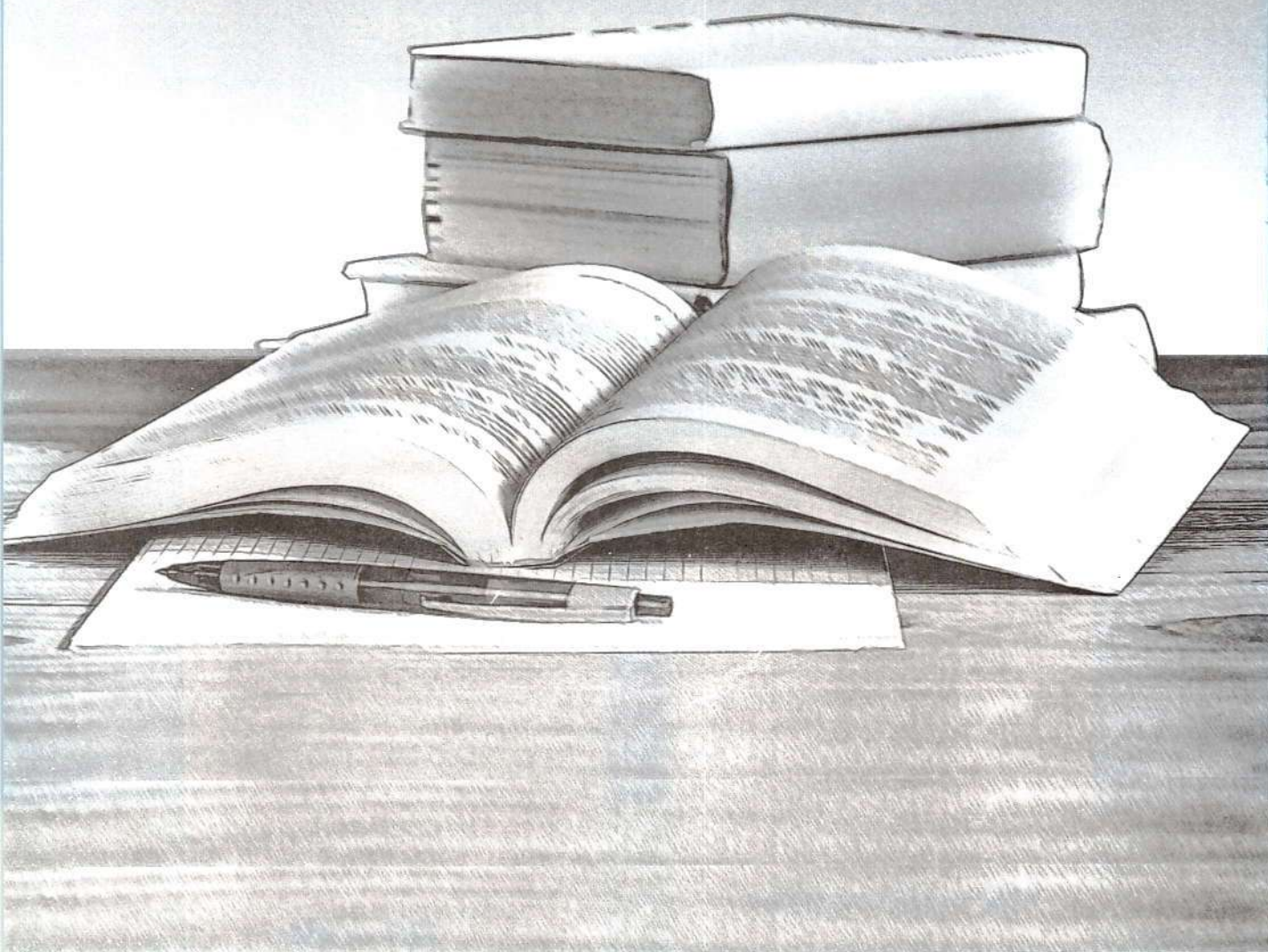


FIRST TERM
2
SEC.
2024

EXAMINATIONS



CONTENTS



- ▶ Accumulative quizzes.
- ▶ Monthly tests.
- ▶ School book examination.
- ▶ Final examinations.
- ▶ Answers.

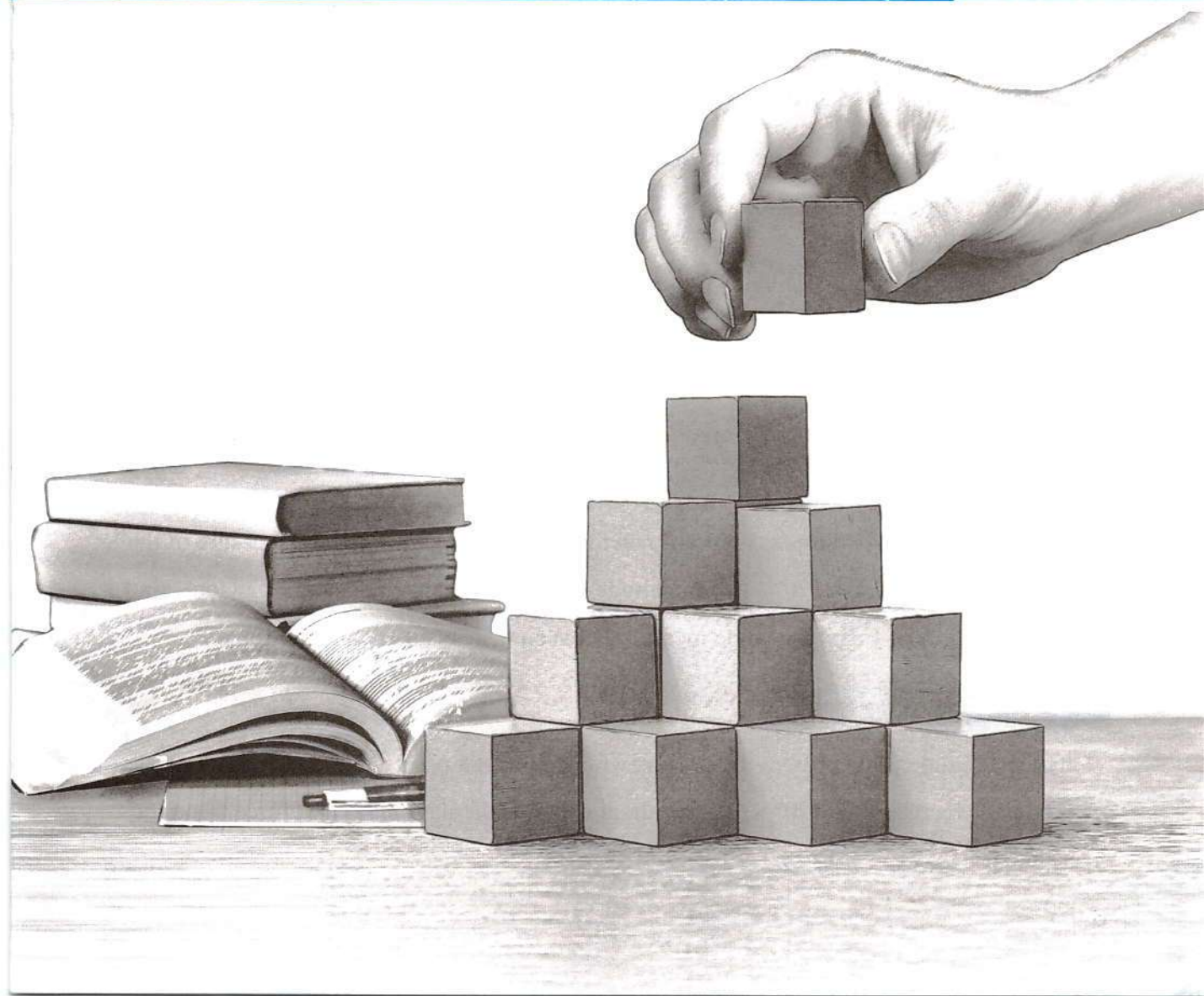
Accumulative Quizzes

FIRST

Accumulative quizzes on statics.

SECOND

Accumulative quizzes on geometry and measurement.



Total mark

Quiz

1

on lesson 1 – unit 1

10

Answer the following questions :

First question

4 marks

1 mark for each item

Choose the correct answer from those given :

(1) $\vec{F}_1 = 2\hat{i} + 3\hat{j}$, $\vec{F}_2 = \hat{i} + \hat{j}$ where F_1 , F_2 are measured in newton then the magnitude of their resultant newton.

(a) $\sqrt{2}$

(b) $\sqrt{5}$

(c) $\sqrt{13}$

(d) 5

(2) Two forces are equal act at a point and the measure of the angle between them is $\frac{\pi}{3}$ and their resultant is 3 newton , then the magnitude of each is newton.

(a) $\frac{3}{2}$

(b) $\sqrt{3}$

(c) 3

(d) $3\sqrt{3}$

(3) The resultant of two forces acting at a point is maximum when the included angle between them is equal to

(a) zero

(b) 60°

(c) 120°

(d) 180°

(4) The magnitude of the resultant of two forces 3 , 5 newton and the measure of their included angle is 60° equals newton.

(a) 2

(b) 6

(c) 7

(d) 8

Second question

3 marks

The magnitude of two forces are F , 4 newton acting at a point , and the measure of the angle between them is 120° , the magnitude of their resultant equals $4\sqrt{3}$ newton , find the magnitude of \vec{F} and the angle measure between their resultant and the force \vec{F}

Third question

3 marks

The magnitude of two forces are 4 , F newton acting at a point , and the measure of the angle between them is 120° , their resultant is perpendicular on the first force. Find the value of F

Quiz

2

till lesson 2 – unit 1

Total mark

10

Answer the following questions :

First question

4 marks

1 mark for each item

Choose the correct answer from those given :

- (1) Two forces of magnitude $3F$ and $2F$ intersecting at a point and their resultant is $5F$, then the measure of the angle between them is

(a) 0° (b) 60° (c) 20° (d) 180°

- (2) As resolving the force \vec{R} into two forces \vec{F}_1 and \vec{F}_2 making with \vec{R} two angles of measure θ_1 and θ_2 on both sides of \vec{R} respectively, then the magnitude of $\vec{F}_1 =$

(a) $\frac{R \sin \theta_1}{\sin (\theta_1 + \theta_2)}$ (b) $\frac{R \sin \theta_2}{\sin (\theta_1 - \theta_2)}$ (c) $\frac{R \sin \theta_2}{\sin (\theta_1 + \theta_2)}$ (d) $\frac{R \sin (\theta_1 + \theta_2)}{\sin \theta_2}$

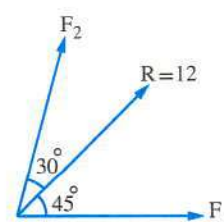
- (3) Two forces of equal magnitudes, inclosing between them an angle of measure 90° . If the magnitude of their resultant is 8 N , then the value of each force measured in newton is

(a) $2\sqrt{2}$ (b) 4 (c) $4\sqrt{2}$ (d) 8

- (4) In the given figure :

$F_1 =$

(a) $12 \cos 75^\circ$ (b) $12 \cos 45^\circ$
(c) $6 \sec 45^\circ$ (d) $6 \csc 75^\circ$



Second question

3 marks

Two forces of magnitudes 4 N and $F\text{ newton}$ act at a point and the measure of their included angle is 135° . Given that their resultant makes angle 45° with the force F , find F and the magnitude of their resultant.

Third question

3 marks

Resolve a force 100 newton in two directions the first inclines by 60° to the force and the other by 30° in the other side of the given force.

Quiz

3

till lesson 3 – unit 1

10

Answer the following questions :**First question**

4 marks

1 mark for each item

Choose the correct answer from those given :

- (1) A body of weight (W) is placed on an inclined plane makes angle of measure θ to the horizontal then the component of its weight in direction of line of greatest slope equals
- (a) $W \cos \theta$ (b) $W \sin \theta$ (c) $W \tan \theta$ (d) W
- (2) Two perpendicular forces of magnitude 12 newton , 5 newton act at a point , then the magnitude of their resultant = newton.
- (a) 17 (b) 7 (c) 13 (d) 14
- (3) Given : $\vec{F}_1 = 3\hat{i} - 2\hat{j}$, $\vec{F}_2 = a\hat{i} - \hat{j}$, $\vec{F}_3 = 4\hat{i} - b\hat{j}$ and their resultant $\vec{R} = 6\hat{i} - 4\hat{j}$, then $a + b = \dots\dots\dots$
- (a) 2 (b) - 2 (c) zero (d) - 1
- (4) Given : $\vec{F}_1 = 5\hat{i}$, $\vec{F}_2 = 7\hat{i} - 5\hat{j}$, \vec{R} is their resultant then $\|\vec{R}\| = \dots\dots\dots$
- (a) $\sqrt{5} + \sqrt{74}$ (b) 49 (c) 13 (d) $\sqrt{12} - \sqrt{5}$

Second question

3 marks

Three coplanar forces of magnitudes 85 , 75 , $50\sqrt{2}$ kg.wt. act at a point , the first acts towards East , the second towards 30° West of the North and the third towards West South. Find the magnitude of their resultant.

Third question

3 marks

Two forces act at a point , the maximum value of their resultant is 32 kg.wt. and the minimum value of their resultant is 12 kg.wt. Find the magnitude of each force , then find the magnitude of their resultant when the angle between the two forces = 60°

Quiz

4

till lesson 4 – unit 1

Total mark

10

Answer the following questions :

First question

4 marks

1 mark for each item

Choose the correct answer from those given :

- (1) Three equal forces in magnitudes act at a point and the forces are in equilibrium , then the measure of the angle between any two forces =
- (a) 60° (b) 90° (c) 120° (d) 150°
- (2) The maximum and minimum value respectively of the resultant of the two forces of magnitudes 8 , 13 newton are newton.
- (a) 13 , 8 (b) 13 , 5 (c) 21 , 8 (d) 21 , 5
- (3) Two forces act at a point of magnitudes 5 , 3 newton and the measure of the angle between them is 60° then the magnitude of their resultant (R) equals newton.
- (a) 2 (b) 7 (c) 8 (d) 5
- (4) Two forces of equal magnitudes , the magnitude of their resultant is 3 newton and the measure of the angle between them is $\frac{\pi}{3}$, then the magnitude of each newton.
- (a) $\sqrt{3}$ (b) 3 (c) $\frac{3}{2}$ (d) $3\sqrt{3}$

Second question

3 marks

A body of weight 300 gm.wt. is placed on a smooth plane inclined to the horizontal with an angle whose tangent equals $\frac{1}{\sqrt{3}}$ The body is prevented from sliding by a force makes with the line of the greatest slope an angle of measure 30° upwards.

Find the magnitude of the force and the reaction of the plane.

Third question

3 marks

If $\vec{F}_1 = 5\vec{i} + 3\vec{j}$, $\vec{F}_2 = a\vec{i} + 6\vec{j}$, $\vec{F}_3 = -14\vec{i} + b\vec{j}$ are three coplanar forces meeting at a point and their resultant is $\vec{R} = (10\sqrt{2}, \frac{3\pi}{4})$, then find the values of a and b

Quiz

5

till lesson 5 – unit 1

Total mark

10

Answer the following questions :**First question**

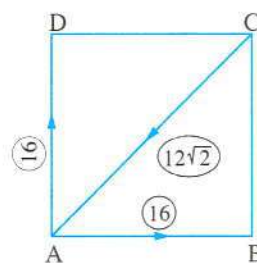
2 marks

The magnitudes of two forces F , $\sqrt{2} F$ newton act at a point and their resultant is perpendicular to the first force. Find the angle between the two forces and prove that the magnitude of their resultant equals F

Second question

2 marks

The opposite figure represents the forces 16, 16, $12\sqrt{2}$ newton which act in the square ABCD in the directions \overrightarrow{AB} , \overrightarrow{AD} , \overrightarrow{CA} respectively. Find the magnitude and direction of their resultant.

**Third question**

4 marks

A smooth sphere of radius length 30 cm. and of weight 10 gm.wt. rests on a vertical smooth wall. It is suspended by a string of length 30 cm. , one of its ends is attached to a point on the surface of the sphere and the other end is fixed at a point on the wall above the tangency point of the sphere and the wall.

Find the magnitudes of the tension in the string and the reaction of the wall

Fourth question

2 marks

Three coplanar forces of magnitudes 5, 10, $4\sqrt{7}$ newton act at a point , the measure of the angle between the first two forces equals 60° , find the greatest and the smallest magnitude of their resultant.

SECOND

Accumulative Quizzes on Geometry and Measurement

Total mark

Quiz

1

on lesson 1 – unit 2

10

Answer the following questions :

First question

5 marks

1 mark for each item

Choose the correct answer from those given :

(1) All the following cases determine a plane except

- (a) a straight line and a point not on it. (b) two different parallel straight lines.
(c) two intersecting straight lines. (d) two skew straight lines.

(2) The number of planes which passes through 3 non-collinear points equals

- (a) 1 (b) 3 (c) 6 (d) infinite numbers.

(3) The skew lines

- (a) never intersect. (b) are not perpendicular.
(c) are not parallel. (d) are neither parallel nor intersecting.

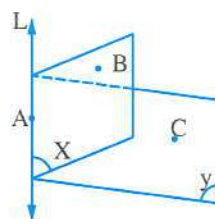
(4) In the opposite figure :

The plane $X \cap$ the plane $Y \cap$ the plane $ABC = \dots\dots\dots$

- (a) $\{A\}$ (b) the straight line L
(c) \overleftrightarrow{AC} (d) \overleftrightarrow{AB}

(5) If $\overleftrightarrow{AB} \parallel$ plane X , then $\overleftrightarrow{AB} \cap X = \dots\dots\dots$

- (a) \overleftrightarrow{AB} (b) \overleftrightarrow{AB} (c) \overleftrightarrow{AB} (d) \emptyset



Second question

5 marks

1 mark for each item

By using the opposite figure state :

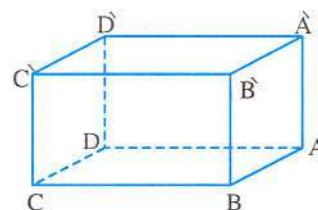
(1) Two parallel planes.

(2) Two intersecting planes.

(3) Two skew straight lines.

(4) A straight line and a plane which are parallel.

(5) The intersection line of the plane $ABB'A'$ with the plane ACD



Quiz

2

till lesson 2 – unit 2

Total mark

10

Answer the following questions :

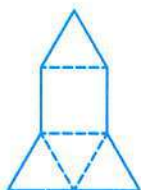
First question

4 marks

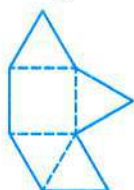
1 mark for each item

Choose the correct answer from those given :

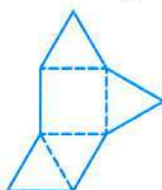
- (1) Which of the following nets does not make a regular quadrilateral pyramid when it folded ?



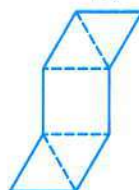
(a)



(b)



(c)



(d)

- (2) The volume of a regular quadrilateral pyramid 12 cm^3 and its height 4 cm. then the length of its base side = cm.

(a) 1

(b) 2

(c) 3

(d) 4

- (3) A regular quadrilateral pyramid , the length of its base side is 10 cm. , and its lateral height is 13 cm. , then its volume in cm^3 =

(a) $\frac{1}{2} \times (10)^2 \times 13$

(b) $\frac{1}{3} \times (10)^2 \times 12$

(c) $\frac{1}{2} \times (12)^2 \times 13$

(d) $\frac{1}{3} \times (13)^2 \times 10$

- (4) If the sum of edge lengths of a triangular regular faces pyramid equals 18 cm. , then its total area = cm^2 .

(a) $\frac{27\sqrt{2}}{4}$

(b) $\frac{27\sqrt{3}}{4}$

(c) $\frac{27\sqrt{3}}{2}$

(d) $9\sqrt{3}$

Second question

3 marks

$1\frac{1}{2}$ marks for each item

The side length of the base of regular quadrilateral pyramid is 20 cm. and its height is $10\sqrt{3}$ cm.

Find : (1) The lateral area. (2) The volume of the pyramid.

Third question

3 marks

A regular hexagonal pyramid , the side length of its base = 12 cm. and its slant height = $10\sqrt{3}$ cm. Find its total area.

Total mark

Quiz

3

till lesson 3 – unit 2

10

Answer the following questions :

First question

4 marks

1 mark for each item

Choose the correct answer from those given :

- (1) The lateral area of a right cone whose base radius length is 6 cm. and its height = 8 cm. is cm^2
- (a) 60π (b) 28π (c) 10π (d) 48π
- (2) A regular quadrilateral pyramid of base side length 10 cm. and its lateral height 13 cm. , its lateral area =
- (a) 260 cm^2 (b) 360 cm^2 (c) 130 cm^2 (d) 520 cm^2
- (3) The number of planes passes through 3 collinear points is
- (a) zero (b) 1 (c) 3 (d) infinite.
- (4) The volume of a regular quadrilateral pyramid whose base perimeter 36 cm. and its height 10 cm. equals cm^3
- (a) 810 (b) 180 (c) 360 (d) 270

Second question

3 marks

The base length of a regular quadrilateral pyramid is 18 cm. , its volume is 1296 cm^3 , Find its lateral height and its lateral area.

Third question

3 marks

Find the radius length of the base of right circular cone whose total area $616\pi\text{ cm}^2$ and the length of its drawer is 30 cm.

Quiz

4

till lesson 4 – unit 2

10

Answer the following questions :

First question

4 marks

1 mark for each item

Choose the correct answer from those given :

(1) The centre of the circle : $x^2 + y^2 - 6x + 8y = 0$ is the point

- (a) (3 , -4) (b) (4 , -3) (c) (-3 , 4) (d) (-4 , 3)

(2) The circumference of a circle whose equation : $(x - 3)^2 + (y + 2)^2 = 25$ equals

- (a) 2π (b) 3π (c) 10π (d) 25π

(3) The lateral area of a right cone whose base radius length 6 cm. and its height 8 cm. equals cm^2

- (a) 60π (b) 28π (c) 40π (d) 48π

(4) The point which lies on the circle : $(x - 2)^2 + y^2 = 13$

- (a) (2 , 3) (b) (3 , -2) (c) (2 , 5) (d) (4 , 3)

Second question

3 marks

Find the general form of the circle whose centre (-2 , 5) and passes through (3 , 2)

Third question

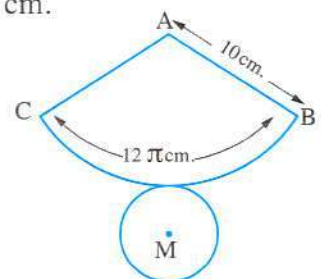
3 marks

The opposite figure represents the net of a solid where $\widehat{BC} = 12\pi \text{ cm}$.

, $AB = 10 \text{ cm}$. , calculate :

(1) The total area of this solid.

(2) The volume of the solid.



Monthly Tests

FIRST

Monthly tests of October.

SECOND

Monthly tests of November.



Contents of October

Statics

From : Unit (1) - Lesson (1) :
Forces - Resultant of two
forces meeting at a point.

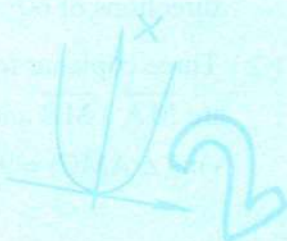
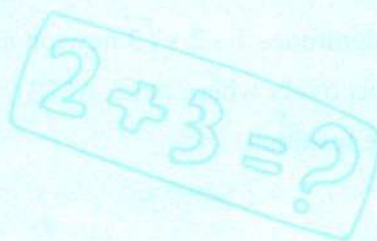
To : Unit (1) - Lesson (3) :
Resultant of coplanar forces
meeting at a point.

Contents of November

Lessons

From : Statics : Equilibrium of rigid
body under the effect of two/
three forces meeting at a point.

To : Geometry : Total surface area
of pyramid and cone.



Test

1

Total mark

10

1 Choose the correct answer from the given ones :

(6 marks)

- (1) Two forces of magnitudes 8 and 16 kg.wt. and the measure of their included angle is 120° . If these two forces act at a body, then the direction of motion of the body makes an angle of measure with the smaller force.
 (a) 30° (b) 90° (c) 60° (d) 45°
- (2) Two forces of equal magnitude and intersecting at a point. The measure of the angle between the two forces is 120° and the magnitude of each is 6 N., then the magnitude of their resultant = N.
 (a) 12 (b) $6\sqrt{3}$ (c) 6 (d) $12\sqrt{3}$
- (3) F N. and K N. are the magnitudes of two forces where $F > K$. If the smallest and the greatest value of their resultant are 5, 9 newton respectively, then $5F - 2K = \dots\dots\dots$ N.
 (a) 53 (b) 31 (c) 49 (d) 4
- (4) A body of weight 20 N. is placed on a smooth inclined plane makes an angle of measure 30° with the horizontal, then the component of the weight in direction perpendicular to the plane = N.
 (a) 10 (b) 20 (c) $10\sqrt{2}$ (d) $10\sqrt{3}$
- (5) Forces of magnitudes 8, $4\sqrt{3}$, $6\sqrt{3}$, 14 newton act at a point. The measure of the angle between the first and second force is 30° and between the second and third is 120° and between the third and fourth is 90° in one cyclic order, then the magnitude of their resultant =
 (a) 4 (b) 6 (c) 8 (d) 7
- (6) Two forces of magnitudes 3, F newton and measure of the angle between them is $\frac{2\pi}{3}$ if their resultant is perpendicular to the first force, then $F = \dots\dots\dots$ newton.
 (a) 1.5 (b) 3 (c) $3\sqrt{2}$ (d) 6

2 Answer the following questions :

- (1) A force of magnitude 18 newton acts in south direction. Find its two components in directions of 60° East of South and 30° West of South. (2 marks)
- (2) Three coplanar forces of magnitudes 1, 2, $\sqrt{3}$ newton act at M, their directions are \overrightarrow{MA} , \overrightarrow{MB} and \overrightarrow{MC} respectively where $m(\angle AMB) = 60^\circ$, $m(\angle BMC) = 30^\circ$, $m(\angle AMC) = 90^\circ$. Find the resultant. (2 marks)

Test

2

Total mark

10

1 Choose the correct answer from the given ones :

(6 marks)

(1) The resultant of two forces 6 , 8 newton is 10 N. , then the measure of the angle between their directions =°

- (a) 60 (b) 90 (c) 120 (d) 150

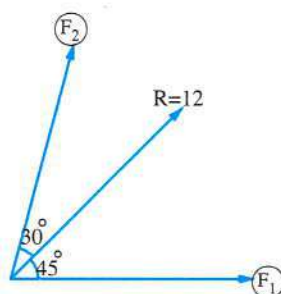
(2) Two forces intersecting at a point , their magnitudes 7 and F newton and their resultant bisects the angle between them , then $(F - 1) = \dots\dots\dots$ N.

- (a) 8 (b) 7 (c) 6 (d) 5

(3) In the opposite figure :

The force \vec{R} is resolved into two components \vec{F}_1 and \vec{F}_2 , then $F_1 = \dots\dots\dots$ newton.

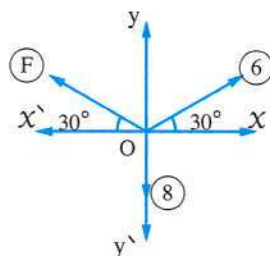
- (a) $12 \cos 75^\circ$ (b) $12 \cos 45^\circ$
(c) $6 \csc 45^\circ$ (d) $6 \csc 75^\circ$



(4) In the opposite figure :

If the resultant of the shown forces acts in direction of y-axis , then $F = \dots\dots\dots$ N.

- (a) 2 (b) 6
(c) 8 (d) 14



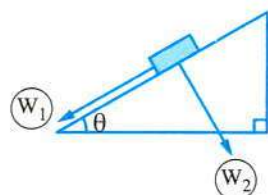
(5) The magnitudes of two forces are 5 and 10 newton and their resultant is perpendicular on the smaller force. If the measure of angle between the two forces is α and their resultant is R , then

- (a) $\alpha = 60^\circ$, $R = 10\sqrt{3}$ N. (b) $\alpha = 120^\circ$, $R = 10\sqrt{3}$ N.
(c) $\alpha = 60^\circ$, $R = 5\sqrt{3}$ N. (d) $\alpha = 120^\circ$, $R = 5\sqrt{3}$ N.

(6) In the opposite figure :

A body of weight 260 gm.wt. and $\tan \theta = \frac{5}{12}$, W_1 , W_2 are magnitudes of the two components in direction of the inclined plane downward and perpendicular to the plane , then

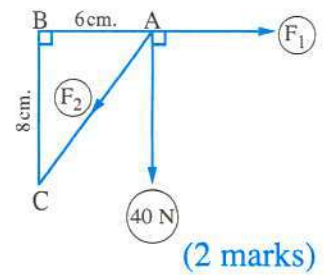
- (a) $W_1 = 120$ gm.wt. , $W_2 = 50$ gm.wt. (b) $W_1 = 260$ gm.wt. , $W_2 = 65$ gm.wt.
(c) $W_1 - W_2 = 70$ gm.wt. (d) $W_1 + W_2 = 340$ gm.wt.



2 Answer the following questions :**(1) In the opposite figure :**

If the force of magnitude 40 N. is resolved into two components \vec{F}_1 and \vec{F}_2 as shown in the figure.

Find the two component magnitudes F_1 , F_2



- (2)** The magnitudes of three forces are 10 , 20 , 30 newton acting at one point. The first acts due east , the second makes an angle of measure 30° west of the north and the third makes an angle of measure 60° south of the west. Find the magnitude and the direction of their resultant. (2 marks)

Test

1

Total mark

10

1 Choose the correct answer from the given ones :

(6 marks)

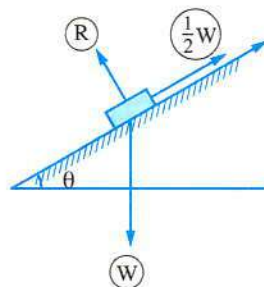
(1) The radius length of the base of a right circular cone = 5 cm. and its total surface area = $90\pi \text{ cm}^2$, then its volume = cm^3 .

- (a) 105π (b) 95π (c) 100π (d) 120π

(2) In the opposite figure :

If the body is in equilibrium under action of the shown forces, then $m(\angle \theta) = \dots\dots\dots$

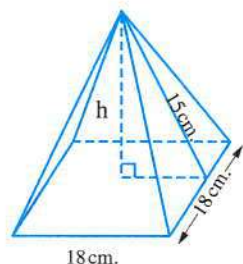
- (a) 30° (b) 60°
(c) 45° (d) 15°



(3) In the opposite figure :

The volume of the regular quadrilateral pyramid in which the side length of its base = 18 cm. and the slant height = 15 cm. is cm^3 .

- (a) 1296 (b) 1620
(c) 540 (d) 1944

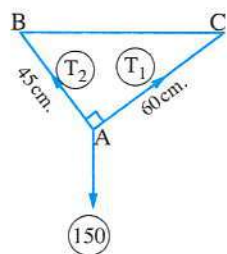


(4) Which of the following statements is not true ?

- (a) Any two points in the space have only one plane passing through them.
(b) Any three non-collinear points in the space determine a plane.
(c) The vertices of a triangle determine a plane.
(d) Every two intersecting straight lines are contained in one plane.

(5) In the opposite figure :

A body of weight 150 gm. wt. is in equilibrium by suspended by two perpendicular strings their lengths are 60 cm. , 45 cm. and the other ends are fixed at C and B on the same horizontal line, then $T_2 - T_1 = \dots\dots\dots$ gm.wt.

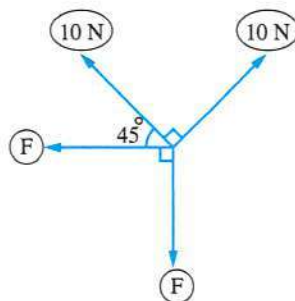


- (a) 120 (b) 90 (c) 60 (d) 30

(6) In the opposite figure :

The condition of equilibrium of the given forces is

- (a) $F = 10$ newton.
- (b) $F = 10\sqrt{2}$ newton.
- (c) $F = 5\sqrt{2}$ newton.
- (d) the system can not be in equilibrium.



2 Answer the following questions :

- (1) A regular quadrilateral pyramid , the side length of its base = 40 cm. , and its slant height is 25 cm. , **find :** (2 marks)
 - (1) Height of the pyramid. (2) The lateral surface area.
 - (3) The total surface area. (4) Its volume.
- (2) \overline{AB} is a uniform rod with length 60 cm. and weight 40 newton is connected to a hinge on the vertical wall at A. If the rod kept in equilibrium horizontally by a light string connected to the rod at B and with point C on the wall above A and at a distance 60 cm. from A. Find the tension in the string and magnitude of the reaction of the hinge at A. (2 marks)

Test

2

Total mark

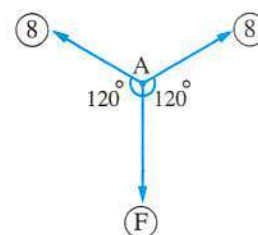
10

1 Choose the correct answer from the given ones :

(6 marks)

(1) In the opposite figure :

Particle A is kept in equilibrium under action of the three forces , as shown in the figure , where \vec{F} is in equilibrium with two forces each of magnitude 8 N. and it makes with each an angle of measure 120° , then $F = \dots\dots\dots$ N.



- (a) zero (b) 8 (c) 16 (d) $8 \sin 120^\circ$

(2) Volume of a regular quadrilateral pyramid is 400 cm^3 and its height is 12 cm. , then its lateral surface area = $\dots\dots\dots \text{ cm}^2$

- (a) 240 (b) 260 (c) 300 (d) 360

(3) The total surface area of a right circular cone which its drawer length equal the diameter length of its base is $\dots\dots\dots$

- (a) $4 \pi r^2$ (b) $3 \pi r^2$ (c) $3 \pi r^3$ (d) $4 \pi r^3$

(4) Any three non-collinear points identify $\dots\dots\dots$

- (a) 1 plane. (b) 2 planes. (c) 3 planes. (d) 4 planes.

(5) A body of weight (W) newton is placed on a smooth plane inclined with the horizontal at an angle of measure 30° and kept in equilibrium by a force of magnitude 36 newton acts in the direction of the line of greatest slope of the plane upwards , then the magnitude of the weight = $\dots\dots\dots$ newton.

- (a) 36 (b) $72\sqrt{3}$ (c) 72 (d) $36\sqrt{3}$

(6) A body of weight 32 newton is suspended at the end of a string with length 10 cm. and the other end of the string is fixed at a point on a vertical wall and the body is pulled by horizontal force to make the body in equilibrium when it is at a distance 6 cm. from the wall , then the magnitude of this force = $\dots\dots\dots$ newton.

- (a) 24 (b) 40 (c) 36 (d) 28

2 Answer the following questions :

(1) The area of base of a right circular cone is $36\pi \text{ cm}^2$ and the length of its drawer is 10 cm. , **find its :**

(1) Lateral surface area.

(2) Total surface area.

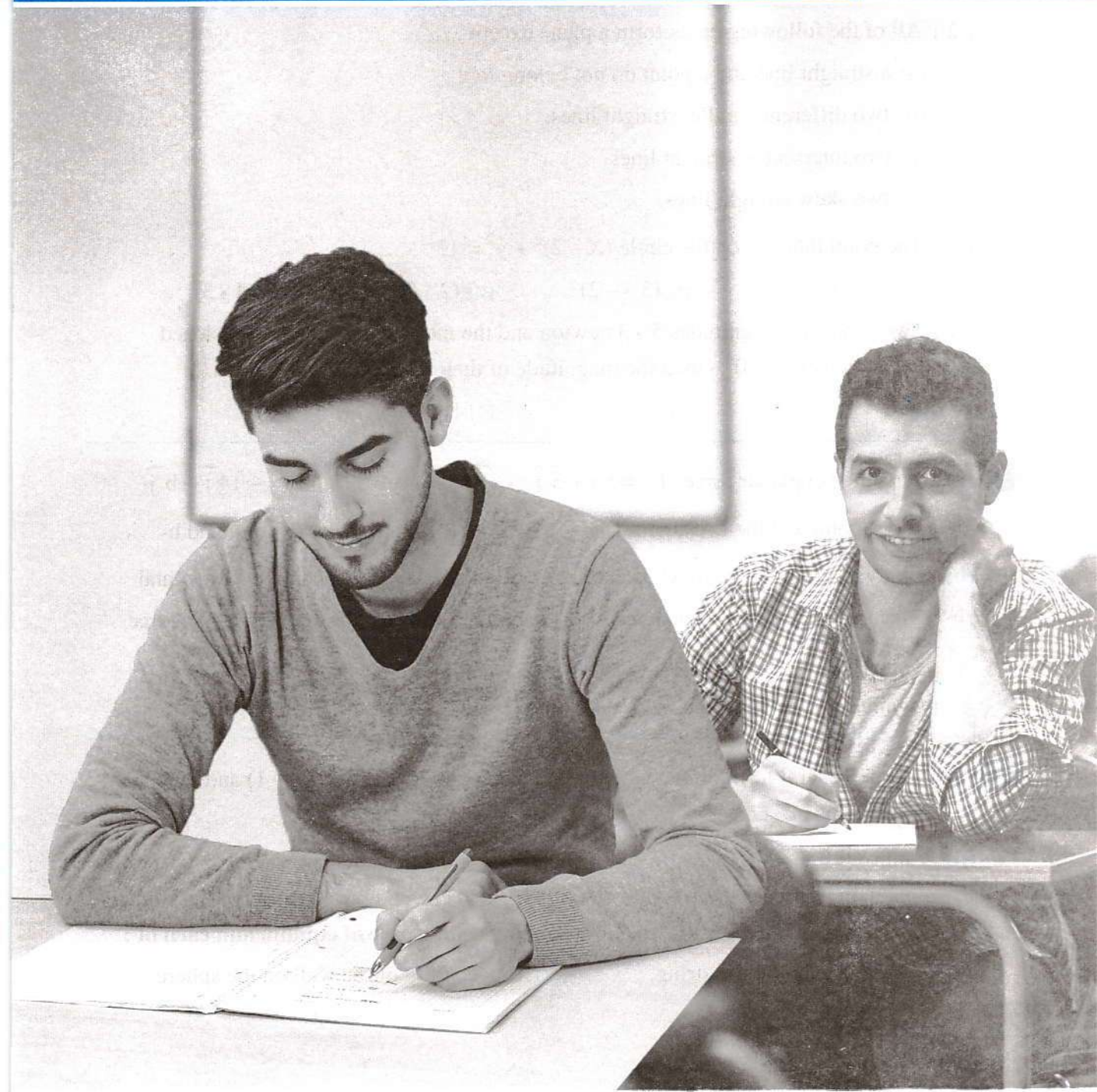
(3) Volume.

(2 marks)

(2) A weight of magnitude 200 gm.wt. is suspended by two strings of lengths 60 cm. and 80 cm. , from two points on the same horizontal line where the distance between them is 100 cm. Find the magnitude of tension in each string.

(2 marks)

School Book Examination



School Book Examination

Answer the following questions :

1 Choose the correct answer from the given ones :

- (1) Two forces of magnitude $3F$, $2F$ and the magnitude of their resultant is $5F$, then the measure of the angle enclosed between the two forces equals
- (a) 0° (b) 60° (c) 20° (d) 180°
- (2) All of the following cases form a plane except
- (a) a straight line and a point do not belong to it.
(b) two different parallel straight lines.
(c) two intersected straight lines.
(d) two skew straight lines.
- (3) The point that lies on the circle $(x-2)^2 + y^2 = 13$
- (a) $(2, 3)$ (b) $(3, -2)$ (c) $(2, 5)$ (d) $(4, 3)$
- (4) Two forces of magnitudes 5 , 3 newton and the measure of the angle enclosed between them is 60° , then the magnitude of their resultant R equals
- (a) 2 (b) 7 (c) 8 (d) 5

The questions of the first term in the school book examinations are collected to form one test.

2 (a) If the three coplanar forces $\vec{F}_1 = 5\hat{i} + 3\hat{j}$, $\vec{F}_2 = a\hat{i} + 6\hat{j}$, $\vec{F}_3 = -14\hat{i} + b\hat{j}$ act at a point and their resultant $\vec{R} = (10\sqrt{2}, \frac{3}{4}\pi)$ Find the values of a and b

(b) A body of weight 300 gm.wt. is placed on a smooth plane inclined to the horizontal with an angle whose tangent equals $\frac{1}{\sqrt{3}}$ the body is prevented from sliding by a force form with the line of the greatest slope an angle of measure 30° upwards. Find the magnitude of the force and the reaction of the plane.

3 (a) Find the general form of the equation of a circle whose centre $(2, -1)$ and the length of its radius is 3 cm.

(b) A uniform smooth sphere of weight 10 gm.wt. and radius length 30 cm. is hanged from a point on its surface by a light string of length 30 cm. and the other end of the string is fixed in a point on a vertical smooth wall. **Find in the case of equilibrium each of :**

- (1) The tension in the string. (2) The reaction of the wall on the sphere.

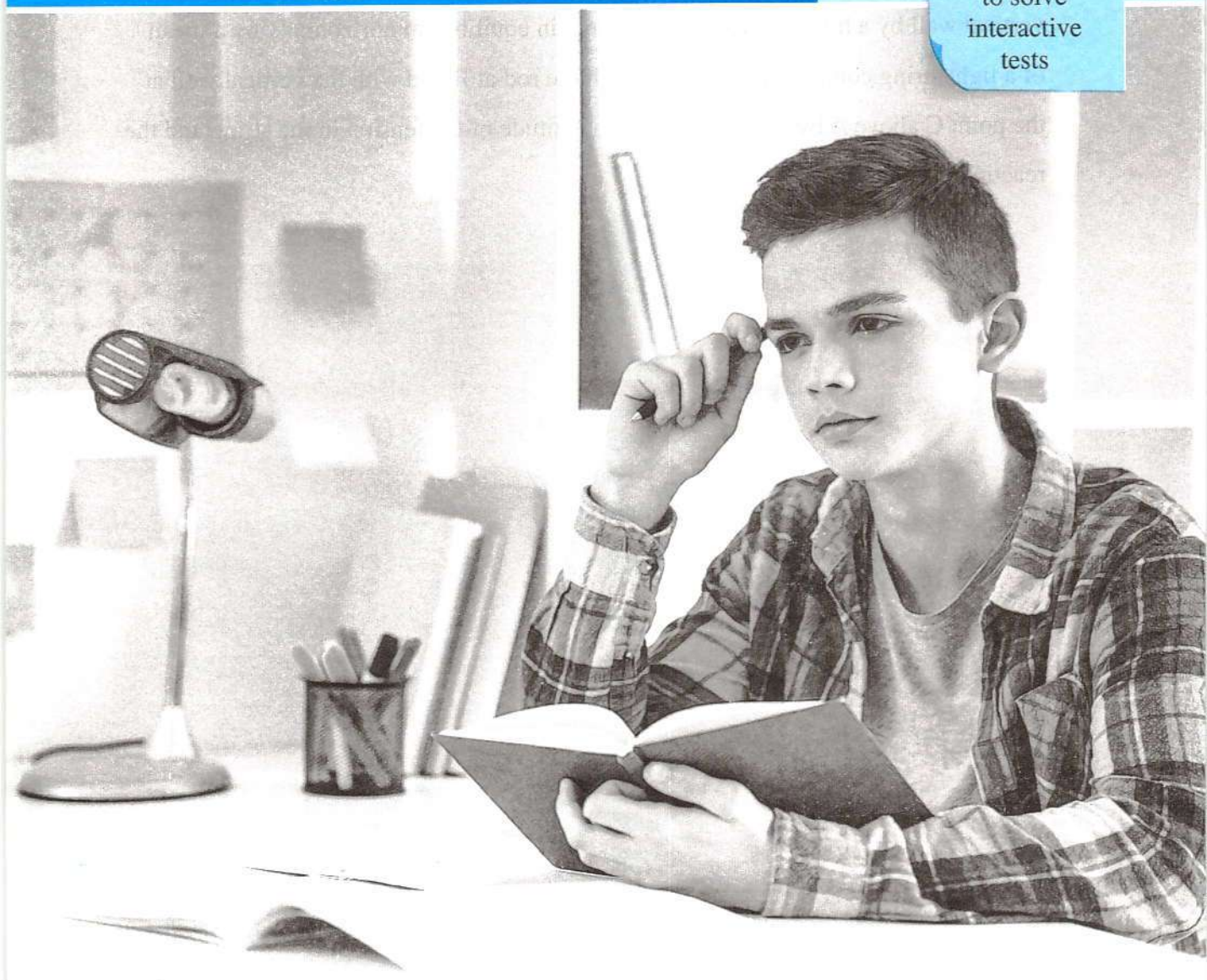
- 4** (a) A cube of wax with edge length 30 cm. transfer into a right circular cone of height 45 cm. Find the length of the radius of the base of the cone, if 8 % of the wax loss during milting and transferring processes.
- (b) A uniform rod of length 100 cm. and weight 150 gm.wt. is suspended freely from its ends by two strings and the other ends of the strings are fixed in one point. If the lengths of the two strings are 80 cm. , 60 cm. , find the tension in the two strings.
-
- 5** (a) ABCDEF is a uniform hexagon , the forces of magnitudes 8 , $6\sqrt{3}$, 5 and $4\sqrt{3}$ newton act on \overrightarrow{AB} , \overrightarrow{AC} , \overrightarrow{AD} , \overrightarrow{AE} respectively. Find the magnitude and the direction of their resultant.
- (b) \overline{AB} is a uniform rod with length 40 cm. and weight 30 newton is attached with a vertical wall by a hinge at A , the rod is kept in equilibrium horizontally by a mean of a light string connected by its ends with the rod at B and with the vertical wall at the point C above A by 40 cm. Find the magnitude of the tension in the string and the reaction of the hinge at A

Final Examinations

some school examinations.



Scan the
QR codes
to solve
interactive
tests



1

Cairo Governorate



El-Salam Educational Zone
Math's Supervision

First

Multiple choice questions



Interactive
test ①

Choose the correct answer from the given ones :

- (1) Two forces of magnitudes $4F$, $5F$ newton , their resultant $9F$ newton , then the measure of the angle included between them =
- (a) 0° (b) 90° (c) 180° (d) 120°
- (2) Two perpendicular forces of magnitudes F , 12 newton , their resultant 13 newton , then $F = \dots\dots\dots N$
- (a) 5 (b) 12 (c) 1 (d) zero
- (3) Two forces of magnitudes F , F newton , their resultant F newton , then the measure of the angle included between them =
- (a) 90° (b) 120° (c) 180° (d) zero
- (4) Two forces of magnitude F , 6 newton , then their resultant perpendicular to the first force , the measure of the angle included between them 120° , then $F = \dots\dots\dots$
- (a) 3 (b) 6 (c) $6\sqrt{2}$ (d) 12
- (5) Two forces of magnitude 3 , 5 newton , then their resultant $\in \dots\dots\dots$
- (a) $[3, 5]$ (b) $]3, 5[$ (c) $[2, 8]$ (d) $]2, 8[$
- (6) A body of weight W is placed on a smooth inclined plane with the horizontal by an angle of measure θ , then its component in the direction of the line of greatest slope
- (a) $W \sin \theta$ (b) $W \cos \theta$ (c) $W \tan \theta$ (d) $W \cot \theta$
- (7) A force of magnitude 12 newtons acts in direction 30° North of the East , then its component in the East direction = newton.
- (a) 6 (b) $6\sqrt{3}$ (c) 12 (d) 24
- (8) Some coplanar forces meeting at a point , and the sum of their components in the direction of X -axis equal 3 newton and the sum of their components in direction of y -axis equal 4 newton , then their resultant = newton.
- (a) 3 (b) 4 (c) 5 (d) 7

- (9) Some coplanar forces act at a point, their resultant makes with positive direction of X -axis an angle of tangent $\frac{3}{4}$ and the sum of components of these forces in direction of X -axis equal 12 newton, then the sum of their component in direction of y -axis = newton

(a) 9 (b) 12 (c) 16 (d) 20

- (10) Two equilibrium forces, $\vec{F}_1 = (4, a)$ and $\vec{F}_2 = (b, -5)$, then $a + b =$

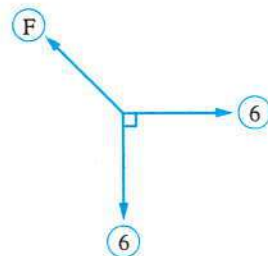
(a) 1 (b) -1 (c) 9 (d) -9

- (11) In the opposite figure :

the forces are in equilibrium

, then $F =$

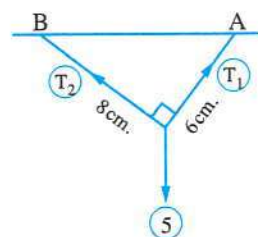
(a) 6 (b) $6\sqrt{2}$
(c) $5\sqrt{2}$ (d) 12



- (12) In the opposite figure :

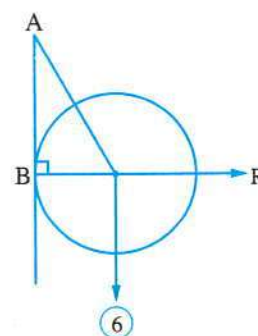
$T_1 \times T_2 =$

(a) 6
(b) $6\sqrt{2}$
(c) $3\sqrt{2}$
(d) 12



- (13) A sphere of weight 6 N attached by a string of length equal the radius length of the sphere, then the reaction of the wall = N

(a) $2\sqrt{3}$ (b) 6
(c) $6\sqrt{3}$ (d) 12



- (14) The two skew lines are

(a) not intersecting. (b) not parallel.
(c) not lie on one plane. (d) all the previous.

- (15) The least number of non-collinear points that determine a plane

(a) one. (b) two. (c) three. (d) four.

- (16) The base of the quadrilateral regular pyramid is a

(a) triangle. (b) square. (c) rectangle. (d) rhombus.

- (17) A triangular pyramid of regular faces, its edge length 6 cm., then its volume = cm^3

(a) 36 (b) 216 (c) $216\sqrt{2}$ (d) $18\sqrt{2}$

(18) A right cone , its lateral area $18\pi \text{ cm}^2$, its drawer length 6 cm. , then the length of radius of its base = cm.

- (a) 3 (b) 6 (c) 9 (d) 12

(19) If the equation of a circle $x^2 + y^2 = 36$, then its area = cm^2 .

- (a) 36 (b) 6π (c) 12π (d) 36π

(20) If the equation of a circle is $x^2 + y^2 + 4x - 6y - 10 = 0$, then its centre is

- (a) (4 , - 6) (b) (2 , - 3) (c) (- 2 , 3) (d) (- 4 , 6)

Second Essay questions

Answer the following questions :

1 A body of weight 12 newton is placed on an inclined plane with the horizontal by an angle of measure 30° , if the body kept in equilibrium under the action of a horizontal force. Find the magnitude of this force and the normal reaction of the plane.

2 A regular quadrilateral pyramid , the perimeter of its base = 40 cm. and its height 13 cm. find its volume.

2 Cairo Governorate



Shoubra Educational Zone
Mathematics Supervision

First Multiple choice questions



Interactive
test ②

Choose the correct answer from the given ones :

(1) Two forces of magnitudes 4 , 5 newton and the cosine of their included angle is $\frac{-2}{5}$, then the magnitude of their resultant = newton.

- (a) 15 (b) 9 (c) 5 (d) 13

(2) If $\vec{F} = 3\vec{i} - 4\vec{j}$, then $\|\vec{F}\| = \dots\dots\dots$ force unit.

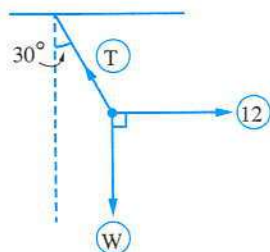
- (a) - 1 (b) 5 (c) 7 (d) 25

(3) A force of magnitude 12 newton , acts in the direction of 30° north of west and is resolved into two perpendicular directions , then the magnitude of its component in the west direction = newton.

- (a) 6 (b) 12 (c) $12\sqrt{3}$ (d) $6\sqrt{3}$

(4) In the opposite figure :

A body is suspended by the end of a string and its other end fixed at the ceiling of a room. A horizontal force of magnitude 12 gm.wt. pulled the body until the string inclines to the vertical by an angle of measure 30° , then the weight of the body = gm.wt.



- (a) $12\sqrt{3}$ (b) $3\sqrt{12}$ (c) 12 (d) 15

(5) If a body is in equilibrium under the action of three coplanar forces, meeting at a point and their magnitudes are 7, 8, 5 newton, then the measure of the angle between the second and the third forces =

- (a) 150° (b) 120° (c) 90° (d) 60°

(6) In the regular pyramid : the height the slant height.

- (a) $<$ (b) $>$ (c) \leq (d) \geq

(7) If the point (5, 2) lies on the circle : $(X - 3)^2 + (y + a)^2 = 13$, then : a =

- (a) ± 5 (b) ± 1 (c) 5 or -1 (d) -5 or 1

(8) Two forces act at a point of magnitudes $2F$, $3F$ newton and the magnitude of their resultant $5F$ newton, then the measure of their included angle =

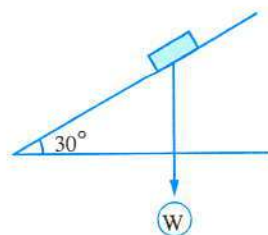
- (a) 0° (b) 60° (c) 120° (d) 180°

(9) If $\vec{F}_1 = 2\hat{i} + 3\hat{j}$, $\vec{F}_2 = \hat{i} + \hat{j}$, then the magnitude of their resultant = force unit.

- (a) $\sqrt{2}$ (b) $\sqrt{13}$ (c) 5 (d) 25

(10) In the opposite figure :

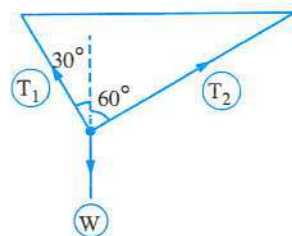
A body of weight (W) is placed on a smooth inclined plane inclines to the horizontal by an angle of measure 30° , then the component of its weight along the greatest slope of the plane is



- (a) W (b) $W \sin 30^\circ$ (c) $W \cos 30^\circ$ (d) $W \tan 30^\circ$

(11) In the opposite figure :

A body of weight 36 kg.wt. is suspended by two strings incline to the vertical by angles of measures 30° , 60° , then : $T_1 + T_2 =$ kg.wt.



- (a) 45 (b) $9 + 18\sqrt{3}$
(c) $36 + 18\sqrt{3}$ (d) $18(1 + \sqrt{3})$

(12) The least number of planes that determine a solid is

- (a) 2 (b) 3 (c) 4 (d) 5

(13) If the side length of the base of a regular quadrilateral pyramid equals 40 cm. and its height equals $20\sqrt{3}$ cm. , then its lateral surface area = cm^2

- (a) 3200 (b) 4300 (c) 6300 (d) 3400

(14) The length of diameter of the circle whose equation is :

$$4x^2 + 4y^2 + 16x - 8y - 16 = 0 \text{ equals length unit.}$$

- (a) 3 (b) 6 (c) 12 (d) 24

(15) Two forces act at a point their magnitudes are 7 , F newton and their resultant bisects the angle between them , then F = newton.

- (a) 49 (b) 14 (c) 7 (d) $7\sqrt{2}$

(16) Two forces of magnitudes 3 , F newton act at a point , include an angle of measure 120° and their resultant perpendicular to the first force , then F = newton.

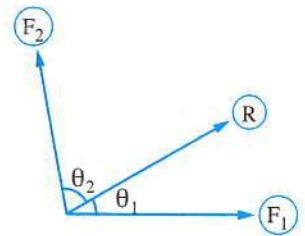
- (a) 0 (b) $3\sqrt{3}$ (c) 1.5 (d) 6

(17) In the opposite figure :

The force R is resolved into two components

$$F_1, F_2, \text{ then : } \frac{F_1}{F_2} = \dots\dots\dots$$

- (a) $\sin(\theta_1 + \theta_2)$ (b) $\frac{\sin \theta_2}{\sin \theta_1}$
(c) $\frac{\sin \theta_1}{\sin \theta_2}$ (d) $\sin \frac{\theta_1}{\theta_2}$



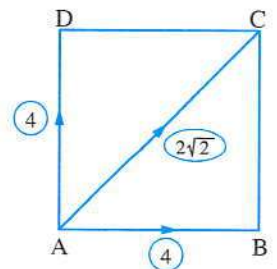
(18) In the opposite figure :

ABCD is a square , the forces 4 , 4 , $2\sqrt{2}$ newton

act in the directions of \overrightarrow{AB} , \overrightarrow{AD} , \overrightarrow{AC} respectively

, then the magnitude of their resultant = newton.

- (a) 6 (b) $10\sqrt{2}$ (c) $6\sqrt{2}$ (d) $6\sqrt{3}$



(19) The two straight lines are skew if they are

- (a) Not contained in one plane (b) Not parallel
(c) Not perpendicular (d) Not intersecting

(20) The volume of a right cone which the circumference of its base equals 44 cm. and its height 15 cm. equals cm^3 . (where $\pi = \frac{22}{7}$)

- (a) 110 (b) 235 (c) 245 (d) 770

Second Essay questions

Answer the following questions :

- 1 A body of weight 100 gm.wt. is suspended by two strings of lengths 60 cm. , 80 cm. , the other two ends are fixed at two points on the same horizontal line and the distance between them equals 100 cm. Find the tension in each string in the equilibrium position.

- 2 Form the general equation of the circle in which \overline{AB} is diameter of it where :

A (6 , -4) , B (0 , 2)

3 Cairo Governorate



Educational Administration
of Al-Shrouk

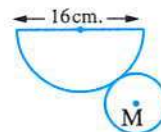
First Multiple choice questions



Interactive
test 3

Choose the correct answer from the given ones :

- (1) The least number of planes that determine a solid is
- (a) 2 (b) 3 (c) 4 (d) 5
- (2) A lamp of weight 30 gm.wt. is attached to the end of a string. It is in equilibrium under the effect of a force (F) perpendicular to the string when it is inclined to the vertical by an angle of measure 60° where T is the tension of the string , then $\frac{F}{T} = \dots\dots\dots$
- (a) 2 (b) $\frac{1}{2}$ (c) $\frac{1}{\sqrt{3}}$ (d) $\sqrt{3}$
- (3) A force of magnitude 150 newton acts in the direction 30° North of the west is resolved into two perpendicular components , then the magnitude of the component in North direction = newton.
- (a) 30 (b) 75 (c) $75\sqrt{3}$ (d) 150
- (4) The ratio between the edge length of the triangular pyramid of regular faces and its height =
- (a) $\sqrt{2} : \sqrt{3}$ (b) $\sqrt{3} : 2$ (c) $\sqrt{6} : 3$ (d) $\sqrt{6} : 2$
- (5) Two forces of magnitudes 6 N and 8 N. the magnitude of their resultant is 10 N. , then the measure of the angle between them =
- (a) 60° (b) 90° (c) 120° (d) 150°
- (6) If we fold the shown net , it becomes a cone.
- Its base radius length is cm.
- (a) 2.5 (b) 4 (c) 8 (d) 16

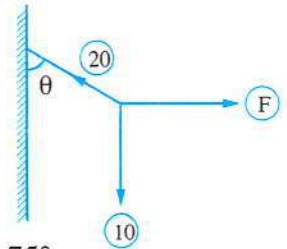


- (7) Two forces of magnitudes F and F act at a particle and the measure of the angle between them is 120° , then their resultant = newton.
 (a) $\sqrt{2} F$ (b) F (c) $2 F$ (d) $2\sqrt{F}$
- (8) A regular quadrilateral pyramid whose base perimeter is 36 and its height 10 cm., then its volume = cm^3
 (a) 180 (b) 270 (c) 360 (d) 810
- (9) Two forces of magnitude 3 and F newton act at a point the measure of the angle between them is $\frac{2\pi}{3}$, if the resultant is perpendicular to the first force, then F = newton.
 (a) 6 (b) 3 (c) $\sqrt{2}$ (d) 6
- (10) The difference between the greatest and smallest values of the resultant of two forces of magnitudes 5 and 8 newton =
 (a) 5 (b) 8 (c) 10 (d) 13
- (11) If the radius length of the base of a right circular cone = 6 cm., and its height = 8 cm., then its lateral surface area = cm^2
 (a) 60π (b) 48π (c) 69π (d) 96π
- (12) If the forces $\vec{F}_1 = a\vec{i} - 6\vec{j}$, $\vec{F}_2 = -3\vec{i} + 4\vec{j}$, $\vec{F}_3 = 9\vec{i} + 2\vec{j}$ are equilibrium, then a =
 (a) 6 (b) -6 (c) 1 (d) 15
- (13) A body of weight 6 newton is placed on a smooth plane inclined to the horizontal at an angle of measure 30° , it is kept in equilibrium by a horizontal force, then the magnitude of the reaction of the plane = N.
 (a) $2\sqrt{3}$ (b) $3\sqrt{3}$ (c) $4\sqrt{3}$ (d) $8\sqrt{3}$
- (14) The circumference of the circle whose equation : $(x - 3)^2 + (y + 5)^2 = 25$ is length units.
 (a) 2π (b) 3π (c) 10π (d) 25π
- (15) ABCD is a rectangle $AB = 4$ cm., $BC = 3$ cm., forces of magnitudes 4, 10, 6 newton. act along \vec{AB} , \vec{AC} , \vec{AD} respectively, the resultant of these forces makes with \vec{AB} an angle of measure
 (a) 45° (b) 60° (c) 30° (d) 90°
- (16) Two forces of magnitudes 8 and 8 N, the measure of the angle between them is 60° , then the magnitude of their resultant is N
 (a) $8\sqrt{3}$ (b) 8 (c) $4\sqrt{3}$ (d) 4

(17) In the opposite figure :

A body of weight 10 N , is suspended by a string which inclines to the vertical by an angle of measure θ , it is in equilibrium under the effect of a horizontal force F , then $\theta = \dots\dots\dots$

- (a) 30° (b) 45° (c) 60° (d) 75°



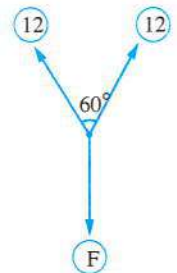
(18) If the total area of a triangular of pyramid of regular faces is $9\sqrt{3} \text{ cm}^2$, then the length of its edge = $\dots\dots\dots$ cm.

- (a) 3 (b) 9 (c) 27 (d) $\sqrt{3}$

(19) In the opposite figure :

The system is in equilibrium , then $F = \dots\dots\dots$ newton.

- (a) $12\sqrt{2}$ (b) $12\sqrt{3}$
(c) 6 (d) 12



(20) Which of the following sets of forces could not be in equilibrium ?

- (a) 11 , 7 , 5 newton. (b) 4 , 6 , 8 newton.
(c) 10 , 10 , 8 newton. (d) 8 , 4 , 14 newton.

Second Essay questions

Answer the following questions :

- 1 Find the equation of the circle whose centre is $(7, -5)$ and passes through the point $(3, -2)$
- 2 A uniform sphere of weight 24 newton and its radius length 6 cm. If it is in equilibrium by a string of length 4 cm. attached to a point of its surface and the other end of the string is fixed at a point in the vertical smooth wall. Find the tension of the string and the reaction of the wall.

4

Giza Governorate



Maths Inspection

First

Multiple choice questions



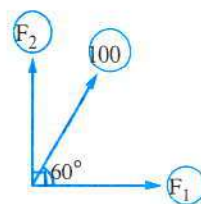
Interactive test 4

Choose the correct answer from the given ones :

- (1)** Two equal forces in magnitude , the magnitude of their resultant $= 7\sqrt{3}$ newton and the measure of the included angle is $\frac{\pi}{3}$, then the magnitude of each of them = $\dots\dots\dots$ newton.
- (a) 3 (b) $5\sqrt{3}$ (c) 5 (d) 7

(2) In the opposite figure :

If the force of magnitude 100 newton
is resolved into two forces \vec{F}_1 and \vec{F}_2 and the force
is measured by newton , then $(F_1, F_2) = \dots\dots\dots$



- (a) $(50, 50\sqrt{3})$ (b) $(50\sqrt{3}, 10)$ (c) $(50, 50)$ (d) $(10, 10)$

(3) If $\vec{F}_1 = 3\vec{i} + 2\vec{j}$, $\vec{F}_2 = a\vec{i} + 7\vec{j}$, $\vec{F}_3 = -12\vec{i} + b\vec{j}$ are three coplanar forces meeting
at a point and the resultant $\vec{R} = (6\sqrt{2}, \frac{3}{4}\pi)$, then $a - b = \dots\dots\dots$

- (a) -3 (b) 3 (c) zero (d) 6

(4) The force which is in equilibrium with two perpendicular forces F, F newton makes with
one of the two forces an angle of measure $\dots\dots\dots^\circ$

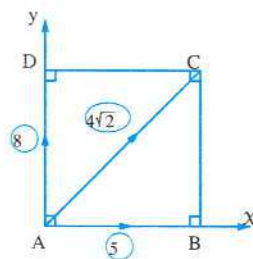
- (a) 90 (b) 120 (c) 135 (d) 150

(5) Two forces act at a point , the magnitude of the two forces are 6 , 3 newton and their
resultant is perpendicular to one of them , then the magnitude of their resultant
= $\dots\dots\dots$ newton.

- (a) 3 (b) $3\sqrt{3}$ (c) 6 (d) $6\sqrt{3}$

(6) In the opposite figure :

ABCD is a square , the forces of magnitudes 5 , 8 , $4\sqrt{2}$ newton
act on \vec{AB} , \vec{AD} and \vec{AC} respectively , then the polar form of the
resultant is $\dots\dots\dots$



- (a) $(5, 54^\circ)$ (b) $(15, 60^\circ)$
(c) $(15, 53^\circ 8')$ (d) $(13, 90^\circ)$

(7) A triangular regular faces pyramid , its edge length 10 cm. , then its total area
equal $\dots\dots\dots \text{cm}^2$

- (a) 40 (b) 100 (c) $100\sqrt{3}$ (d) $25\sqrt{3}$

(8) If the length of the diameter of the base of a right circular cone is 12 cm. and its height
8 cm. , then its lateral area equal $\dots\dots\dots \text{cm}^2$

- (a) 60π (b) 28π (c) 10π (d) 48π

(9) The area of the circle whose equation is : $(x - 5)^2 + (y + 4)^2 = 7$
equals $\dots\dots\dots$ square unit.

- (a) 3.5π (b) 7π (c) 12.25π (d) 49π

(10) The equation of the circle whose centre (4, 3) and touches X-axis is

(a) $(x - 3)^2 + (y - 4)^2 = 16$

(b) $(x - 4)^2 + (y - 3)^2 = 9$

(c) $(x + 3)^2 + (y + 4)^2 = 9$

(d) $(x + 3)^2 + (y - 4)^2 = 16$

(11) Two forces are equal in magnitude and each of them equal F newton if the magnitude of the resultant is F newton, then the measure of the included angle =

(a) 0

(b) 30°

(c) 60°

(d) 120°

(12) A force of magnitude $10\sqrt{2}$ newton acts in the direction of East it is resolved into two perpendicular components, one in the direction of eastern north, then the components of the force in the perpendicular direction is newton.

(a) 10

(b) 20

(c) $10\sqrt{3}$

(d) $10\sqrt{2}$

(13) Three coplanar forces $\vec{F}_1 = 6\hat{i} + 7\hat{j}$, $\vec{F}_2 = a\hat{i} - 9\hat{j}$, $\vec{F}_3 = 5\hat{i} + b\hat{j}$ act at a particle and they are in equilibrium, then $a + 2b = \dots\dots\dots$

(a) -9

(b) 5

(c) 7

(d) -7

(14) In the opposite figure :

A body of weight 90 gm.wt. is attached to the end of a string of 30 cm. long the body is pulled by a horizontal force.

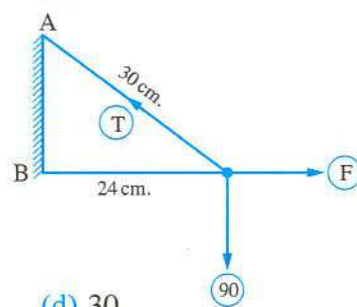
It comes to equilibrium when it is 24 cm. apart from wall \overline{AB} , then $T - F = \dots\dots\dots$ gm.wt.

(a) 150

(b) 120

(c) 50

(d) 30



(15) In the opposite figure :

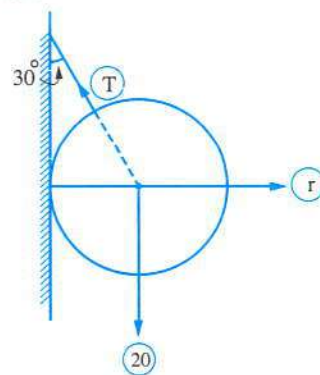
A smooth sphere of weight 20 newton rests against a smooth vertical wall. It is suspended at a point on its surface by means of a string and the other end is fixed to the wall at a point lies directly above the point of tangency of the sphere and the wall, if the string makes with the vertical an angle of measure 30° , then in case of equilibrium $T : r = \dots\dots\dots$

(a) 2 : 1

(b) 1 : 2

(c) $\sqrt{3} : 1$

(d) $2 : \sqrt{3}$



(16) If $\vec{F}_1 = \hat{i} - \hat{j}$, $\vec{F}_2 = 2\hat{i} - 4\hat{j}$, $\vec{R} = 2a\hat{i} - 3b\hat{j}$, then $a + b = \dots\dots\dots$

(a) 3

(b) $3\frac{1}{3}$

(c) $3\frac{1}{6}$

(d) 12

(17) If the total area of a triangular pyramid of regular faces = $36\sqrt{3} \text{ cm}^2$, then the sum of its edges lengths = cm.

(a) 6

(b) 12

(c) 18

(d) 36

- (18) A right circular cone , the length of its drawer equals the length of the diameter of its base , then its total area = cm^2
 (a) $3 \pi r^2$ (b) $3 \pi r^3$ (c) $4 \pi r^2$ (d) $4 \pi r^3$
- (19) Three equal forces in magnitude meeting at a point and they are in equilibrium , then the measure of the angle between each two forces =
 (a) 60° (b) 90° (c) 120° (d) 150°
- (20) Number of planes that are passing through two different parallel straight lines =
 (a) 1 (b) 2 (c) 3 (d) an infinite number.

Second Essay questions

Answer the following questions :

- 1 A regular quadrilateral pyramid whose base area is 9 cm^2 and the length of its lateral edge is 5 cm. Find its volume.
- 2 A smooth sphere of weight 15 newton is on a smooth vertical wall and suspended by a light string from a point on its surface. The other end of the string is attached to a point on the wall above the point of contact between the wall and the sphere. If the length of the string equals the radius length of the sphere. Find the pressure on the wall and the tension in the string in case of equilibrium.

5

Giza Governorate



Awseem Eductional Directorate
Mathematics Inspection

First Multiple choice questions



Interactive
test 5

Choose the correct answer from the given ones :

- (1) Two forces of magnitudes $2 F$, $5 F$ newton and the magnitude of their resultant is $3 F$ newton , then the measure of the angle between the two forces = $^\circ$
 (a) zero (b) 60 (c) 90 (d) 180
- (2) Two forces are of magnitudes 8 , F gm.wt. and their resultant bisects the angle between them then $F =$ gm.wt.
 (a) 4 (b) 16 (c) 2 (d) 8
- (3) Two forces of magnitudes 3 , F newton and the measure of the angle between them is 120° and their resultant is perpendicular to the first force , then $F =$ N
 (a) 1.5 (b) 3 (c) $3\sqrt{3}$ (d) 6

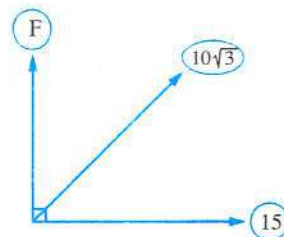
- (4) A force of magnitude 6 newton acts in the North direction is resolved into two perpendicular components, then its component in the Eastern North direction = newton.

(a) zero (b) 3 (c) $3\sqrt{2}$ (d) 6

- (5) In the opposite figure :

A force of magnitude $10\sqrt{3}$ newton is resolved into two perpendicular components, the magnitude of one of them is 15 newton, then the magnitude of the other component = newton.

(a) 5 (b) $5\sqrt{3}$ (c) 10 (d) 15

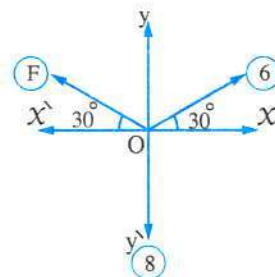


- (6) If the resultant of the two forces $\vec{F}_1 = 2\hat{i} - 2\hat{j}$, $\vec{F}_2 = 4\hat{i} - 8\hat{j}$ is $\vec{R} = 2a\hat{i} - 5b\hat{j}$, then $a + b = \dots\dots\dots$

(a) 3 (b) 2 (c) 5 (d) 1

- (7) If the resultant of the forces in the opposite figure is in the direction of y-axis, then $F = \dots\dots\dots$ newton.

(a) 2 (b) 6
(c) 8 (d) 14



- (8) If three forces are equal in magnitude, meeting at a point and in equilibrium, then the measure of the angle between any two of them =

(a) 60° (b) 90° (c) 120° (d) 150°

- (9) Three forces are meeting at a point and are in equilibrium, if 7, 3 are the magnitudes of two of them, then the magnitude of the third could be newton.

(a) 3 (b) 5 (c) 11 (d) 2

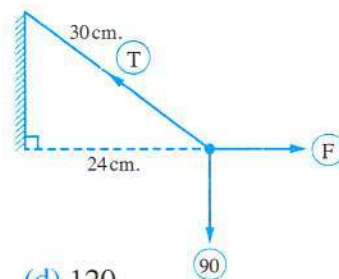
- (10) If the force of magnitude F is in equilibrium with the two forces of magnitudes 5, 3 and enclosing an angle between them of measure 60° , then $F = \dots\dots\dots$ newton.

(a) $\sqrt{34}$ (b) $\sqrt{19}$ (c) 7 (d) 15

- (11) In the opposite figure :

A body of weight 90 gm.wt. is attached to a string of length 30 cm. the body is pulled by a horizontal force to be in equilibrium at a distance 24 cm. from the wall, then $T = \dots\dots\dots$ gm.wt.

(a) 50 (b) 30 (c) 150 (d) 120



- (12) The least number of unequal forces could be in equilibrium is
 (a) 1 (b) 2 (c) 3 (d) 4
- (13) If the force of magnitude F is in equilibrium with the two perpendicular forces of magnitudes 8, 15, then $F = \dots\dots\dots$ newton.
 (a) 7 (b) 21 (c) 23 (d) 17
- (14) The number of planes that pass through two given points is
 (a) 1 (b) 2 (c) 3 (d) an infinite number.
- (15) MABCD is a regular quadrilateral pyramid the side length of its base is 10 cm. and its height is 12 cm. , then its volume = cm^3 .
 (a) 300 (b) 400 (c) 450 (d) 120
- (16) A regular quadrilateral pyramid of lateral surface area 30 cm^2 and its lateral height is 5 cm. , then the perimeter of its base is cm.
 (a) 12 (b) 24 (c) 36 (d) 40
- (17) A right circular cone its base radius length is 6 cm. and the length of its drawer is 10 cm. , then its volume is cm^3 .
 (a) 32π (b) 64π (c) 96π (d) 288π
- (18) A right circular cone its base radius length is 15 cm. and the length of its drawer is 25 cm. , then its lateral surface area is cm^2 .
 (a) 375π (b) 600π (c) 1500π (d) 1875π
- (19) The centre of the circle : $x^2 + y^2 - 6x + 8y = 0$ is the point
 (a) (4, -3) (b) (-3, 4) (c) (3, -4) (d) (-4, 3)
- (20) The circumference of the circle whose equation is : $x^2 + y^2 = 16$ is
 (a) 4π (b) 8π (c) 10π (d) 16π

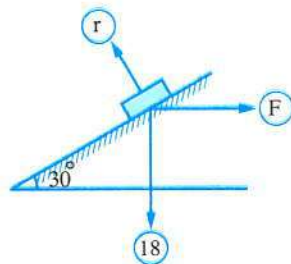
Second Essay questions

Answer the following questions :

1 In the opposite figure :

A body of weight 18 newton is placed on a smooth inclined plane to the horizontal at an angle of measure 30° is in equilibrium under the effect of a horizontal force \vec{F}

Find : the value of each of F , r



- 2 Write the general form of the equation of the circle whose center is $(-2, 3)$ and the length of its diameter is 18 length units.



First Multiple choice questions



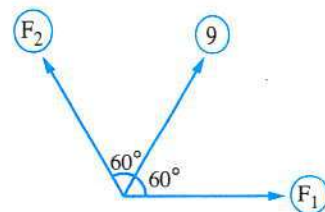
Interactive
test 6

Choose the correct answer from the given ones :

- (1) If the resultant of two forces acting at a point reached its minimum value , then the measure of the angle between them =
- (a) zero° (b) 60° (c) 120° (d) 180°
- (2) A triangular regular faces pyramid , its edge length l cm. , then the total surface area = cm^2
- (a) l^2 (b) $\sqrt{3} l^2$ (c) $2\sqrt{3} l$ (d) $4 l^2$
- (3) A body of weight 10 newton , is placed on a smooth plane inclines to the horizontal by an angle of measure 30° is kept in equilibrium by a force \vec{F} in direction of greatest slope upward , then the magnitude of the reaction of the plane on the body = newton.
- (a) 5 (b) $\frac{5\sqrt{3}}{2}$ (c) $10\sqrt{3}$ (d) $5\sqrt{3}$
- (4) If two straight lines are parallel to the third in the space , then they are
- (a) perpendicular. (b) intersecting.
(c) parallel. (d) Not in the same plane.
- (5) Two forces of magnitudes $(5 F + 30)$, $(7 F + 10)$ newton acting at a point and the resultant bisect the angle between the two forces then $F = \dots\dots\dots$ newton
- (a) 10 (b) 30 (c) 40 (d) 4
- (6) $\vec{F} = \left(6, \frac{2\pi}{3}\right)$, then $\|\vec{F}\| = \dots\dots\dots$ unit of forces.
- (a) - 2 (b) 1 (c) 6 (d) $\frac{2\pi}{3}$
- (7) The length of the diameter of the circle : $2x^2 + 2y^2 + 8x - 4y - 8 = 0$ equals
- (a) 3 (b) 12 (c) 24 (d) 6
- (8) The lateral surface area of a right cone the diameter length of its base = 10 cm. and its height = 12 cm. equals cm^2
- (a) 65π (b) 120π (c) 65 (d) 120
- (9) If we fold the sector its central angle is θ where $180^\circ > \theta > 0^\circ$ and L is cone drawer , r is radius length of its base cone , then
- (a) $L > 2r$ (b) $L = 2r$ (c) $L < 2r$ (d) $L < r$

(10) In the opposite figure :

a force of magnitude 9 newton is resolved into two component F_1 and F_2 , then $F_1 = \dots\dots\dots$ newton.



- (a) 4.5 (b) $4.5\sqrt{3}$ (c) 9 (d) 10

(11) The volume of the quadrilateral pyramid , where its base as rhombus of diagonals length 6 cm. , 5 cm. and its height 4 cm. equals $\dots\dots\dots$ cm^3 .

- (a) 40 (b) 25 (c) 30 (d) 20

(12) The circumference of the circle of its equation : $(x - 3)^2 + (y + 2)^2 = 25$ equals $\dots\dots\dots$ length unit.

- (a) 2π (b) 3π (c) 10π (d) 25π

(13) $\vec{F}_1 = 4\vec{i} - 3\vec{j}$, $\vec{F}_2 (2, -7)$ and $\vec{F}_3 = -\vec{i} + 22\vec{j}$ and \vec{R} is their resultant , then $\|\vec{R}\| = \dots\dots\dots$

- (a) 13 (b) 5 (c) 12 (d) 17

(14) The resultant of the two perpendicular forces 6 newton and 8 newton is $\dots\dots\dots$ newton

- (a) 14 (b) 2 (c) 48 (d) 10

(15) If the ratio between the maximum and minimum value of the resultant of two forces is 3 : 2 , then the ratio between the two forces is $\dots\dots\dots$

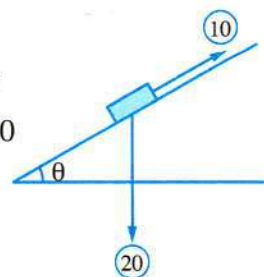
- (a) 1 : 1 (b) 2 : 3 (c) 5 : 1 (d) 1 : 2

(16) Three forces are equals in magnitude and acting at a point and in equilibrium , then the measure of the angle between any two forces = $\dots\dots\dots^\circ$

- (a) 60 (b) 120 (c) 150 (d) 180

(17) In the opposite figure :

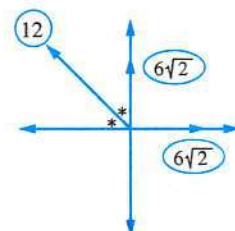
the body is placed on a smooth plane and it is kept in equilibrium by a force acting along the line of greatest slope upward of magnitude 10 , then the measure of $\theta = \dots\dots\dots^\circ$



- (a) 30 (b) 45
(c) 60 (d) 75

(18) In the opposite figure :

The resultant is in direction of $\dots\dots\dots$



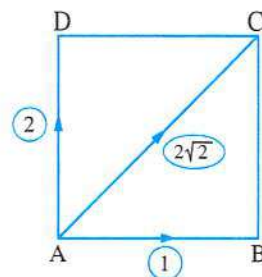
- (a) South. (b) East.
(c) West. (d) North.

- (19) $\vec{F}_1 = 7\hat{i} - 4\hat{j}$, $\vec{F}_2 = k\hat{i} + 3\hat{j}$ and $\vec{F}_3 = -9\hat{i} + m\hat{j}$ and \vec{R} is their resultant and $\vec{R} = (5\sqrt{2}, \frac{\pi}{4})$, then $m + k = \dots\dots\dots$

(a) 13 (b) 5 (c) 7 (d) 6

- (20) ABCD is a square , then the resultant is

(a) $(5, 36^\circ 52')$
 (b) $(5, 53^\circ 8')$
 (c) $(5, 52^\circ 8')$
 (d) $(6, 36^\circ 52')$



Second Essay questions

Answer the following questions :

- 1 The weight of a body is 200 gm.wt. It is tied by two perpendicular strings their lengths are 60 cm. , 80 cm. and the other ends are fixed on the same horizontal line , find the difference between the tensions in the two strings.
- 2 Determine the position of the circle $C_1 : (X - 5)^2 + (y + 2)^2 = 4$ with respect to the circle $C_2 : (X + 7)^2 + (y - 3)^2 = 1$

7

El-Kalyoubia Governorate



Maths Inspection

First Multiple choice questions



Interactive test 7

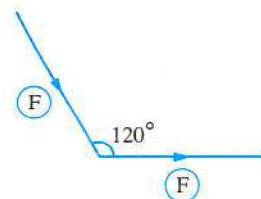
Choose the correct answer from the given ones :

- (1) The magnitude of two forces are $2F$ newton and $6F$ newton and its resultant is $8F$ newton , then the angle between them is°
- (a) 0 (b) 60 (c) 90 (d) 180
- (2) If \vec{R} is the resultant of the forces \vec{F}_1 and \vec{F}_2 , where $R \in [10, 22]$, $F_1 < F_2$, then $(F_1, F_2) = \dots\dots\dots$
- (a) $(10, 22)$ (b) $(6, 16)$ (c) $(12, 32)$ (d) $(5, 11)$
- (3) The magnitude of two forces are $7, F$ newton and their resultant bisects the angle between them , then $F = \dots\dots\dots$ newton.
- (a) $7\sqrt{2}$ (b) 3.5 (c) 7 (d) 14
- (4) If \vec{R} is the resultant of the forces \vec{F}_1 and \vec{F}_2 where $\vec{R} \perp \vec{F}_2$, then $F_1^2 = \dots\dots\dots$
- (a) $R^2 - F_2^2$ (b) $F_2^2 - R^2$ (c) $R^2 + F_2^2$ (d) $R + F_2$

(5) In the opposite figure :

The resultant of two forces F and F is

- (a) $\frac{1}{2} F$ (b) F
(c) $\sqrt{3} F$ (d) $\sqrt{5} F$



(6) A force of magnitude 20 newton act in direction 30° north of east is resolved into two perpendicular components , then the magnitude of its component in direction the east is

- (a) 10 (b) 20 (c) $10\sqrt{2}$ (d) $10\sqrt{3}$

(7) A body of weight 15 N. is placed on a smooth plane inclines to the horizontal by an angle of measure θ° , the body is kept in equilibrium by a horizontal force of magnitude $15\sqrt{3}$ N. , then $\theta =$

- (a) 22.5 (b) 30 (c) 45 (d) 60

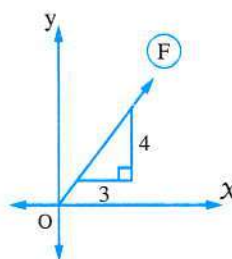
(8) If $\vec{F}_1 = 5\hat{i} + 2\hat{j}$, $\vec{F}_2 = a\hat{i} + 6\hat{j}$, $\vec{F}_3 = -14\hat{i} + b\hat{j}$, are three coplanar forces acting at a point and its resultant $\vec{R} = (10\sqrt{2}, \frac{3\pi}{4})$, then $a + b =$

- (a) -1 (b) 1 (c) 0 (d) 14

(9) In the opposite figure :

If the horizontal component of the force F is 60 newton , then the vertical component is newton.

- (a) 45 (b) 60 (c) 75 (d) 80



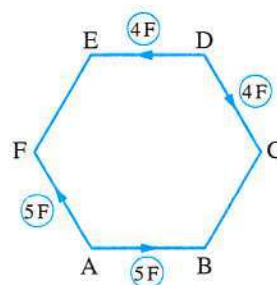
(10) Two forces the difference between their magnitudes 2 and the product of their magnitudes is 48 , and the magnitude of its resultant is $2\sqrt{13}$ newton , then the measure of the angle between their lines of action is

- (a) 90 (b) 120 (c) 135 (d) 150

(11) In the opposite figure :

ABCDEF is a regular hexagon , then the resultant of these forces act in direction

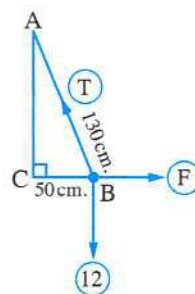
- (a) \vec{AD} (b) \vec{DA}
(c) \vec{AC} (d) \vec{EA}



(12) In the opposite figure :

If the body B is in equilibrium
 , then $T - F = \dots\dots\dots$

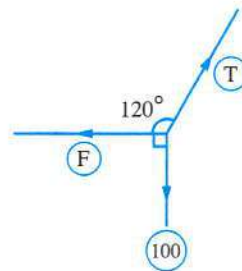
- (a) 18 (b) 12
 (c) 8 (d) 5



(13) In the opposite figure :

If the forces are in equilibrium
 , then $F + T = \dots\dots\dots$ N

- (a) 300 (b) $300\sqrt{3}$
 (c) 100 (d) $100\sqrt{3}$



(14) If the points A , B and C represent a plane , then which of the following is always true ?

- (a) $AB = BC = CA$ (b) $AB + BC = CA$ (c) $AB + BC > CA$ (d) $AB + BC < CA$

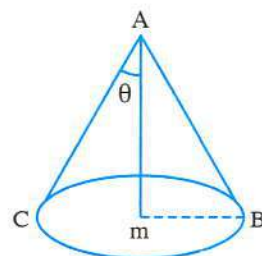
(15) If the total area of triangular pyramid of regular faces = $36\sqrt{3}$ cm² , then the sum of its edges = $\dots\dots\dots$ cm.

- (a) 6 (b) 12 (c) 18 (d) 36

(16) In the opposite figure :

If $\sin \theta = \frac{3}{5}$ and the height of the cone = 12 cm.
 , then the total area of the cone = $\dots\dots\dots \pi$ cm²

- (a) 144 (b) 169
 (c) 216 (d) 612



(17) The straight line $y = 2$ cuts the circle $(x - 3)^2 + (y - 2)^2 = 25$ in the two points A and B , then AB = $\dots\dots\dots$ length unit.

- (a) 7 (b) 8 (c) $\sqrt{13}$ (d) 10

(18) The equation $(a - 1)x^2 + 2y^2 + (b - 3)x + (c - 4)y + (d - 5)xy + 2 = 0$ represents a circle its center (3 , - 1) , then $a + b + c + d = \dots\dots\dots$

- (a) 17 (b) 11 (c) 7 (d) 5

(19) If the ΔOAB is rotate complete rotation about X-axis where the equation of \overleftrightarrow{AB} is $\frac{x}{4} + \frac{y}{3} = 1$, then the volume of the resultant solid is $\dots\dots\dots \pi$ cm³

- (a) $\frac{16}{3}$ (b) 16 (c) 12 (d) 6

(20) A regular quadrilateral pyramid the area of each of its lateral faces equals the area of its base , and the perimeter of the base is 24 cm. , then the volume of the pyramid = cm^3

- (a) 36 (b) $6\sqrt{3}$ (c) $36\sqrt{15}$ (d) $72\sqrt{3}$

Second Essay questions

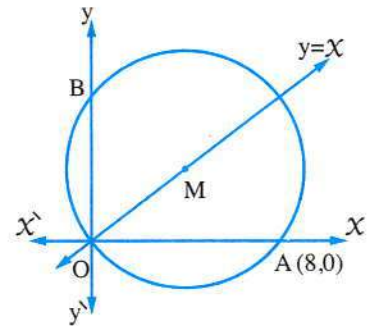
Answer the following questions :

1 The forces of magnitudes F , 6 , $4\sqrt{2}$, $5\sqrt{2}$ and K measured in newton are act at a point in the directions east , north , north west , west south and south respectively. Find the values of F and K if the resultant of forces = 2 newton act in north direction.

2 In the opposite figure :

A circle its center $M \in$ the straight line $y = x$

Find the equation of the circle.



8

El-Monofia Governorate



Menouf Educational Administration
Mathematics Inspection

First Multiple choice questions

Choose the correct answer from the given ones :

- (1) Two perpendicular forces of magnitudes 12 newton , 5 newton , act at point , then the magnitude of their resultant
- (a) 7 (b) 13 (c) 14 (d) 17
- (2) Two forces of equal magnitudes , enclosing between them an angle of measure $\frac{\pi}{2}$ if their resultant is 8 newton , then the value of each force is newton.
- (a) 4 (b) 8 (c) $2\sqrt{2}$ (d) $4\sqrt{2}$
- (3) Three forces are equal in magnitude and meeting at a point are in equilibrium , then the measure of the angle between any two of them is°
- (a) 60 (b) 90 (c) 120 (d) 150

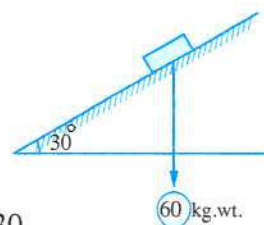


Interactive
test 8

(4) In the opposite figure :

A body of weight 60 kg.wt. is placed on a smooth inclined plane by an angle of measure 30° with the horizontal , then the component in the perpendicular direction on the plane

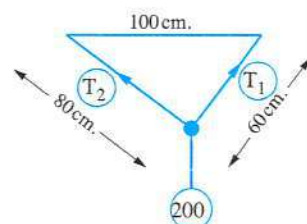
- (a) 60 (b) $30\sqrt{3}$ (c) $30\sqrt{2}$ (d) 30



(5) In the opposite figure :

A weight of a magnitude 200 gm.wt. is suspended by two strings of lengths 60 cm. and 80 cm. , from two points on one horizontal line where the distance between them is 100 cm. , then $T_1 - T_2 = \dots\dots\dots$

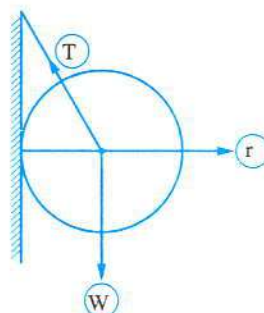
- (a) 160 (b) 120 (c) 80 (d) 40



(6) In the opposite figure :

A solid uniform sphere of weight 15 kg.wt. and radius length 5 cm. is in equilibrium by a string of length 5 cm. attached to a point of its surface and the other end of the string is fixed at a point in the vertical smooth plane above the tangency point , then $\frac{r}{T} = \dots\dots\dots$

- (a) 1 : 2 (b) 1 : 3 (c) $1 : \sqrt{2}$ (d) $1 : \sqrt{3}$



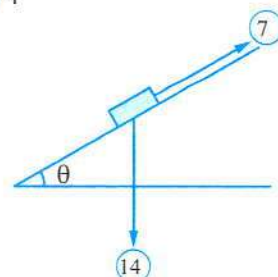
(7) $\vec{F}_1 = \hat{i} - \hat{j}$, $\vec{F}_2 = 2\hat{i} - 3\hat{j}$, then the magnitude of their resultant

- (a) 12 (b) 7 (c) 5 (d) 4

(8) In the opposite figure :

The body equilibrium on a smooth inclined plane , then : $\theta = \dots\dots\dots^\circ$

- (a) 60 (b) 90
(c) 45 (d) 30



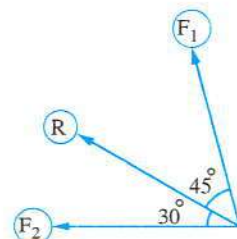
(9) Two forces meeting at a point their magnitudes 5 , 3 newton , then their resultant $\in \dots\dots\dots$

- (a) $[2, 8]$ (b) $]2, 8[$ (c) $[2, 8[$ (d) $]2, 8]$

(10) In the opposite figure :

The resultant $R = 12$ newton , then $F_1 = \dots\dots\dots$

- (a) $12 \cos 75^\circ$ (b) $12 \csc 75^\circ$
(c) $6 \csc 75^\circ$ (d) $6 \cos 75^\circ$



- (11) Three coplanar forces meeting at a point are in equilibrium, the magnitude of two forces of them are 3 and 7 newton, then the magnitude of third could be newton.
 (a) 2 (b) 3 (c) 5 (d) 11
- (12) If three forces meeting at a point and acting up on a particle are in equilibrium, then the magnitude of each force is proportional to the of the included angle between the two other forces.
 (a) sin. (b) cosin. (c) tangent. (d) cotangent.
- (13) Two forces of magnitudes : $3F - 1$, $F + 5$ newton, if their resultant bisects the angle between them, then the value of $F =$ newton.
 (a) 2 (b) 3 (c) 4 (d) 5
- (14) A right circular cone, the length of its slant height 10 cm. and its height 8 cm., then the volume cm^3 .
 (a) 30π (b) 40π (c) 80π (d) 96π
- (15) A triangular pyramid of regular faces, length of its edge is 12 cm., then its total surface area = cm^2 .
 (a) 144 (b) $144\sqrt{2}$ (c) $144\sqrt{3}$ (d) $144\sqrt{6}$
- (16) All the following cases determine a plane except
 (a) A straight line and a point does not belong to it.
 (b) Two parallel and not coincident straight lines.
 (c) Two intersecting straight lines.
 (d) Two skew straight lines.
- (17) The point which lies on the circle : $(x - 3)^2 + (y - 4)^2 = 25$ is
 (a) (3, 4) (b) (3, 0) (c) (0, 4) (d) (0, 0)
- (18) A regular quadrilateral pyramid the perimeter of its base = 40 cm. and its height 12 cm., then lateral surface area = cm^2 .
 (a) 200 (b) 240 (c) 260 (d) 320
- (19) The solid formed from the rotation of a right-angle triangle a complete rotation about one of its right sides as an axis is called
 (a) cube. (b) pyramid. (c) cone. (d) cuboid.
- (20) The circumference of the circle whose equation : $(x - 3)^2 + (y + 2)^2 = 25$ is
 (a) 5π (b) 10π (c) 15π (d) 25π

Second Essay questions

Answer the following questions :

- 1 ABCDHE is a regular hexagon. Forces of magnitudes 2 , $4\sqrt{3}$, 8 , $2\sqrt{3}$ and 4 kg.wt. act at point A in directions \overrightarrow{AB} , \overrightarrow{AC} , \overrightarrow{AD} , \overrightarrow{AH} , \overrightarrow{AE} respectively. Find the magnitude and the direction of their resultant.
- 2 Find the equation of the circle which the straight line : $3x + 4y + 23 = 0$ touches it and its centre is $(1, 1)$.

9 El-Dakahlia Governorate



Maths Supervision

First Multiple choice questions



Interactive test 9

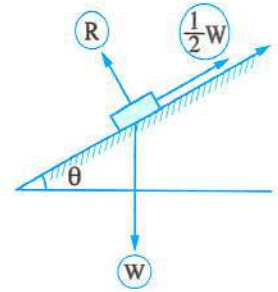
Choose the correct answer from the given ones :

- (1) The volume of the right cone is $27\pi \text{ cm}^3$ and the circumference of its base is $6\pi \text{ cm}$, then its height is cm.
 (a) 27 (b) 18 (c) 9 (d) 6
- (2) Right circular cone , area of its base = $25\pi \text{ cm}^2$, length of its drawer = 13 cm , then its lateral area = cm^2
 (a) 50π (b) 65π (c) 90π (d) 100π
- (3) Two forces of magnitudes $8\sqrt{3}$ and 8 newton act at a point the angle between them of measure 150° , then the magnitude of the resultant of the two forces = newton.
 (a) 64 (b) 32 (c) 16 (d) 8
- (4) A ball of pendulum of weight 600 dyne is in equilibrium when the string makes an angle of measure 30° with the vertical under the effect of a force perpendicular to the string , then the magnitude of the force = dyne.
 (a) 1200 (b) 300 (c) $300\sqrt{2}$ (d) $300\sqrt{3}$
- (5) Force of magnitude $4\sqrt{2}$ acts in east direction it was resolved into two perpendicular component , then the magnitude of the component in direction of eastern north equals newton.
 (a) 4 (b) $4\sqrt{2}$ (c) 8 (d) $8\sqrt{2}$
- (6) If the equation of a circle is $(2a + 1)x^2 + (a + 2)y^2 + (b - 1)xy - 6ax + 12by - 12 = 0$, then its radius length equals length unit.
 (a) 3 (b) 4 (c) 5 (d) 6

(7) In the opposite figure :

If the body is in equilibrium under acting forces , then $m (\angle \theta) = \dots\dots\dots$

- (a) 30° (b) 15°
(c) 60° (d) 45°



(8) A uniform smooth sphere of weight 1.5 gm.wt. and radius length 25 cm.

is suspended at a point on its surface by a light string of length 25 cm. and the other end of the string is fixed at a point in vertical smooth wall , if the sphere is in equilibrium , then the tension in the string = $\dots\dots\dots$ gm.wt.

- (a) $2\sqrt{2}$ (b) $\sqrt{3}$ (c) 3 (d) 6

(9) If the resultant of two forces acting on point is zero , then the angle between them = $\dots\dots\dots$

- (a) 180° (b) 0° (c) 45° (d) 90°

(10) If a force of magnitude (F) is in equilibrium with two forces of magnitudes 5 and 3 newton and the measure of the angle between them is 60° , then $F = \dots\dots\dots$ newton.

- (a) $\sqrt{19}$ (b) $\sqrt{34}$ (c) 7 (d) 15

(11) The equation of the circle which is the image of the circle : $x^2 + y^2 - 12x + 6y + 20 = 0$ by translation $(x + 2, y - 2)$

- (a) $x^2 + y^2 - 10x + 4y + 20 = 0$ (b) $x^2 + y^2 - 16x + 10y + 20 = 0$
(c) $(x - 8)^2 + (y + 5)^2 = 25$ (d) $(x - 6)^2 + (y + 3)^2 = 20$

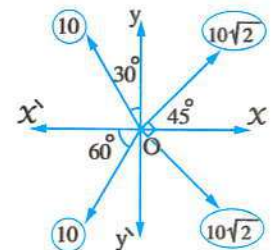
(12) Two forces F , F act at a particle and the magnitude of their resultant is F , then the measure of the included angle between the two forces = $\dots\dots\dots$

- (a) 60° (b) 45° (c) 120° (d) 135°

(13) In the opposite figure :

The resultant of the system of forces R = $\dots\dots\dots$ newton.

- (a) 20 (b) $10\sqrt{2}$
(c) 10 (d) zero



(14) Three coplanar forces intersecting at one point and in equilibrium. If 3 N. and 7 N. are magnitudes of two forces of them , then the magnitude of the third force could be equals $\dots\dots\dots$ N.

- (a) 11 (b) 2 (c) 5 (d) 3

- (15) The force \vec{R} is resolved into two forces \vec{F}_1, \vec{F}_2 which make with the force \vec{R} two angles of measure θ_1, θ_2 from two sides respectively, then the magnitude of $\vec{F}_1 = \dots\dots\dots$

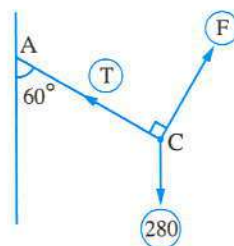
(a) $\frac{R \sin \theta_1}{\sin (\theta_1 + \theta_2)}$ (b) $\frac{R \sin \theta_2}{\sin (\theta_1 - \theta_2)}$ (c) $\frac{R \sin (\theta_1 + \theta_2)}{\sin \theta_2}$ (d) $\frac{R \sin \theta_2}{\sin (\theta_1 + \theta_2)}$

- (16) Two perpendicular forces of magnitudes 6 N. , 8 N. , then the sine of angle between the resultant and first force = $\dots\dots\dots$

(a) $\frac{3}{5}$ (b) $\frac{4}{5}$ (c) $\frac{3}{4}$ (d) $\frac{4}{3}$

- (17) In the opposite figure :

A lamp of weight 280 gm.wt. is attached to the end of a string. It is in equilibrium under the effect of a force perpendicular to the string when it is inclined to the vertical by an angle of measure 60° , then $\frac{F}{T} = \dots\dots\dots$



(a) 2 (b) $\frac{1}{2}$ (c) $\frac{1}{\sqrt{3}}$ (d) $\sqrt{3}$

- (18) The center of the circle : $x^2 + y^2 - 6x + 8y = 0$ is the point $\dots\dots\dots$

(a) (3 , -4) (b) (4 , -3) (c) (-4 , 3) (d) (-3 , 4)

- (19) The lateral surface area of the right cone whose base radius is 6 cm. and the height of the cone is 8 cm. equals = $\dots\dots\dots$ cm²

(a) 28π (b) 10π (c) 60π (d) 48π

- (20) The number of planes that could be passes through three non-collinear points is $\dots\dots\dots$

(a) 1 (b) 2 (c) 3 (d) 4

Second Essay questions

Answer the following questions :

- 1 A metal sphere of weight 400 kg.wt acts in its centre , placed between two smooth planes , one of them is vertical and the other inclined at angle of measure 60° with vertical , then find the reaction of each plane.

- 2 A regular quadrilateral pyramid , the side length of its base is 18 cm. , if its volume is 1296 cm³. Find the slant height and lateral surface area.

10**Damietta Governorate****Maths Inspection****First****Multiple choice questions**Interactive
test 10**Choose the correct answer from the given ones :**

- (1) The resultant of two forces 6 newton and 8 newton could be newton.
 (a) 20 (b) 15 (c) 12 (d) 1
- (2) Two forces of equal magnitudes , enclosing between them an angle of measure $\frac{\pi}{2}$ if the magnitude of their resultant 8 newton , then the value of each force measured in newton is
 (a) $2\sqrt{2}$ (b) 4 (c) $4\sqrt{2}$ (d) 8
- (3) All different vertical straight lines in the space are
 (a) parallel. (b) skew.
 (c) contained in the same plane. (d) intersecting.
- (4) Two forces of magnitudes 3 , F newton and the measure of the angle between them is 120° . If their resultant is perpendicular to the first force , so the value of F in newton is
 (a) 1.5 (b) 3 (c) $3\sqrt{3}$ (d) 6
- (5) The magnitude of two perpendicular forces are $(2F - 5)$ and $(F + 2)$ newton and the magnitude of their resultant is $3\sqrt{5}$ newton , then $F =$ newton.
 (a) 7 (b) 4 (c) 6 (d) 3
- (6) A regular quadrilateral pyramid whose volume is 480 cm^3 , and its base length is 12 cm. , then the length of its height = cm.
 (a) 10 (b) 20 (c) 30 (d) 15
- (7) Two forces of magnitudes 3 F and F newton and their resultant is 4 F newton , then the measure of the angle between them =°
 (a) 60 (b) 0 (c) 180 (d) 90
- (8) Two forces of magnitudes 4 and 6 newton the measure of the angle between them is 90° , then the tangent of the angle between the resultant and the first force equal
 (a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) $2\sqrt{13}$ (d) $\frac{\sqrt{6}}{2}$

- (9) If a body of weight (W) is placed on an inclined smooth plane makes an angle of measure (θ) with the vertical, then its weight component in direction of the plane is

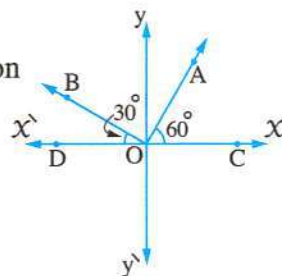
(a) $W \sin \theta$ (b) $W \cos \theta$ (c) W (d) $W \tan \theta$

- (10) The height of a right circular cone is 6 cm. and the circumference of its base is 16π cm. , then its lateral area = cm^2

(a) 144π (b) 64π (c) 60π (d) 80π

- (11) In the opposite figure :

The magnitude of four coplanar forces are 1, 2, $4\sqrt{3}$, $3\sqrt{3}$ newton act at point O in the direction of \vec{OX} , \vec{OA} , \vec{OB} and \vec{Oy} , $m\angle(AOC) = 60^\circ$, $m\angle(BOD) = 30^\circ$, then magnitude and the direction of the resultant of the forces is



(a) (4, 180°) (b) (4, 0°) (c) (3, 0°) (d) (5, 90°)

- (12) If a body is kept in equilibrium under action of several forces, then the least number of forces could cause equilibrium equals

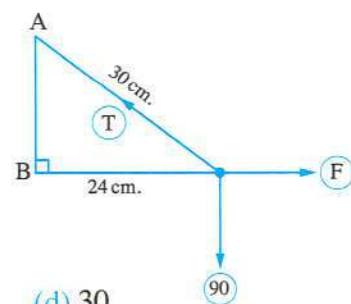
(a) 1 (b) 2 (c) 3 (d) 4

- (13) If the equation : $2x^2 + ay^2 + bxy - 5 = 0$ represents a circle, then its area = square unit.

(a) 5π (b) $\sqrt{5}\pi$ (c) $\frac{5}{2}\pi$ (d) $5\sqrt{2}\pi$

- (14) In the opposite figure :

A body of weight 90 gm.wt. is attached to the end of a string of 30 cm. long. The body is pulled by horizontal force. It comes to equilibrium when it is 24 cm. apart from the wall \overline{AB} , then $T - F =$ gm.wt.



(a) 150 (b) 120 (c) 50 (d) 30

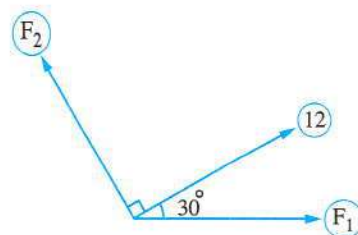
- (15) Two forces of magnitudes 5, 3 newton and the measure of the angle enclosed between them is 60° , then the magnitude of their resultant R equals

(a) 2 (b) 5 (c) 7 (d) 8

- (16) If the circle whose equation : $x^2 + y^2 - 6x + 8y + c = 0$ touches y-axis, then $c =$

(a) -9 (b) 9 (c) 16 (d) -16

- (17) The force of magnitude 12 newton is resolved into two components \vec{F}_1, \vec{F}_2 make angles of measures $30^\circ, 90^\circ$ with it, then $F_2 = \dots\dots\dots$ newton.



- (a) 10
(b) $10\sqrt{3}$
(c) $6\sqrt{3}$
(d) $4\sqrt{3}$

- (18) The radius length of the base of a right circular cone = 5 cm, and its total surface area = $90\pi \text{ cm}^2$, then its volume = $\dots\dots\dots \text{ cm}^3$

- (a) 105π (b) 95π (c) 100π (d) 120π

- (19) If $\vec{F}_1 = (2, -2)$, $\vec{F}_2 = (4, -8)$ and their resultant $\vec{R} = (2a, -3b)$, then $a + b = \dots\dots\dots$

- (a) 3 (b) $\frac{10}{3}$ (c) $6\frac{1}{3}$ (d) 12

- (20) The general form of the equation of a circle its centre is $(5, -4)$ and touches X-axis is $\dots\dots\dots$

- (a) $x^2 + y^2 - 10x + 8y + 25 = 0$ (b) $x^2 + y^2 - 5x + 4y = 0$
(c) $x^2 + y^2 - 10x + 8y = 25$ (d) $x^2 + y^2 + 10x - 8y + 25 = 0$

Second Essay questions

Answer the following questions :

- 1 A regular quadrilateral pyramid the length of its base is 20 cm, and its height is $10\sqrt{3}$ cm.
Find : Its lateral surface area
- 2 A body of weight 20 kg.wt. is placed on a smooth plane inclined to the horizontal with an angle of measure θ where $\cos \theta = \frac{4}{5}$ the body of kept in equilibrium by a horizontal force of magnitude F. **Find :** F and the reaction of the plane.

11

El-Beheira Governorate



Maths Inspection

First Multiple choice questions

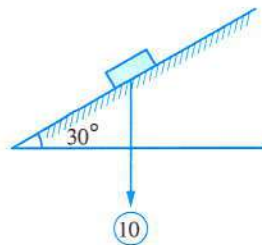
Choose the correct answer from the given ones :

- (1) If A, B and C are three points determine a plane, then $\dots\dots\dots$
- (a) $AB = BC = AC$ (b) $AB + BC = AC$
(c) $AB + BC > AC$ (d) $AB + BC < AC$

- (2) A triangular regular faces pyramid , its edge length 10 cm.
 , then its total area = cm^2
 (a) 40 (b) 100 (c) $100\sqrt{3}$ (d) $25\sqrt{3}$
- (3) A regular quadrilateral pyramid , the area of its base = 100 cm^2 , and its height 12 cm.
 , then its lateral area equal cm^2
 (a) 260 (b) 520 (c) 130 (d) 360
- (4) A regular quadrilateral pyramid whose total area = 70 cm^2 , and its lateral area = 45 cm^2
 , then its height = cm.
 (a) 2.5 (b) 5 (c) $\sqrt{14}$ (d) 4.5
- (5) The volume of a right circular cone is $9\pi \text{ cm}^3$, and the length of its base radius equal
 the length of its height , then its base area = cm^2
 (a) 9π (b) 3π (c) 27π (d) 12π
- (6) The diameter length of the circle : $4x^2 + 4y^2 + 16x - 8y - 16 = 0$, is length unit.
 (a) 3 (b) 6 (c) 12 (d) 24
- (7) The point (2 , 3) lies the circle $x^2 + y^2 = 9$
 (a) on (b) inside (c) outside (d) in the center
- (8) The magnitude of two forces F , 2 newton and the measure of their included angle = $\frac{2\pi}{3}$
 , the magnitude of their resultant is F newton , then F = newton.
 (a) 2 (b) 3 (c) 4 (d) $2\sqrt{2}$
- (9) The magnitude of two forces 2 F , 5 F newton , and the measure of their included angle
 is θ and their resultant is 3 F , then $\theta = \dots\dots\dots^\circ$
 (a) zero (b) 60 (c) 90 (d) 180
- (10) A force of magnitude 40 newton acts vertically upwards is resolved into two components
 , one of them is horizontal of magnitude 20 newton , then the magnitude of the
 other = newton.
 (a) 20 (b) $20\sqrt{3}$ (c) $20\sqrt{5}$ (d) $10\sqrt{3}$

(11) In the opposite figure :

If a body of weight 10 newton is placed on a smooth plane inclined
 to the horizontal at an angle of measure 30° , then the components
 of the weight in direction of line of the greatest
 slope downward = newton.



- (a) $5\sqrt{2}$ (b) $5\sqrt{3}$ (c) 5 (d) $10\sqrt{5}$

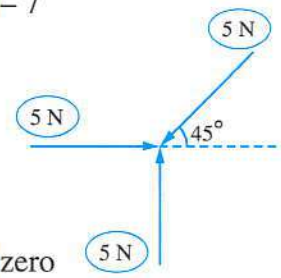
- (12) Three coplanar forces $\vec{F}_1 = 6\hat{i} + 7\hat{j}$, $\vec{F}_2 = a\hat{i} - 9\hat{j}$, $\vec{F}_3 = 5\hat{i} + b\hat{j}$ act at a particle and they are in equilibrium, then $a + 2b = \dots\dots\dots$

(a) -9 (b) 5 (c) 7 (d) -7

- (13) In the opposite figure :

Some forces meeting at a point
, then the magnitude of the resultant of these
forces = $\dots\dots\dots$ newton.

(a) $15\sqrt{2}$ (b) 5 (c) $5\sqrt{2} - 5$ (d) zero



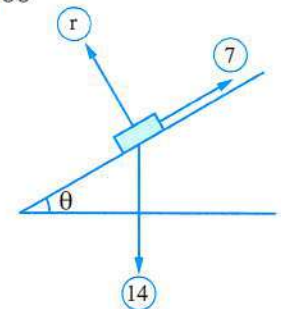
- (14) Three coplanar forces of magnitudes 60, 88 and 60 gm.wt., act at a point, the first is toward north, the second is in the direction 30° south of west and the third in the direction 30° south of east, then the magnitude of the resultant of these forces is $\dots\dots\dots$ gm.wt.

(a) 28 (b) 24 (c) 30 (d) 60

- (15) In the opposite figure :

If the body is in equilibrium when it is
placed on an inclined smooth plane
, then $m(\angle \theta) = \dots\dots\dots^\circ$

(a) 60 (b) 30
(c) 45 (d) 75



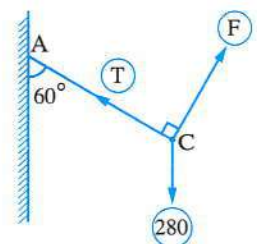
- (16) Three coplanar forces not on the same straight line meeting at a point are in equilibrium, the magnitude of two forces of them are 7 and 3 newton, then the magnitude of the third could be $\dots\dots\dots$ newton.

(a) 10 (b) 4 (c) 5 (d) 3

- (17) In the opposite figure :

A lamp of weight 280 gm.wt. is attached to the end
of a string. It is in equilibrium under the effect of a
force perpendicular to the string when it is inclined to
the vertical by an angle of measure 60° , then $\frac{F}{T} = \dots\dots\dots$

(a) 2 (b) $\frac{1}{2}$ (c) $\frac{1}{\sqrt{3}}$ (d) $\sqrt{3}$



- (18) A uniform rod of weight 20 newton which is movable around a hinge at one of its ends is pulled a side by a horizontal force of magnitude 10 newton acting on the other end, then the measure of the angle of inclination of the rod to the vertical when it is in equilibrium = $\dots\dots\dots^\circ$

(a) 60 (b) 45 (c) 30 (d) 90

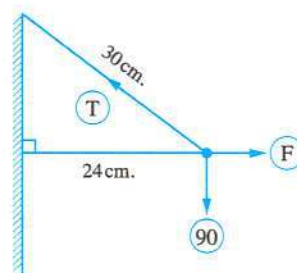
- (19) A metallic sphere of weight 15 gm.wt. is put such that it touches two smooth planes , one of them is vertical and the other inclines to the vertical by an angle of measure 30° , then the reaction on the vertical plane = newton.

(a) $15\sqrt{3}$ (b) 30 (c) 15 (d) $30\sqrt{3}$

- (20) In the opposite figure :

$T - F = \dots\dots\dots$ gm.wt.

(a) 150 (b) 30
(c) 50 (d) 120



Second Essay questions

Answer the following questions :

- 1 A smooth sphere of weight 20 newton is on a smooth vertical wall and suspended by a light string from a point on its surface. The other end of the string is attached to a point on the wall above the point of contact between the wall and the sphere. If the length of the string equal the diameter of the sphere. Find the pressure on the wall and the tension in the string in case of equilibrium.
- 2 ABC is an equilateral triangle , its side length 6 cm. , if the triangle is rotated a complete rotation around BC. Find the volume of the solid which formed from the rotation in terms of π

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Beni-Suef Governorate



Maths Inspection

First Multiple choice questions

Choose the correct answer from the given ones :

- (1) If the magnitude of the resultant of two forces act at a point is maximum value , then the measure of the angle between their line of actions equals
- (a) 0° (b) 60° (c) 120° (d) 180°
- (2) Two forces act at a point the magnitude of the two forces are 6 , 3 newton and their resultant is perpendicular to one of them , then the magnitude of their resultant = newton.
- (a) 3 (b) $3\sqrt{3}$ (c) 6 (d) $6\sqrt{3}$

- (3) Two forces of magnitudes 8 and F gm.wt. the measure of the angle between them is $\theta \in]0, \pi[$, their resultant bisects the included angle between them, then $F = \dots\dots\dots$ gm.wt.

(a) 4 (b) $2\sqrt{2}$ (c) 8 (d) 16

- (4) Two forces of magnitudes 4 and 6 newton act at a point, the measure of the angle between them is 90° , then the tangent of the angle between the resultant and the first force equals $\dots\dots\dots$

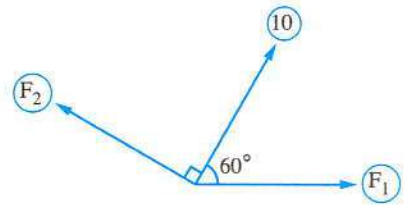
(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) $2\sqrt{13}$ (d) $\frac{\sqrt{6}}{2}$

- (5) The magnitude of a force is 6 newton and acts towards the North. It is resolved into two perpendicular components, then its component in direction of Eastern North is of magnitude $\dots\dots\dots$ newton.

(a) zero (b) 6 (c) $3\sqrt{2}$ (d) $2\sqrt{3}$

- (6) In the opposite figure :

If the force of magnitude 10 newton is resolved into two components \vec{F}_1 and \vec{F}_2 inclined to the force by two angles of measures 60° and 90° respectively, then $F_2 = \dots\dots\dots$ newton.



(a) $5\sqrt{3}$ (b) 10 (c) $10\sqrt{3}$ (d) 20

- (7) If a body of weight 10 newton is placed on a smooth plane inclined to the horizontal at an angle of measure 30° , then the component of the weight in direction of line of the greatest slope downward = $\dots\dots\dots$ newton.

(a) $5\sqrt{2}$ (b) 5 (c) $5\sqrt{3}$ (d) $10\sqrt{3}$

- (8) If $\vec{F}_1 = \vec{i} - \vec{j}$, $\vec{F}_2 = 2\vec{i} - 4\vec{j}$, their resultant $\vec{R} = 2a\vec{i} - 3b\vec{j}$, then $a + b = \dots\dots\dots$

(a) 3 (b) $3\frac{1}{3}$ (c) $3\frac{1}{6}$ (d) 12

- (9) If $\vec{F}_1 = 5\vec{i}$, $\vec{F}_2 = 7\vec{i} - 5\vec{j}$, \vec{R} is their resultant, then $\|\vec{R}\| = \dots\dots\dots$ force unit.

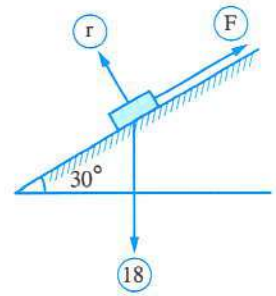
(a) 13 (b) $\sqrt{5} + \sqrt{74}$ (c) 49 (d) $\sqrt{12} - \sqrt{5}$

- (10) If \vec{F} is in equilibrium with two forces of magnitudes 5 and 3 newton and the measure of the angle between them is 60° , then $F = \dots\dots\dots$ newton.

(a) $\sqrt{19}$ (b) $\sqrt{34}$ (c) 7 (d) 15

(11) In the opposite figure :

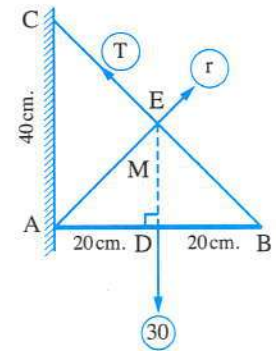
A body of weight 18 newton is placed on a smooth plane inclined to the horizontal by an angle of measure 30° , it is kept in equilibrium by a force of magnitude F newton in the direction of the plane upward, then $F + r = \dots\dots\dots$ newton.



- (a) $6\sqrt{3}$ (b) $9\sqrt{3}$
(c) $18\sqrt{3}$ (d) $9 + 9\sqrt{3}$

(12) In the opposite figure :

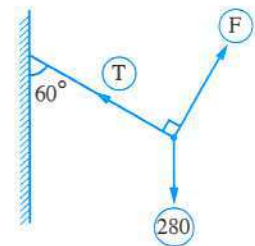
\overline{AB} is a uniform rod with length 40 cm. and weight 30 newton is connected to a hinge at A if the rod kept in equilibrium horizontally by a light string connected to the rod at B and C where C is located on the wall just above A, $AC = 40$ cm., then the reaction of the hinge $r = \dots\dots\dots$ newton.



- (a) 20 (b) $15\sqrt{2}$ (c) 30 (d) $40\sqrt{2}$

(13) In the opposite figure :

A lamp of weight 280 gm.wt. is attached to the end of a string. It is in equilibrium under the effect of a force perpendicular to the string when it is inclined to the vertical by an angle of measure 60° , then $\frac{F}{T} = \dots\dots\dots$

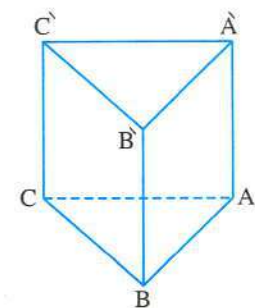


- (a) $\frac{1}{\sqrt{3}}$ (b) $\frac{1}{2}$ (c) $\sqrt{3}$ (d) 2

(14) In the opposite figure :

The plane $ABC \cap$ the plane $\hat{A}\hat{B}\hat{C} = \dots\dots\dots$

- (a) \overleftrightarrow{BB}
(b) \emptyset
(c) \overleftrightarrow{AB}
(d) \overleftrightarrow{AA}



(15) Number of planes that are passing through three non-collinear points is

- (a) 1 (b) 2 (c) 3 (d) an infinite number

- (16) A regular quadrilateral pyramid whose volume is 480 cm^3 and its base length is 12 cm. , then the length of its height = cm.
 (a) 10 (b) 15 (c) 20 (d) 30
- (17) A triangular regular faces pyramid , its edge length 10 cm. , then its total area equal cm^2 .
 (a) 40 (b) 100 (c) $100\sqrt{3}$ (d) $25\sqrt{3}$
- (18) The center of the circle whose equation : $x^2 + y^2 - 6x + 8y = 0$ is the point
 (a) (3 , - 4) (b) (4 , - 3) (c) (- 3 , 4) (d) (- 4 , 3)
- (19) Which of the following points does lie on the circle whose equation : $(x - 2)^2 + y^2 = 13$?
 (a) (2 , 3) (b) (3 , - 2) (c) (2 , 5) (d) (4 , 3)
- (20) The equation of the circle whose center (4 , 3) and touches x -axis is
 (a) $(x - 3)^2 + (y - 4)^2 = 16$ (b) $(x - 4)^2 + (y - 3)^2 = 9$
 (c) $(x + 3)^2 + (y + 4)^2 = 9$ (d) $(x + 3)^2 + (y - 4)^2 = 16$

Second Essay questions

Answer the following questions :

- 1 ABCDEF is a regular hexagon , the forces of magnitudes $6, 2\sqrt{3}, 6, 2\sqrt{3}$ newton act on $\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD}$ and \overrightarrow{AE} respectively. Find the magnitude of the resultant of these forces.
- 2 Find to the nearest tenth , the total area of the right circular cone in which the diameter length of its base is 10 cm. and its height is 12 cm.

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El-Menia Governorate



Maths Inspection

First Multiple choice questions

Choose the correct answer from the given ones :

- (1) The case that doesn't determine a plane is
 (a) two intersecting straight lines. (b) two different parallel straight lines.
 (c) three points not collinear. (d) straight line and point on it.
- (2) Two forces of magnitudes 8 , F newton , the angle between them $\theta \in]0, \pi[$ their resultant bisects the angle between them , then F = newton.
 (a) 4 (b) 8 (c) 16 (d) $2\sqrt{3}$

(3) If the circle whose equation : $x^2 + y^2 - 6x + 8y + c = 0$ touches x -axis , then $c = \dots\dots\dots$

- (a) 6 (b) -6 (c) 9 (d) -9

(4) If θ is the measure of the angle between two forces of magnitudes 2 N , 6 N and R is the resultant between them by newton where $4 \leq R < 8$, then angle between them $\in \dots\dots\dots$

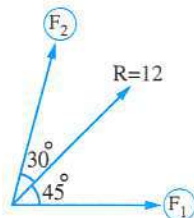
- (a) $[0, \pi[$ (b) $]0, \pi]$ (c) $[\frac{\pi}{2}, \pi]$ (d) $]0, \pi[$

(5) In the opposite figure :

$R = 12$ newton

, then $F_1 = \dots\dots\dots$ newton.

- (a) $12 \cos 45^\circ$ (b) $12 \sin 45^\circ$
(c) $6 \csc 45^\circ$ (d) $6 \csc 75^\circ$



(6) Two forces of magnitudes 12 N , 15 N acting at a point and angle between them θ° where $\cos \theta^\circ = \frac{-4}{5}$, then the angle between resultant and first force = $\dots\dots\dots$

- (a) zero (b) 30° (c) 60° (d) 90°

(7) If $\vec{F}_1, \vec{F}_2, \vec{F}_3$ are three forces intersect at a point and equilibrium where $\vec{F}_1 = (2, -5)$, $\vec{F}_2 = (-3, 2)$, then $\vec{F}_3 = \dots\dots\dots$

- (a) $(-1, -3)$ (b) $(1, 3)$ (c) $(-6, -10)$ (d) $(6, 10)$

(8) If $\vec{F}_1 = 3\hat{i} - 2\hat{j}$, $\vec{F}_2 = a\hat{i} - \hat{j}$, $\vec{F}_3 = 4\hat{i} - b\hat{j}$, and the resultant $\vec{R} = 6\hat{i} - 4\hat{j}$, then $(a, b) = \dots\dots\dots$

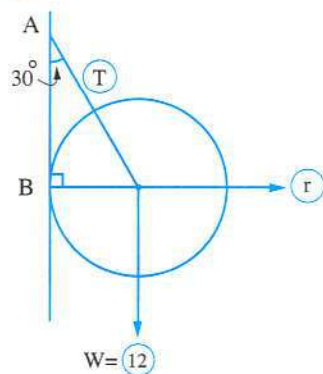
- (a) $(1, -1)$ (b) $(-1, 1)$ (c) $(-1, -1)$ (d) $(1, 1)$

(9) In the opposite figure :

If the sphere is in equilibrium

, then $(T, r) = \dots\dots\dots$ newton.

- (a) $(4, 8)$ (b) $(12, 8)$
(c) $(4\sqrt{3}, 8\sqrt{3})$ (d) $(8\sqrt{3}, 4\sqrt{3})$



(10) The volume of triangular regular faces pyramid its edge length 6 cm. = $\dots\dots\dots$ cm^3

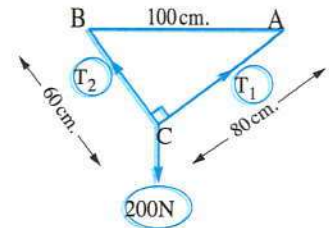
- (a) $18\sqrt{2}$ (b) $54\sqrt{2}$ (c) $27\sqrt{3}$ (d) $36\sqrt{3}$

(11) A right circular cone the length of its drawer 25 cm. and its lateral area 550 cm^2 , then its volume = $\dots\dots\dots$ cm^3 ($\pi = \frac{22}{7}$)

- (a) 1223 (b) 1232 (c) 1322 (d) 3122

(12) In the opposite figure :

A body its weight 200 N is hanged by two strings
 , then the magnitude of the tension
 in the two strings = N



- (a) 120 , 160 (b) 180 , 12 (c) 150 , 160 (d) 100 , 130

**(13) If the length of the radius of right circular cone 3 cm. and its height 4 cm.
 , then its total area = cm^2**

- (a) 9π (b) 10π (c) 21π (d) 24π

(14) Three coplanar forces of magnitude 5 , 6 , 7 newton act at a particle if the forces are in equilibrium , then the cosine of the angle between the second and the third force =

- (a) $\frac{7}{5}$ (b) $-\frac{5}{7}$ (c) $\frac{15}{17}$ (d) $\frac{1}{2}$

(15) The point that lies on the circle : $(x + 2)^2 + y^2 = 13$ from the following is

- (a) $(-2, 0)$ (b) $(0, -2)$ (c) $(1, 2)$ (d) $(-1, -2)$

(16) Any four points don't lie in one plane determine

- (a) one plane. (b) two planes. (c) three planes. (d) four planes.

(17) Three coplanar forces not on the same straight line meeting at a point are in Equilibrium the magnitudes of them are 4 , 8 , F , then F could be N

- (a) 3 (b) 4 (c) 8 (d) 13

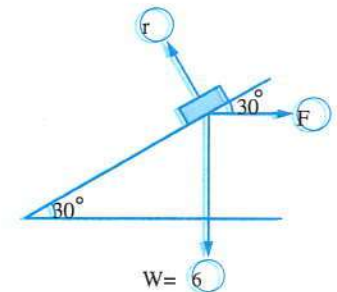
**(18) A body of weight 6 newton is placed on smooth plane inclined to the horizontal at an angle 30° it kept in equilibrium by horizontal force of magnitude F , then
 F = newton.**

- (a) $2\sqrt{3}$ (b) $3\sqrt{3}$ (c) $4\sqrt{3}$ (d) $6\sqrt{3}$

(19) In the opposite figure :

$r = \dots\dots\dots$

- (a) $2\sqrt{3}$ (b) $3\sqrt{3}$
 (c) $4\sqrt{3}$ (d) $6\sqrt{3}$



(20) In the previous figure :

The component of the weight in the direction
 of the greatest slope to the bottom = N

- (a) 3 (b) $3\sqrt{3}$ (c) $4\sqrt{3}$ (d) $6\sqrt{3}$

Second

Essay questions

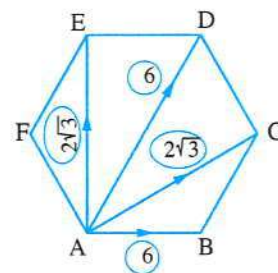
Answer the following questions :

- 1 Regular quadrilateral pyramid , the length of its base side is 10 cm. , and area of one of its lateral faces is 60 cm^2 **Find :** Its total area.

- 2 **In the opposite figure :**

ABCDEF is a regular hexagon , forces of magnitudes 6 , $2\sqrt{3}$, 6 , $2\sqrt{3}$ newton act along \overrightarrow{AB} , \overrightarrow{AC} , \overrightarrow{AD} and \overrightarrow{AE} respectively

Find their resultant.



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Assiut Governorate



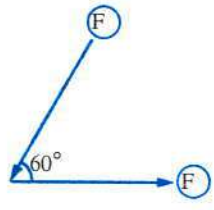
Maths Inspection

First

Multiple choice questions

Choose the correct answer from the given ones :

- (1) Two forces of equal magnitudes enclosing between them an angle of measure $\frac{\pi}{2}$ if the magnitude of their resultant is 8 newton , then the value of each force is newton.
- (a) $2\sqrt{2}$ (b) 4 (c) $4\sqrt{2}$ (d) 8
- (2) A force of magnitude $10\sqrt{2}$ gm.wt. acts in the Eastern South direction , is resolved into two perpendicular components , then the magnitude of the component in the south direction = gm.wt.
- (a) 5 (b) 10 (c) $10\sqrt{2}$ (d) $5\sqrt{2}$
- (3) If $\vec{F}_1 = 4\hat{i}$, $\vec{F}_2 = 8\hat{i} - \hat{j}$, $\vec{F}_3 = 4\hat{i} - 5\hat{j}$, then $\|\vec{R}\| = \dots\dots\dots$ force unit.
- (a) 12 (b) 5 (c) 13 (d) $2\sqrt{73}$
- (4) Two forces act at a point the magnitude of the two forces $8\sqrt{3}$, 8 newton and the measure of the included angle between them 150° , then the magnitude of their resultant = newton.
- (a) 64 (b) 32 (c) 16 (d) 8
- (5) Two forces of magnitudes F , 2 F newton act at a point if their resultant is perpendicular to one of them , then R =
- (a) $\sqrt{5} F$ (b) $\sqrt{3} F$ (c) 3 F (d) F

- (6) The magnitude of a force is 8 newton and acts in East direction. It is resolved into two components, the angle between the two components is 120° , then its component in south direction = newton.
 (a) 16 (b) 8 (c) $8\sqrt{3}$ (d) $\frac{8\sqrt{3}}{3}$
- (7) The resultant of two forces of magnitudes 6 newton and 8 newton could be newton.
 (a) 20 (b) 15 (c) 12 (d) 1
- (8) The magnitude of the resultant of the two forces shown in the opposite figure is
 (a) $\frac{1}{2} F$ (b) F
 (c) $\sqrt{3} F$ (d) $\sqrt{5} F$
- 
- (9) The magnitude of the resultant of two forces act at a point is maximum value, then the measure of the angle between the two forces equal
 (a) 180° (b) 120° (c) zero (d) 60°
- (10) Three equal forces in magnitude meeting at a point and they are in equilibrium, then the measure of the angle between each two forces is
 (a) 60° (b) 90° (c) 120° (d) 150°
- (11) The least number of coplanar unequal in magnitude forces could be in equilibrium is
 (a) 1 (b) 2 (c) 3 (d) 4
- (12) The weight of a body is 20 kg.wt. it is placed on a smooth inclined plane makes an angle of measure θ to the horizontal, where $\sin \theta = \frac{3}{5}$ and it prevent from sliding by a horizontal force F , then $F =$ kg.wt.
 (a) 30 (b) 15 (c) 10 (d) $5\sqrt{3}$
- (13) Number of planes that are passing through three non-collinear points is
 (a) 1 (b) 2 (c) 3 (d) an infinite number.
- (14) A regular quadrilateral pyramid whose volume is 480 cm^3 and its base length is 12 cm, then the length of its height = cm.
 (a) 10 (b) 20 (c) 30 (d) 15
- (15) The right circular cone is generated by folding a paper in the shape of
 (a) an equilateral triangle. (b) a circular segment.
 (c) a right-angled triangle. (d) a circular sector.

- (16) The radius length of the base of a right circular cone where its total area $616 \pi \text{ cm}^2$ and the length of its drawer is 30 cm. is cm.
 (a) 44 (b) 14 (c) 30 (d) 34
- (17) The radius length of the circle whose equation : $x^2 + y^2 - 4x + 2y - 4 = 0$ is length unit.
 (a) 2 (b) 3 (c) 4 (d) 9
- (18) The circumference of the circle whose equation : $(x - 3)^2 + (y + 2)^2 = 25$ equal length unit.
 (a) 2π (b) 3π (c) 10π (d) 25π
- (19) The measure of the smallest rotation angle of an isosceles triangle around its axis of symmetry to form a right circular cone is
 (a) 90° (b) 180° (c) 270° (d) 60°
- (20) The point which lies on the circle : $(x - 2)^2 + y^2 = 13$ is
 (a) (2 , 3) (b) (3 , -2) (c) (2 , 0) (d) (4 , 3)

Second Essay questions

Answer the following questions :

- 1 Two forces of magnitude 2 and F newton , the angle between them is of measure 120° find F if the resultant is perpendicular to the second force.
- 2 Four coplanar forces act on a particle the first of magnitude 4 newton act in the East direction , the second of magnitude 2 newton acts in direction 60° North of the East , the third of magnitude 5 newton acts in the direction 60° North of the West and the fourth of magnitude $3\sqrt{3}$ newton acts in direction 60° West of the South find the magnitude of the resultant and its direction.

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Qena Governorate



Maths Inspection

First

Multiple choice questions

Choose the correct answer from the given ones :

- (1) The circle which equation : $x^2 + y^2 = 25$ its center
 (a) (0 , 0) (b) (5 , 5) (c) (0 , 1) (d) (1 , 0)

- (2) A regular quadrilateral pyramid its height 4 cm. , and its slant height 5 cm. , then length side of its base cm.
 (a) 5 (b) 3 (c) 4 (d) 6
- (3) Two forces F , 16 newton act on a particle if their resultant 26 newton and angle between their directions 120° , then $F =$ newton.
 (a) 30 (b) 41 (c) 16 (d) 26
- (4) If $\vec{F}_1, \vec{F}_2, \vec{F}_3$ are three forces meeting at a point they are in equilibrium , then magnitude of resultant of the two forces \vec{F}_1, \vec{F}_2 is the magnitude of
 (a) \vec{F}_1 (b) $\vec{F}_1 + \vec{F}_2$ (c) 0 (d) \vec{F}_3
- (5) Two straight lines L_1, L_2 are parallel if
 (a) $L_1 \cap L_2 = \emptyset$
 (b) L_1, L_2 lie in the same plane.
 (c) $L_1 \cap L_2 = \emptyset$ and L_1, L_2 lie in the same plane.
 (d) $L_1 \cap L_2 = \emptyset$ and L_1, L_2 don't lie in the same plane.
- (6) Three forces of magnitudes 60 , 120 , K newton meeting at a point they are in equilibrium if measure of the angle between first and second forces 120° and between second and third 150° , then $K =$ newton.
 (a) 120 (b) $60\sqrt{3}$ (c) 150 (d) 60
- (7) Right circular cone , radius length of its base 9 cm. , its height 14 cm. , then volume = cm^3 ($\pi = \frac{22}{7}$)
 (a) 3564 (b) 396 (c) 1188 (d) 1782
- (8) Regular quadrilateral pyramid length side of its base 10 cm. , its height 12 cm. , then its volume cm^3
 (a) 300 (b) 400 (c) 600 (d) 120
- (9) Two equal forces , magnitude of each 6 gm.wt. and magnitude of their resultant 6 gm.wt. , then measure of the angle between directions of the two forces is
 (a) 60° (b) 120° (c) 30° (d) 45°
- (10) The weight of a body is 10 newton it is placed on smooth inclined plane make an angle 30° to the horizontal , then the component of the weight in perpendicular direction to the plane = newton
 (a) 5 (b) 10 (c) $5\sqrt{3}$ (d) 2

(11) Ratio between edge length of triangular pyramid of regular faces : its height =

- (a) $\sqrt{2} : \sqrt{3}$ (b) $\sqrt{3} : \sqrt{2}$ (c) $\sqrt{3} : 2$ (d) $\sqrt{3} : 3$

(12) Force of magnitude 6 newton act in direction east it is resolved into to perpendicular components so its component in direction of north newton.

- (a) 0 (b) $3\sqrt{2}$ (c) 6 (d) 3

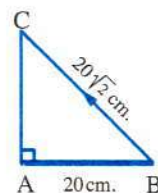
(13) The minimum value of the resultant of two forces 10 , 7 newton meeting at point = newton.

- (a) 17 (b) 10 (c) 7 (d) 3

(14) A body of weight 60 newton is placed on smooth plane inclined with the horizontal at angle of measure 30° and tied up by string in direction of line of greatest slope of the plane upward , then value tension of string =

- (a) 30 (b) $30\sqrt{3}$ (c) 60 (d) $60\sqrt{3}$

(15) \overline{AB} is uniform rod with length 20 cm. and weight 30 newton connected to a hinge on the vertical wall at A if the rod kept in equilibrium horizontally by light string connected to rod at B of length $20\sqrt{2}$ cm. , fixed at point C on the wall just above A , then the reaction of the hinge



- (a) In direction of \overrightarrow{AB} (b) bisect \overline{BC}
(c) Its magnitude 15 newton. (d) Its line of action far from wall by 10 cm.

(16) If \vec{F} is in equilibrium with two perpendicular forces of magnitudes 3 , 4 newton , then F = newton.

- (a) 4 (b) 5 (c) 6 (d) 25

(17) If $\vec{F}_1 = 4\hat{i} + 3\hat{j}$, $\vec{F}_2 = -\hat{i} + 5\hat{j}$, $\vec{F}_3 = 2\hat{i} - 20\hat{j}$ are three forces , then magnitude of resultant = unit force.

- (a) 13 (b) 0 (c) 17 (d) 7

(18) Radius length of the base of right circular cone 15 cm. , and length of its drawer 25 cm. , then lateral surface area = cm^2

- (a) 375π (b) 15π (c) 25π (d) 187.5π

(19) The forces of magnitudes $7\sqrt{3}$, 7 , K newton act on particle in the directions East , North , 30° South of West respectively if magnitude of the resultant 8 newton in direction 30° North of East , then K = newton.

- (a) $7\sqrt{3}$ (b) 6 (c) 7 (d) 12

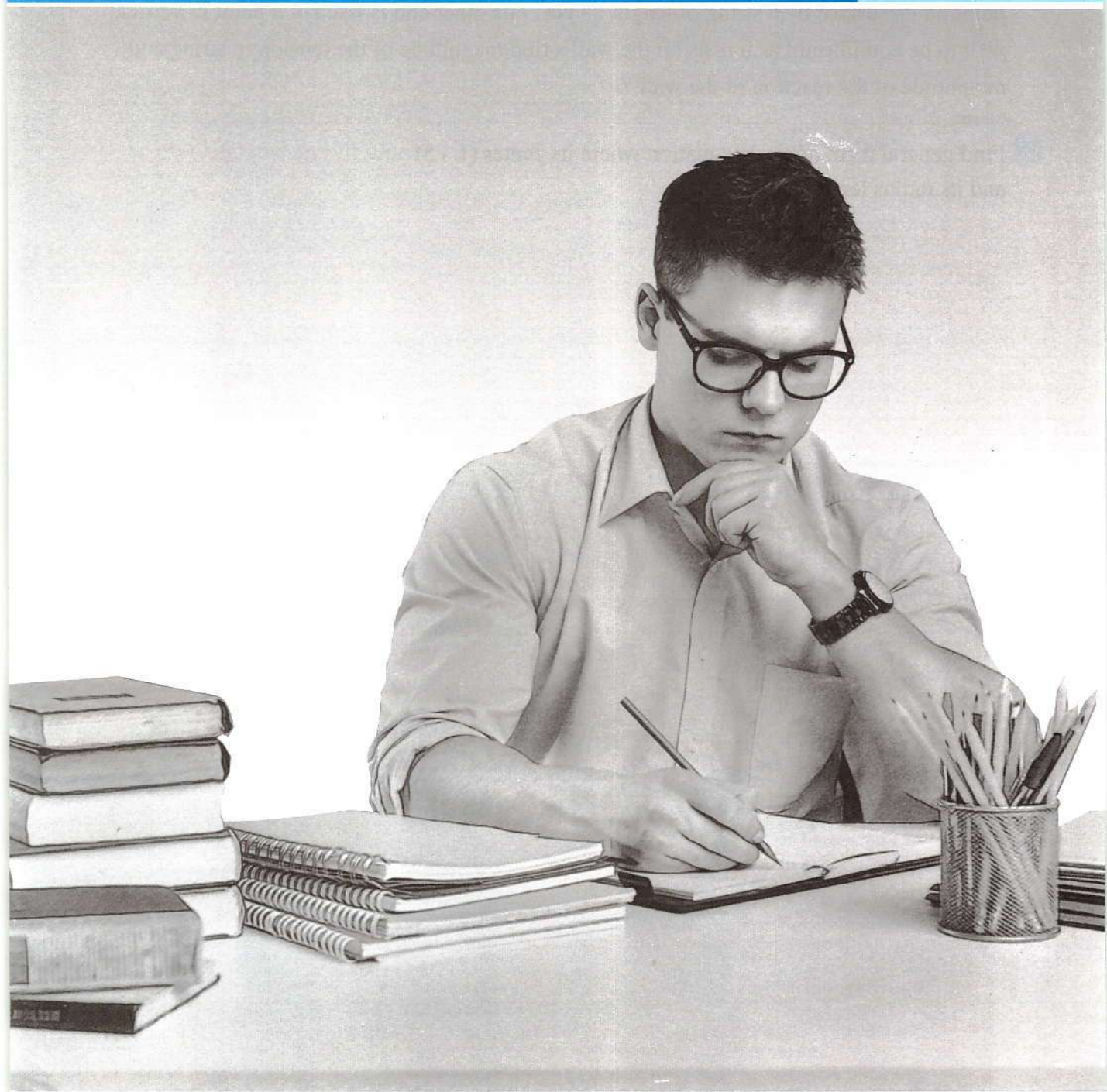
- (20) A weight of 100 gm.wt. is suspended by two string of length 30 cm. , 40 cm. , the two other ends are fixed at two points on horizontal line such that the string parts are perpendicular to each other then magnitude of the tension in first string gm.wt.
- (a) 80 (b) 100 (c) $60\sqrt{3}$ (d) 60

Second Essay questions

Answer the following questions :

- 1 A metallic sphere of weight 1.5 kg.wt. and of radius length 25 cm. , is suspended at a point on its surface by a string of length 25 cm. , its other end is fixed at a point in vertical wall to be equilibrium as it rests on the wall , find magnitude of the tension in string and magnitude of the reaction of the wall ?
- 2 Find general form of circle equation where its center (1 , 5) and its radius length 6 unit length.

Answers



Answers of accumulative quizzes on Statics

Accumulative quiz 1

1 (1) (d) (2) (b) (3) (a) (4) (c)

2

$F = 8$ newton, the measure of the inclination angle of the resultant on $\vec{F} = 30^\circ$

3

$F = 8$ newton

Accumulative quiz 2

1 (1) (a) (2) (c) (3) (c) (4) (d)

2

$F = 4\sqrt{2}$ newton, $R = 4$ newton.

3

$50, 50\sqrt{3}$ newton.

Accumulative quiz 3

1 (1) (b) (2) (c) (3) (c) (4) (c)

2

$R \approx 15.16$ kg.wt., $\theta \approx 99^\circ 30'$

3

$22, 10$ kg.wt., $R = 2\sqrt{201}$ kg.wt.

Accumulative quiz 4

1 (1) (c) (2) (d) (3) (b) (4) (a)

2

$100\sqrt{3}$ gm.wt., $100\sqrt{3}$ gm.wt.

3

$a = -1, b = 1$

Accumulative quiz 5

1

135° , prove by yourself.

2

$R = 4\sqrt{2}$ newton, in direction of \vec{AC}

3

$T = \frac{20\sqrt{3}}{3}$ gm.wt., $r = \frac{10\sqrt{3}}{3}$ gm.wt.

4

$9\sqrt{7}$ newton, $\sqrt{7}$ newton.

Answers of accumulative quizzes on Geometry and Measurement

Accumulative quiz 1

1 (1) (d) (2) (a) (3) (d) (4) (a) (5) (d)

2

(1) The two planes $ABCD, \vec{A}\vec{B}\vec{C}\vec{D}$
(there are other solutions)

(2) The two planes $ABCD, \vec{A}\vec{B}\vec{B}\vec{A}$
(there are other solutions)

(3) The two straight lines \vec{AB}, \vec{BC}
(there are other solutions)

(4) \vec{AB} , the plane $\vec{A}\vec{B}\vec{C}\vec{D}$
(there are other solutions)

(5) \vec{AB}

Accumulative quiz 2

1 (1) (b) (2) (c) (3) (b) (4) (d)

2

(1) Lateral area = 800 cm^2

(2) The volume = $\frac{4000\sqrt{3}}{3}\text{ cm}^3$

3

Total area = $576\sqrt{3}\text{ cm}^2$

Accumulative quiz 3

1

(1) (a) (2) (a) (3) (d) (4) (d)

2

Lateral height = 15 cm ,

lateral area = 540 cm^2

3

14 cm .

Accumulative quiz 4

1

(1) (a) (2) (c) (3) (a) (4) (d)

2

$x^2 + y^2 + 4x - 10y - 5 = 0$

3

(1) Total area = $96\pi\text{ cm}^2$

(2) Volume = $96\pi\text{ cm}^3$

Answers of October tests

Answers of Test 1

1

- (1) b (2) c (3) b
(4) d (5) a (6) d

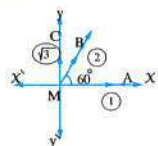
2

- (1) ∴ The two components are perpendicular

$$\therefore F_1 = 18 \cos 60^\circ = 9 \text{ newton}$$

$$\therefore F_2 = 18 \sin 60^\circ = 9\sqrt{3} \text{ newton}$$

(2)



Consider \vec{OX} is the direction of the first force.

$$X = 1 \times \cos 0^\circ + 2 \cos 60^\circ + \sqrt{3} \cos 90^\circ$$

$$= 1 \times 1 + 2 \times \frac{1}{2} + \sqrt{3} \times 0 = 2$$

$$Y = 1 \times \sin 0^\circ + 2 \times \sin 60^\circ + \sqrt{3} \sin 90^\circ$$

$$= 1 \times 0 + 2 \times \frac{\sqrt{3}}{2} + \sqrt{3} \times 1 = 2\sqrt{3}$$

$$\therefore \vec{R} = 2\hat{i} + 2\sqrt{3}\hat{j}, R = \sqrt{(2)^2 + (2\sqrt{3})^2} = 4 \text{ newton}$$

$$\therefore \tan \theta = \frac{2\sqrt{3}}{2} = \sqrt{3}$$

$$\therefore X > 0, Y > 0 \therefore \theta = 60^\circ$$

∴ The magnitude of \vec{R} = 4 newton and its direction is \vec{MB}

Answers of Test 2

1

- (1) b (2) c (3) d
(4) b (5) d (6) d

2

- (1) From the figure

$$\sin \theta = 0.8, \cos \theta = 0.6$$

$$F_1 = \frac{40 \sin (90^\circ - \theta)}{\sin (180^\circ - \theta)}$$

$$= \frac{40 \cos \theta}{\sin \theta} = \frac{40 \times 0.6}{0.8} = 30 \text{ N.}$$

$$\therefore F_2 = \frac{40 \sin 90^\circ}{\sin (180^\circ - \theta)} = \frac{40 \times 1}{0.8} = 50 \text{ N.}$$

- (2) $X = 10 \cos 0^\circ + 20 \cos 120^\circ$

$$+ 30 \cos 240^\circ = -15$$

$$Y = 10 \sin 0^\circ + 20 \sin 120^\circ$$

$$+ 30 \sin 240^\circ = -5\sqrt{3}$$

$$\therefore \vec{R} = -15\hat{i} - 5\sqrt{3}\hat{j}$$

$$\therefore R = \sqrt{225 + 75}$$

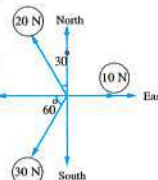
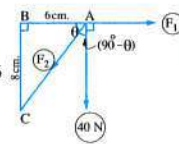
$$= 10\sqrt{3} \text{ N.}$$

$$\tan \theta = \frac{y}{x} = \frac{-5\sqrt{3}}{-15} = \frac{1}{\sqrt{3}}$$

$$\therefore X < 0, Y < 0$$

$$\therefore \theta = 180^\circ + 30^\circ = 210^\circ$$

i.e. In direction 30° South of West.



Answers of November tests

Answers of Test 1

1

- (1) c (2) a (3) a
(4) a (5) d (6) d

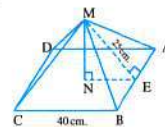
2

- (1) (1) In $\triangle MNE$:

$$MN = \sqrt{(25)^2 - (20)^2} = 15 \text{ cm.}$$

i.e. height = 15 cm.

(2)



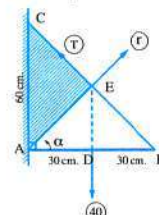
$$\text{The lateral area} = \frac{1}{2} \times \text{base perimeter} \times \text{slant height}$$

$$= \frac{1}{2} \times (4 \times 40) \times 25 = 2000 \text{ cm}^2$$

$$(3) \text{ The total area} = 2000 + (40)^2 = 3600 \text{ cm}^2$$

$$(4) \text{ The volume} = \frac{1}{3} \times (40)^2 \times 15 = 8000 \text{ cm}^3$$

(2)



∴ The set of forces are in equilibrium.

∴ The line of action of \vec{r} passes through the point E

∴ D is the midpoint of \overline{AB} , $\overline{DE} \parallel \overline{AC}$

∴ E is the midpoint of \overline{BC}

$BC = 60\sqrt{2} \text{ cm.}$ (Pythagoras theorem)

$\triangle AEC$ is the triangle of forces where:

$$AE = \frac{1}{2} BC = 30\sqrt{2} \text{ cm.}$$

$$\therefore EC = 30\sqrt{2} \text{ cm.}, AC = 60 \text{ cm.}$$

$$\therefore \frac{r}{30\sqrt{2}} = \frac{T}{30\sqrt{2}} = \frac{40}{60}$$

$$\therefore r = T = 20\sqrt{2} \text{ newton}$$

Answers of Test 2

1

- (1) b (2) b (3) b
(4) a (5) c (6) a

2

- (1) (1) The area of the base = πr^2

$$\therefore 36\pi = \pi r^2$$

$$\therefore r = 6 \text{ cm.}$$

$$\therefore \text{the lateral area} = \pi r L = \pi \times 6 \times 10$$

$$= 60\pi \text{ cm}^2$$

$$(2) \text{ The total area} = \pi r (L + r) = \pi \times 6 (10 + 6)$$

$$= 96\pi \text{ cm}^2$$

$$(3) h = \sqrt{(10)^2 - (6)^2} = 8 \text{ cm.}$$

$$\therefore \text{Volume} = \frac{1}{3} \pi r^2 h = \frac{1}{3} \pi \times 6^2 \times 8$$

$$= 96\pi \text{ cm}^3$$

- (2) ∴ $(60)^2 + (80)^2 = (100)^2$

∴ $\triangle ACB$ is right-angled at C

From lami's rule

$$\therefore \frac{200}{\sin 90^\circ} = \frac{T_1}{\sin \theta_1} = \frac{T_2}{\sin \theta_2}$$

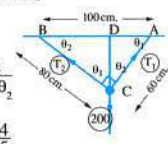
$$\therefore \sin \theta_1 = \frac{BC}{AB} = \frac{80}{100} = \frac{4}{5}$$

$$\therefore \sin \theta_2 = \frac{AC}{AB} = \frac{60}{100} = \frac{3}{5}$$

$$\therefore \frac{200}{1} = \frac{T_1}{\frac{4}{5}} = \frac{T_2}{\frac{3}{5}}$$

$$\therefore T_1 = 200 \times \frac{5}{4} = 250 \text{ gm.wt.}$$

$$\therefore T_2 = 200 \times \frac{5}{3} = 333 \text{ gm.wt.}$$



Answers of school book examination

1

(1) (a) (2) (d) (3) (d) (4) (b)

2

$$\begin{aligned} \text{(a)} \quad 5 + a - 14 &= 10\sqrt{2} \cos 135^\circ = -10 & \therefore a &= -1 \\ 3 + 6 + b &= 10\sqrt{2} \sin 135^\circ = 10 & \therefore b &= 1 \end{aligned}$$

(b) Let the angle between the inclined plane and the horizontal be θ

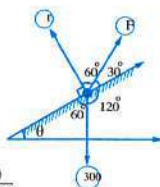
$$\therefore \tan \theta = \frac{1}{\sqrt{3}}$$

$$\therefore \theta = 30^\circ$$

$$\therefore \frac{F}{\sin 150^\circ} = \frac{r}{\sin 150^\circ} = \frac{300}{\sin 60^\circ}$$

$$\therefore \frac{F}{\frac{1}{2}} = \frac{r}{\frac{1}{2}} = \frac{300}{\frac{\sqrt{3}}{2}}$$

$$\therefore F = r = \frac{300 \times \frac{1}{2}}{\frac{\sqrt{3}}{2}} = 100\sqrt{3} \text{ gm.wt.}$$



3

$$\text{(a)} (X-2)^2 + (y+1)^2 = 3^2$$

$$\therefore X^2 + y^2 - 4X + 2y - 4 = 0$$

(b) $\triangle MAB$ is the triangle of forces

where $AM = 60 \text{ cm}$.

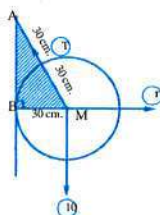
$MB = 30 \text{ cm}$.

$$\begin{aligned} \therefore AB &= \sqrt{(60)^2 - (30)^2} \\ &= 30\sqrt{3} \end{aligned}$$

Applying the triangle of forces rule:

$$\frac{r}{30} = \frac{T}{60} = \frac{10}{30\sqrt{3}}$$

$$\therefore r = \frac{10\sqrt{3}}{3} \text{ gm.wt.}, T = \frac{20\sqrt{3}}{3} \text{ gm.wt.}$$



4

(a) Volume of the wax = volume of cube

$$= (30)^3 = 27000 \text{ cm}^3$$

$\therefore 8\%$ of wax had been lost during the melting and transferring

\therefore The volume of the cone = $92\% \times 27000$

$$= 24840 \text{ cm}^3$$

$$\therefore \text{volume of the cone} = \frac{1}{3} \pi r^2 h$$

$$\therefore \frac{1}{3} \times \pi \times r^2 \times 45 = 24840$$

$$\therefore r = 22.959 \text{ cm.}$$

(b) $\therefore (AB)^2 = (BC)^2 + (AC)^2$

$$\therefore m(\angle ACB) = 90^\circ \therefore CD = \frac{1}{2} AB = 50 \text{ cm.}$$

$$\therefore CD = DB \therefore m(\angle B) = \theta_1$$

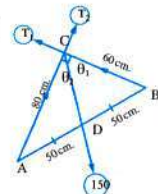
$$\therefore CD = AD \therefore m(\angle A) = \theta_2$$

$$\begin{aligned} \frac{T_1}{\sin(90^\circ + \theta_1)} &= \frac{T_2}{\sin(90^\circ + \theta_2)} \\ &= \frac{150}{\sin 90^\circ} \end{aligned}$$

$$\therefore \frac{T_1}{\cos \theta_1} = \frac{T_2}{\cos \theta_2} = \frac{150}{1}$$

$$\therefore \frac{T_1}{\frac{6}{10}} = \frac{T_2}{\frac{8}{10}} = 150$$

$$\therefore T_1 = 90 \text{ gm.wt.}, T_2 = 120 \text{ gm.wt.}$$



5

$$\text{(a)} X = 8 \cos 0^\circ + 6\sqrt{3} \cos 30^\circ$$

$$+ 5 \cos 60^\circ + 4\sqrt{3} \cos 90^\circ$$

$$= 8 \times 1 + 6\sqrt{3} \times \frac{\sqrt{3}}{2}$$

$$+ 5 \times \frac{1}{2} + 4\sqrt{3} \times 0$$

$$= \frac{39}{2}$$

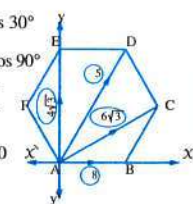
$$Y = 8 \sin 0^\circ + 6\sqrt{3} \sin 30^\circ + 5 \sin 60^\circ + 4\sqrt{3} \sin 90^\circ$$

$$= 8 \times 0 + 6\sqrt{3} \times \frac{1}{2} + 5 \times \frac{\sqrt{3}}{2} + 4\sqrt{3} \times 1 = \frac{19\sqrt{3}}{2}$$

$$\therefore \vec{R} = \frac{39}{2} \hat{i} + \frac{19\sqrt{3}}{2} \hat{j}$$

$$\therefore R = \sqrt{\left(\frac{39}{2}\right)^2 + \left(\frac{19\sqrt{3}}{2}\right)^2} = \sqrt{651} \text{ newton}$$

$$\tan \theta = \frac{19}{39} \sqrt{3} \therefore \theta \approx 40^\circ \hat{\jmath}$$



(b) \therefore The set of forces are in equilibrium

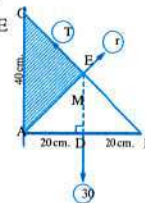
$\therefore \vec{r}$ passes through the point E

$\therefore D$ is the midpoint of

$$\overline{AB}, \overline{DE} \parallel \overline{AC}$$

$\therefore E$ is the midpoint of \overline{CB}

$$\begin{aligned} \therefore BC &= \sqrt{(40)^2 + (40)^2} \\ &= 40\sqrt{2} \end{aligned}$$



$$\therefore CE = 20\sqrt{2} \text{ and } AE = 20\sqrt{2}$$

$\therefore \triangle AEC$ is triangle of forces

$$\therefore \frac{r}{20\sqrt{2}} = \frac{T}{20\sqrt{2}} = \frac{30}{40}$$

$$\therefore r = T = 15\sqrt{2} \text{ newton.}$$

Answers of schools examinations

1 Cairo

First Multiple choice questions

- (1) (a) (2) (a) (3) (b) (4) (a)
 (5) (c) (6) (a) (7) (b) (8) (c)
 (9) (a) (10) (a) (11) (b) (12) (d)
 (13) (a) (14) (d) (15) (c) (16) (b)
 (17) (d) (18) (a) (19) (d) (20) (c)

Second Essay questions

1

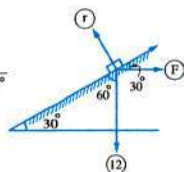
Applying lami's rule

$$\therefore \frac{F}{\sin 150^\circ} = \frac{r}{\sin 90^\circ} = \frac{12}{\sin 120^\circ}$$

$$\therefore \frac{F}{\frac{1}{2}} = \frac{r}{1} = \frac{12}{\frac{\sqrt{3}}{2}}$$

$$\therefore F = \frac{12 \times \frac{1}{2}}{\frac{\sqrt{3}}{2}} = 4\sqrt{3} \text{ N.}$$

$$\therefore r = \frac{12}{\frac{\sqrt{3}}{2}} = 8\sqrt{3} \text{ N.}$$

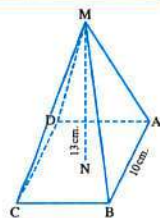


2

The side length of the base

$$= 40 \div 4 = 10 \text{ cm.}$$

$$\begin{aligned} \text{Volume} &= \frac{1}{3} \times \text{base area} \times h \\ &= \frac{1}{3} \times (10)^2 \times 13 \\ &= \frac{1300}{3} \text{ cm}^3. \end{aligned}$$



2 Cairo

First Multiple choice questions

- (1) (c) (2) (b) (3) (d) (4) (a)
 (5) (b) (6) (a) (7) (d) (8) (a)
 (9) (c) (10) (b) (11) (d) (12) (c)
 (13) (a) (14) (b) (15) (c) (16) (d)
 (17) (b) (18) (c) (19) (a) (20) (d)

Second Essay questions

1

$$\therefore (100)^2 = (60)^2 + (80)^2$$

$\therefore \Delta ABC$ is right-angled triangle at C

From lami's rule :

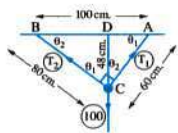
$$\frac{100}{\sin 90^\circ} = \frac{T_1}{\sin \theta_1} = \frac{T_2}{\sin \theta_2}$$

$$\therefore \sin \theta_1 = \frac{BC}{AB} = \frac{80}{100} = \frac{4}{5}$$

$$\therefore \sin \theta_2 = \frac{AC}{AB} = \frac{60}{100} = \frac{3}{5}$$

$$\therefore \frac{100}{1} = \frac{T_1}{(\frac{4}{5})} = \frac{T_2}{(\frac{3}{5})} \quad \therefore T_1 = 100 \times \frac{4}{5} = 80 \text{ gm.wt.}$$

$$\therefore T_2 = 100 \times \frac{3}{5} = 60 \text{ gm.wt.}$$



2

The centre of the circle is the midpoint of \overline{AB}

$$= \left(\frac{6+0}{2}, \frac{-4+2}{2} \right) = (3, -1)$$

$$\text{The diameter length} = \sqrt{(0-6)^2 + (2+4)^2} = 6\sqrt{2}$$

$$\therefore \text{The radius } r = 3\sqrt{2}$$

\therefore The equation of the circle is :

$$(x-3)^2 + (y+1)^2 = (3\sqrt{2})^2$$

$$\therefore x^2 + y^2 - 6x + 2y - 8 = 0$$

3 Cairo

First Multiple choice questions

- (1) (c) (2) (d) (3) (b) (4) (d)
 (5) (b) (6) (b) (7) (b) (8) (b)
 (9) (d) (10) (c) (11) (a) (12) (b)
 (13) (c) (14) (c) (15) (a) (16) (a)
 (17) (c) (18) (a) (19) (b) (20) (d)

Second Essay questions

1

$$\text{The radius} = \sqrt{(3-7)^2 + (-2-(-5))^2} = 5 \text{ length unit.}$$

\therefore The centre is $(7, -5)$

\therefore The equation of the circle is :

$$(x-7)^2 + (y+5)^2 = 25$$

2

In ΔABC :

$$(AB) = \sqrt{10^2 - 6^2} = 8 \text{ cm.}$$

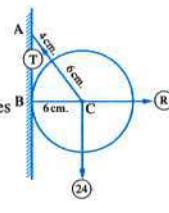
$\therefore \Delta ABC$ is a triangle of forces

$$\therefore \frac{R}{6} = \frac{24}{AB} = \frac{T}{AC}$$

$$\therefore \frac{R}{6} = \frac{24}{8} = \frac{T}{10}$$

$$\therefore \text{The tension } T = 10 \times \frac{24}{8} = 30 \text{ newton}$$

$$\therefore \text{The reaction of the wall } R = 6 \times \frac{24}{8} = 18 \text{ newton}$$



4 Giza

First Multiple choice questions

- (1) (d) (2) (a) (3) (d) (4) (c)
 (5) (b) (6) (c) (7) (c) (8) (a)
 (9) (b) (10) (b) (11) (d) (12) (a)
 (13) (d) (14) (d) (15) (a) (16) (c)
 (17) (d) (18) (a) (19) (c) (20) (a)

Second Essay questions

1

\therefore The quadrilateral pyramid is regular

$\therefore ABCD$ is a square

$$\therefore \text{The area of the square } ABCD = 9 \text{ cm}^2$$

$$\therefore AB = BC = 3 \text{ cm.}$$

$\therefore \Delta ABC$ is right angled at B

$$\therefore AC = \sqrt{3^2 + 3^2} = 3\sqrt{2} \text{ cm.}$$

$\therefore N$ is a midpoint of \overline{AC}

$$\therefore AN = \frac{3\sqrt{2}}{2} \text{ cm.}$$

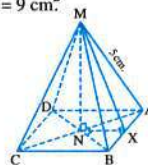
$\therefore \overline{MN} \perp \overline{AC}$

$\therefore \Delta MNA$ is a right angled at N

$$\therefore MN = \sqrt{5^2 - \left(\frac{3\sqrt{2}}{2}\right)^2} = \frac{\sqrt{82}}{2} \text{ cm.}$$

\therefore The volume of the pyramid = $\frac{1}{3} \times \text{base area} \times h$

$$= \frac{1}{3} \times 9 \times \frac{\sqrt{82}}{2} = \frac{3\sqrt{82}}{2} \text{ cm}^3$$



2

\therefore The wall is smooth

$\therefore \overline{R} \perp$ the wall

\therefore The set of forces are in equilibrium

$\therefore \overline{T}$ passes through M the point M

$\therefore \Delta ABM$ is the triangle of forces

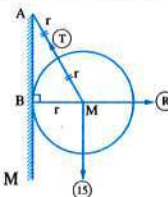
$$AM = 2r, MB = r, AB = \sqrt{(2r)^2 - r^2} = \sqrt{3}r$$

$$\therefore \frac{T}{2r} = \frac{R}{r} = \frac{15}{\sqrt{3}r}$$

$$\therefore R = 5\sqrt{3} \text{ newton}$$

$$\therefore T = 10\sqrt{3} \text{ newton}$$

$$\therefore P = R = 5\sqrt{3} \text{ newton}$$



5 Giza

First Multiple choice questions

- (1) (d) (2) (d) (3) (d) (4) (c)
 (5) (b) (6) (c) (7) (b) (8) (c)
 (9) (b) (10) (c) (11) (c) (12) (c)
 (13) (d) (14) (d) (15) (b) (16) (a)
 (17) (c) (18) (a) (19) (c) (20) (b)

Second Essay questions

1

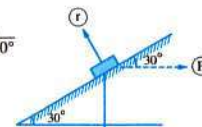
Apply lami's rule :

$$\frac{18}{\sin 120^\circ} = \frac{F}{\sin 150^\circ} = \frac{r}{\sin 90^\circ}$$

$$\therefore F = \frac{18 \sin 150^\circ}{\sin 120^\circ}$$

$$= 6\sqrt{3} \text{ newton}$$

$$\therefore r = \frac{18 \sin 90^\circ}{\sin 120^\circ} = 12\sqrt{3} \text{ newton}$$



2

\therefore The length of the diameter = 18 length units.

$\therefore r = 9$ length units.

\therefore centre is $(-2, 3)$

\therefore The equation of the circle is :

$$(x-(-2))^2 + (y-3)^2 = 9^2$$

$$\therefore x^2 + 4x + 4 + y^2 - 6y + 9 = 81$$

$$\therefore x^2 + y^2 + 4x - 6y - 68 = 0$$

6 Alexandria

First Multiple choice questions

- (1) (d) (2) (b) (3) (d) (4) (c)
 (5) (a) (6) (c) (7) (d) (8) (a)
 (9) (a) (10) (c) (11) (d) (12) (c)
 (13) (a) (14) (d) (15) (c) (16) (b)
 (17) (a) (18) (d) (19) (a) (20) (b)

Second Essay questions

1

ΔABC is a right angled triangle at C

$$\therefore (AB)^2 = 60^2 + 80^2$$

$$\therefore AB = 100 \text{ cm.}$$

$$\text{From lami's rule}$$

$$\therefore \frac{200}{\sin 90^\circ} = \frac{T_1}{\sin \theta_1} = \frac{T_2}{\sin \theta_2}$$

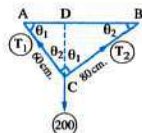
$$\therefore \sin \theta_1 = \frac{BC}{AB} = \frac{80}{100} = \frac{4}{5}, \sin \theta_2 = \frac{AC}{AB} = \frac{60}{100} = \frac{3}{5}$$

$$\therefore \frac{200}{1} = \frac{T_1}{(\frac{4}{5})} = \frac{T_2}{(\frac{3}{5})}$$

$$\therefore T_1 = 200 \times \frac{4}{5} = 160 \text{ gm.wt.}$$

$$\therefore T_2 = 200 \times \frac{3}{5} = 120 \text{ gm.wt.}$$

$$\therefore |T_1 - T_2| = |160 - 120| = 40$$



2

The centre of C_1 is $(5, -2)$ and its radius $r_1 = 2$

the centre of C_2 is $(-7, 3)$ and its radius $r_2 = 1$

the distance between their centres

$$= \sqrt{(5 - (-7))^2 + (-2 - 3)^2} = 13, r_1 + r_2 = 3$$

\therefore The distance between the two centres $> r_1 + r_2$

\therefore The two circles are distant.

7 El-Kalyoubia

First Multiple choice questions

- (1) (a) (2) (b) (3) (c) (4) (c)
 (5) (c) (6) (d) (7) (d) (8) (b)
 (9) (d) (10) (b) (11) (a) (12) (b)
 (13) (d) (14) (c) (15) (d) (16) (c)
 (17) (d) (18) (c) (19) (c) (20) (c)

Second Essay questions

1

$$X = F \cos 0^\circ + 6 \cos 90^\circ + 4\sqrt{2} \cos 135^\circ$$

$$+ 5\sqrt{2} \cos 225^\circ + k \cos 270^\circ$$

$$\therefore X = F - 9$$

$$Y = F \sin 0^\circ + 6 \sin 90^\circ$$

$$+ 4\sqrt{2} \sin 135^\circ$$

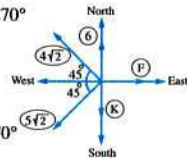
$$+ 5\sqrt{2} \sin 225^\circ + k \sin 270^\circ$$

$$\therefore Y = 5 - k$$

\therefore The resultant = 2 in direction of north

$$\therefore X = 0 \quad \therefore F - 9 = 0 \quad \therefore F = 9$$

$$\therefore Y = 2 \quad \therefore 5 - k = 2 \quad \therefore k = 3$$



2

Draw $\overline{MC} \perp \overline{OA}$

$$\therefore MA = MO = r$$

$\therefore C$ is the midpoint of \overline{OA}

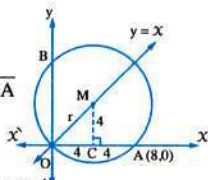
$$\therefore C = \left(\frac{0+8}{2}, \frac{0+0}{2}\right)$$

$$C = (4, 0)$$

$\therefore M$ lies on the straight line $y = x$

$$\therefore \text{The coordinates of } M \text{ is } (4, 4), r = \sqrt{4^2 + 4^2} = 4\sqrt{2}$$

$$\therefore \text{The equation of the circle is : } (X-4)^2 + (y-4)^2 = 32$$



8 El-Monofia

First Multiple choice questions

- (1) (b) (2) (d) (3) (c) (4) (b)
 (5) (d) (6) (a) (7) (c) (8) (d)
 (9) (a) (10) (c) (11) (c) (12) (a)
 (13) (b) (14) (d) (15) (c) (16) (d)
 (17) (d) (18) (c) (19) (c) (20) (b)

Second Essay questions

1

Let \overline{AB} in the direction of \overline{OX}

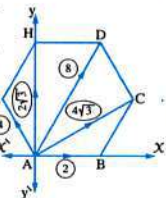
$$\therefore X = 2 \cos 0^\circ + 4\sqrt{3} \cos 30^\circ$$

$$+ 8 \cos 60^\circ + 2\sqrt{3} \cos 90^\circ$$

$$+ 4 \cos 120^\circ$$

$$= 2 \times 1 + 4\sqrt{3} \times \frac{\sqrt{3}}{2} + 8 \times \frac{1}{2}$$

$$+ 2\sqrt{3} \times 0 + 4 \times \frac{-1}{2} = 10$$



$$\therefore Y = 2 \sin 0^\circ + 4\sqrt{3} \sin 30^\circ + 8 \sin 60^\circ$$

$$+ 2\sqrt{3} \sin 90^\circ + 4 \sin 120^\circ$$

$$= 2 \times 0 + 4\sqrt{3} \times \frac{1}{2} + 8 \times \frac{\sqrt{3}}{2} + 2\sqrt{3} \times 1 + 4 \times \frac{\sqrt{3}}{2}$$

$$= 10\sqrt{3}$$

$$\therefore \vec{R} = 10\hat{i} + 10\sqrt{3}\hat{j}$$

$$\therefore R = \sqrt{(10)^2 + (10\sqrt{3})^2} = 20 \text{ kg.wt.}$$

$$\therefore \tan \theta = \frac{10\sqrt{3}}{10} = \sqrt{3} \quad \therefore \theta = 60^\circ$$

\therefore The magnitude of $\vec{R} = 20$ kg.wt. and its direction makes an angle of measure 60° with \vec{AB}

2

The length of the perpendicular drawn from the centre $(1, 1)$ to the straight line

$$= \frac{|3(1) + 4(1) + 23|}{\sqrt{3^2 + 4^2}} = 6 = r$$

$$\therefore \text{The equation of the circle is } (X-1)^2 + (y-1)^2 = 36$$

9 El-Dakahlia

First Multiple choice questions

- (1) (c) (2) (b) (3) (d) (4) (b)
 (5) (a) (6) (a) (7) (a) (8) (b)
 (9) (a) (10) (c) (11) (c) (12) (c)
 (13) (c) (14) (c) (15) (d) (16) (b)
 (17) (d) (18) (a) (19) (c) (20) (a)

Second Essay questions

1

\therefore The two planes are smooth

$\therefore r_1$ and r_2 are

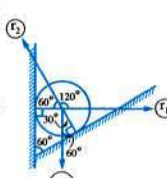
perpendicular to the two planes and passing through the center of the sphere

Applying lami's rule

$$\therefore \frac{r_1}{\sin 150^\circ} = \frac{r_2}{\sin 90^\circ} = \frac{400}{\sin 120^\circ}$$

$$\therefore r_1 \text{ (The reaction of the vertical plane)} = \frac{400\sqrt{3}}{3} \text{ kg.wt.}$$

$$\therefore r_2 \text{ (The reaction of the inclined plane)} = \frac{800\sqrt{3}}{3} \text{ kg.wt.}$$



2

Volume of the pyramid = $\frac{1}{3}$ base area \times height

$$\therefore 1296 = \frac{1}{3} \times (18)^2 \times \text{height}$$

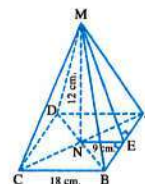
height = 12 cm.

$$\text{The slant height} = \sqrt{9^2 + (12)^2}$$

$$= 15 \text{ cm.}$$

The lateral area

$$= \frac{1}{2} \times (4 \times 18) \times 15 = 540 \text{ cm}^2$$



10 Damietta

First Multiple choice questions

- (1) (c) (2) (c) (3) (a) (4) (d)
 (5) (b) (6) (a) (7) (b) (8) (b)
 (9) (b) (10) (d) (11) (a) (12) (b)
 (13) (c) (14) (d) (15) (c) (16) (c)
 (17) (d) (18) (c) (19) (c) (20) (a)

Second Essay questions

1

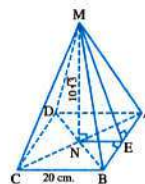
The slant height

$$= \sqrt{(10)^2 + (10\sqrt{3})^2}$$

$$= 20 \text{ cm.}$$

The lateral area

$$= \frac{1}{2} \times (4 \times 20) \times 20 = 800 \text{ cm}^2$$



2

Applying lami's rule

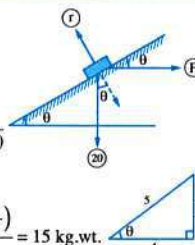
$$\frac{20}{\sin (90^\circ + \theta)} = \frac{r}{\sin 90^\circ}$$

$$= \frac{F}{\sin (180^\circ - \theta)}$$

$$\therefore \frac{20}{\cos \theta} = \frac{r}{1} = \frac{F}{\sin \theta}$$

$$\therefore F = \frac{20 \times \sin \theta}{\cos \theta} = \frac{20 \times (\frac{3}{5})}{(\frac{4}{5})} = 15 \text{ kg.wt.}$$

$$\therefore r = \frac{20}{\cos \theta} = \frac{20}{(\frac{4}{5})} = 25 \text{ kg.wt.}$$



11 El-Beheira

First Multiple choice questions

- (1) (c) (2) (c) (3) (a) (4) (c)
 (5) (a) (6) (b) (7) (c) (8) (a)
 (9) (d) (10) (c) (11) (c) (12) (d)
 (13) (c) (14) (a) (15) (b) (16) (c)
 (17) (d) (18) (b) (19) (a) (20) (b)

Second Essay questions

1

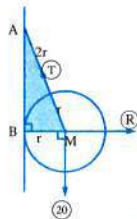
ΔABM is the triangle of forces

$$\therefore AB = \sqrt{(3r)^2 - r^2} = 2\sqrt{2}r$$

$$\therefore \frac{R}{r} = \frac{20}{2\sqrt{2}r} = \frac{T}{3r}$$

$$\therefore P = R = 5\sqrt{2} \text{ newton}$$

$$\therefore T = 15\sqrt{2} \text{ newton.}$$



2

The formed solid is two cones

with common base and equal heights

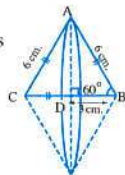
$$\therefore r = 6 \sin 60^\circ = 3\sqrt{3} \text{ cm.}$$

$$\therefore h = 3 \text{ cm.}$$

\therefore The volume of each cone

$$= \frac{1}{3} \pi (3\sqrt{3})^2 \times 3 = 27\pi \text{ cm}^3$$

$$\therefore \text{The volume of the whole solid} = 54\pi \text{ cm}^3$$



12 Beni-Suef

First Multiple choice questions

- (1) (a) (2) (b) (3) (c) (4) (b)
 (5) (c) (6) (c) (7) (b) (8) (c)
 (9) (a) (10) (c) (11) (d) (12) (b)
 (13) (c) (14) (b) (15) (a) (16) (a)
 (17) (c) (18) (a) (19) (d) (20) (b)

Second Essay questions

1

$$X = 6 \cos 0^\circ + 2\sqrt{3} \cos 30^\circ$$

$$+ 6 \cos 60^\circ + 2\sqrt{3} \cos 90^\circ$$

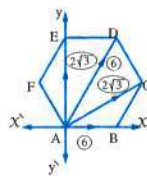
$$\therefore X = 12 \text{ newton}$$

$$\therefore Y = 6 \sin 0^\circ + 2\sqrt{3} \sin 30^\circ$$

$$+ 6 \sin 60^\circ + 2\sqrt{3} \sin 90^\circ$$

$$\therefore Y = 6\sqrt{3}$$

$$R = \sqrt{(12)^2 + (6\sqrt{3})^2} = 6\sqrt{7} \text{ newton}$$



2

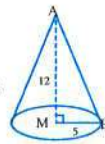
$$AB = \sqrt{12^2 + 5^2} = 13 \text{ cm.}$$

$$\therefore \text{The total area} = \pi r^2 + \pi r l$$

$$= \pi (5)^2 + \pi \times 5 \times 13$$

$$= 90\pi \text{ cm}^2$$

$$\approx 282.7 \text{ cm}^2$$



13 El-Menia

First Multiple choice questions

- (1) (d) (2) (b) (3) (c) (4) (b)
 (5) (d) (6) (d) (7) (b) (8) (b)
 (9) (d) (10) (a) (11) (b) (12) (a)
 (13) (d) (14) (b) (15) (c) (16) (d)
 (17) (c) (18) (a) (19) (c) (20) (a)

Second Essay questions

1

\therefore The quadrilateral Pyramid is regular

\therefore The base is a square its side length = 10

$$\therefore \text{The base area} = (10)^2 = 100 \text{ cm}^2$$

$$\therefore \text{The lateral area} = 60 \times 4 = 240 \text{ cm}^2$$

$$\therefore \text{The total area} = 240 + 100 = 340 \text{ cm}^2$$

2

$$X = 6 \cos 0^\circ + 2\sqrt{3} \cos 30^\circ$$

$$+ 6 \cos 60^\circ + 2\sqrt{3} \cos 90^\circ$$

$$\therefore X = 12 \text{ newton.}$$

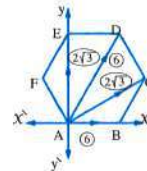
$$\therefore Y = 6 \sin 0^\circ + 2\sqrt{3} \sin 30^\circ$$

$$+ 6 \sin 60^\circ + 2\sqrt{3} \sin 90^\circ$$

$$\therefore Y = 6\sqrt{3} \text{ newton.}$$

$$\therefore R = \sqrt{(12)^2 + (6\sqrt{3})^2}$$

$$= 6\sqrt{7} \text{ newton}$$



14 Assiut

First Multiple choice questions

- (1) (c) (2) (b) (3) (d) (4) (d)
 (5) (b) (6) (d) (7) (c) (8) (b)
 (9) (c) (10) (c) (11) (c) (12) (b)
 (13) (a) (14) (a) (15) (d) (16) (b)
 (17) (b) (18) (c) (19) (b) (20) (d)

Second Essay questions

1

$$\therefore \vec{R} \perp \vec{F}$$

$$\therefore F + 2 \cos 120^\circ = 0$$

$$\therefore F = 1$$

2

$$\therefore X = 4 \cos 0^\circ + 2 \cos 60^\circ$$

$$+ 5 \cos 120^\circ$$

$$+ 3\sqrt{3} \cos 210^\circ$$

$$= 4 \times 1 + 2 \times \frac{1}{2} + 5 \times -\frac{1}{2}$$

$$+ 3\sqrt{3} \times -\frac{\sqrt{3}}{2} = -2$$

$$\therefore Y = 4 \sin 0^\circ + 2 \sin 60^\circ + 5 \sin 120^\circ + 3\sqrt{3} \sin 210^\circ$$

$$= 4 \times 0 + 2 \times \frac{\sqrt{3}}{2} + 5 \times \frac{\sqrt{3}}{2} + 3\sqrt{3} \times -\frac{1}{2} = 2\sqrt{3}$$

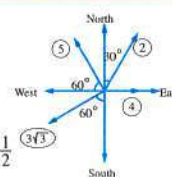
$$\therefore \vec{R} = -2\vec{i} + 2\sqrt{3}\vec{j}$$

$$\therefore R = \sqrt{(-2)^2 + (2\sqrt{3})^2} = 4 \text{ newton}$$

$$\therefore \tan \theta = \frac{2\sqrt{3}}{-2} = -\sqrt{3}$$

$$\therefore X < 0, Y > 0$$

$$\therefore \theta = 120^\circ$$



15 Qena

First Multiple choice questions

- (1) (a) (2) (d) (3) (a) (4) (d)
 (5) (c) (6) (b) (7) (c) (8) (b)
 (9) (b) (10) (c) (11) (b) (12) (a)
 (13) (d) (14) (a) (15) (b) (16) (b)
 (17) (a) (18) (a) (19) (b) (20) (a)

Second Essay questions

1

The wall is smooth

$$\therefore \vec{R} \perp \text{the wall}$$

\therefore The set of forces are in equilibrium

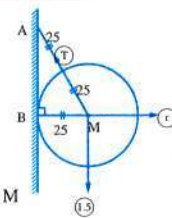
$\therefore T$ passes through the point M

$\therefore \Delta ABM$ is the triangle of forces

$$\therefore AB = \sqrt{50^2 - 25^2} = 25\sqrt{3}$$

$$\therefore \frac{T}{50} = \frac{r}{25} = \frac{1.5}{25\sqrt{3}}$$

$$\therefore T = \sqrt{3} \text{ kg.wt.}, r = \frac{\sqrt{3}}{2} \text{ kg.wt.}$$



2

The equation of the circle is $(X-1)^2 + (Y-5)^2 = 6^2$

$$\therefore X^2 + Y^2 - 2X - 10Y - 10 = 0$$

SCIENTIFIC SECTION

Mathematics

Applications

By a group of supervisors



FIRST TERM

2

SEC.
2024

GUIDE ANSWERS



EL-MOASSER

Answers of Unit One



Answers of accumulative exercise on vectors

1

- (1) c (2) b (3) c (4) c (5) c
 (6) b (7) d (8) d (9) c (10) c
 (11) a (12) c (13) d (14) b (15) d
 (16) c (17) a (18) a

2

$$18\hat{e} + 45\hat{e} - 50\hat{e}$$

3

$$\overrightarrow{AC}, 2\overrightarrow{DM} \text{ or } \overrightarrow{DB}, \overrightarrow{O}, \overrightarrow{AD}, \overrightarrow{MB}$$

Exercise 1

First Multiple choice questions

- (1) d (2) c (3) d (4) c (5) c
 (6) b (7) b (8) c (9) b (10) c
 (11) b (12) b (13) a (14) a (15) c
 (16) d (17) b (18) d (19) d (20) b
 (21) c (22) c (23) c (24) c (25) d
 (26) b (27) a (28) c (29) d (30) c
 (31) a (32) a (33) b (34) c (35) b
 (36) b (37) c (38) b (39) d (40) c
 (41) a (42) a (43) b (44) c (45) d
 (46) d (47) b (48) c (49) c

Second Essay questions

1

$$R = \sqrt{(8)^2 + (15)^2} = 17 \text{ kg.wt.} \quad \tan \theta = \frac{15}{8}$$

$$\therefore \theta = 61^\circ 53' 39''$$

2

$$\therefore \text{Let the two forces be } F_1, F_2 \text{ newton}$$

$$\therefore (50)^2 = F_1^2 + F_2^2 \quad \therefore F_1^2 + F_2^2 = 2500 \quad (1)$$

$$\therefore \tan \theta = \frac{F_2}{F_1} \quad \therefore \frac{F_2}{F_1} = \tan 30^\circ$$

$$\therefore \frac{F_2}{F_1} = \frac{1}{\sqrt{3}} \quad \therefore F_1 = \sqrt{3} F_2 \quad (2)$$

$$\therefore F_1^2 = 3 F_2^2 \quad (3)$$

Substituting from (3) in (1):

$$\therefore 3 F_2^2 + F_2^2 = 2500 \quad \therefore 4 F_2^2 = 2500$$

$$\therefore F_2^2 = \frac{2500}{4} = 625 \quad \therefore F_2 = 25 \text{ newton}$$

Substituting in (2): $\therefore F_1 = 25\sqrt{3}$ newton

\therefore The magnitudes of the two forces are:

$$25\sqrt{3}, 25 \text{ newton.}$$

3

$$(26)^2 = (30)^2 + (16)^2 + 2 \times 30 \times 16 \cos \alpha$$

$$\therefore \cos \alpha = -\frac{1}{2} \quad \therefore \alpha = 120^\circ$$

4

$$(3\sqrt{7})^2 = (9)^2 + (6)^2 + 2 \times 9 \times 6 \cos \alpha$$

$$\therefore \cos \alpha = -\frac{1}{2} \quad \therefore \alpha = 120^\circ$$

$$\tan \theta = \frac{6 \sin 120^\circ}{9 + 6 \cos 120^\circ} = \frac{\sqrt{3}}{2} \quad \therefore \theta \approx 40^\circ 53' 36''$$

5

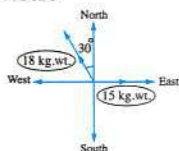
$$R = \sqrt{(15)^2 + (18)^2} + 2 \times 15 \times 18 \cos 120^\circ$$

$$= 3\sqrt{31} \text{ kg.wt.}$$

$$\tan \theta = \frac{18 \sin 120^\circ}{15 + 18 \cos 120^\circ}$$

$$= \frac{3\sqrt{3}}{2}$$

$$\therefore \theta = 68^\circ 56' 54''$$



6

$$\alpha = 120^\circ, \theta = 30^\circ$$

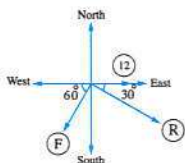
$$\tan 30^\circ = \frac{F \sin 120^\circ}{12 + F \cos 120^\circ}$$

$$\therefore \frac{1}{\sqrt{3}} = \frac{F \times \frac{\sqrt{3}}{2}}{12 + F \times -\frac{1}{2}}$$

$$\therefore \frac{3}{2} F = 12 - \frac{1}{2} F \quad \therefore F = 6 \text{ kg.wt.}$$

$$\therefore R = \sqrt{(12)^2 + 6^2} + 2 \times 12 \times 6 \times \cos 120^\circ$$

$$\therefore R = 6\sqrt{3} \text{ kg.wt.}$$



Another solution :

∴ The resultant is perpendicular to the second force.

$$\therefore \cos 120 = -\frac{F}{12} \quad \therefore F = 6 \text{ kg.wt.}$$

$$\therefore R = \sqrt{(12)^2 - (6)^2} = 6\sqrt{3} \text{ kg.wt.}$$

7

Let F_1 be the smaller force

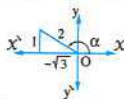
and F_2 is the greater force.

∵ the resultant is perpendicular to F_1

$$\therefore F_1 + 30 \cos \alpha = 0$$

$$\therefore F_1 = 30 \times \frac{\sqrt{3}}{2} = 15\sqrt{3} \text{ kg.wt.}$$

$$\therefore R = \sqrt{(15\sqrt{3})^2 + (30)^2 + 2 \times 15\sqrt{3} \times 30 \times \left(-\frac{\sqrt{3}}{2}\right)} \\ = 15 \text{ kg.wt.}$$

**8**

$$(1) R = \sqrt{(5\sqrt{2})^2 + (5)^2 + 2 \times 5\sqrt{2} \times 5 \cos 45^\circ}$$

$$\therefore R = 5\sqrt{5} \text{ newton}$$

$$\therefore \tan \theta = \frac{5 \sin 45^\circ}{5\sqrt{2} + 5 \cos 45^\circ} = \frac{1}{3}$$

$$\theta \approx 18^\circ 26' 26''$$

$$(2) R = \sqrt{(3)^2 + (3\sqrt{2})^2 + 2 \times 3 \times 3\sqrt{2} \times \cos 45^\circ}$$

$$\therefore R = 3\sqrt{5} \text{ newton}$$

$$\therefore \tan \theta = \frac{3\sqrt{2} \sin 45^\circ}{3 + 3\sqrt{2} \cos 45^\circ} = \frac{1}{2}$$

$$\theta \approx 26^\circ 33' 54''$$

$$(3) R = \sqrt{(150)^2 + (150)^2 + 2 \times 150 \times 150 \times \cos 120^\circ}$$

$$\therefore R = 150 \text{ newton}$$

Another solution :

$$R = 2 \times 150 \times \cos 60^\circ = 150 \text{ newton}$$

∵ the resultant bisects the angle between the two forces.

9

$$(4\sqrt{3})^2 = F^2 + (4)^2 + 2 \times F \times 4 \cos 120^\circ$$

$$\therefore F^2 - 4F - 32 = 0$$

$$\therefore (F+4)(F-8) = 0$$

$$\therefore F = 8 \text{ newton}$$

$$\tan \theta = \frac{4 \sin 120^\circ}{8 + 4 \cos 120^\circ} = \frac{1}{\sqrt{3}} \quad \therefore \theta = 30^\circ$$

10

∵ The resultant is perpendicular to the first force

$$\therefore \sqrt{3}F + 2F \cos \alpha = 0 \quad \therefore \cos \alpha = -\frac{\sqrt{3}}{2}$$

$$\therefore \alpha = 150^\circ, \text{ when } F = 15$$

$$\therefore R = \sqrt{(15\sqrt{3})^2 + (30)^2 + 2 \times 15\sqrt{3} \times 30 \cos 150^\circ} \\ = 15 \text{ newton}$$

11

∵ The resultant is perpendicular to the second force

$$\therefore R^2 = F_1^2 - F_2^2 \quad \therefore 2 = 8 - F_2^2$$

$$\therefore F_2^2 = 6 \quad \therefore F_2 = \sqrt{6} \text{ newton}$$

$$\therefore F_2 + F_1 \cos \alpha = 0 \quad \therefore \sqrt{6} + 2\sqrt{2} \cos \alpha = 0$$

$$\therefore \cos \alpha = \frac{-\sqrt{6}}{2\sqrt{2}} = -\frac{\sqrt{3}}{2} \quad \therefore \alpha = 150^\circ$$

12

$$\therefore \tan 30^\circ = \frac{F \sin 120^\circ}{16 + F \cos 120^\circ} \quad \therefore \frac{1}{\sqrt{3}} = \frac{\frac{\sqrt{3}}{2} F}{16 - \frac{1}{2} F}$$

$$\therefore 16 - \frac{1}{2} F = \frac{3}{2} F \quad \therefore 2F = 16$$

$$\therefore F = 8 \text{ kg.wt.}$$

$$\therefore R = \sqrt{(16)^2 + (8)^2 + 2 \times 16 \times 8 \cos 120^\circ} = 8\sqrt{3} \text{ kg.wt.}$$

13

The resultant of 1st and 2nd forces

$$= \sqrt{(5)^2 + (10)^2 + 2 \times 5 \times 10 \cos 60^\circ} = 5\sqrt{7} \text{ newton}$$

∵ The maximum value of the resultant of

$$\text{the three forces} = 5\sqrt{7} + 4\sqrt{7} = 9\sqrt{7} \text{ newton}$$

The minimum value of the resultant of the three

$$\text{forces} = 5\sqrt{7} - 4\sqrt{7} = \sqrt{7} \text{ newton}$$

14

$$(1) \therefore (3F)^2 = (2F)^2 + (3F)^2 + 2 \times 2F \times 3F \cos \theta$$

$$\therefore 9F^2 = 13F^2 + 12F^2 \cos \theta$$

$$\therefore \cos \theta = -\frac{1}{3} \quad \therefore \theta = 109^\circ 28' 16''$$

$$(2) \therefore \text{The resultant} = F = 3F - 2F$$

∵ The measure of the angle between the two forces = 180°

(3) \therefore The resultant = $5F = 2F + 3F$

\therefore The measure of the angle between the two forces = zero

(4) $(\sqrt{13}F)^2 = (2F)^2 + (3F)^2 + 2 \times 2F \times 3F \cos \theta$

$\therefore 13F^2 = 4F^2 + 9F^2 + 12F^2 \cos \theta$

$\therefore \cos \theta = \text{zero} \quad \therefore \theta = 90^\circ$

15

(1) \therefore The direction of the resultant is perpendicular to the second force

$\therefore F + 2 \cos 120^\circ = 0 \quad \therefore F = 1 \text{ newton}$

(2) $\therefore \tan 45^\circ = \frac{2 \sin 120^\circ}{F + 2 \cos 120^\circ} \quad \therefore \sqrt{3} = F - 1$

$\therefore F = (\sqrt{3} + 1) \text{ newton}$

16

$\therefore R \in [2, 10]$

\therefore The minimum value of $R = 2 \text{ newton}$

$\therefore F_1 - F_2 = 2$

and the maximum value of $R = 10 \text{ newton}$

$\therefore F_1 + F_2 = 10$

adding (1) and (2): $\therefore 2F_1 = 12$

$\therefore F_1 = 6 \text{ newton} \quad \therefore F_2 = 4 \text{ newton}$

$\therefore R = \sqrt{6^2 + 4^2 + 2 \times 6 \times 4 \cos 120^\circ} = 2\sqrt{7} \text{ newton}$

17

Let the two forces be F and $F + 3$

\therefore The resultant is perpendicular to the smaller force.

$\therefore F + (F + 3) \cos \alpha = 0 \quad \therefore \cos \alpha = \frac{-F}{F + 3}$

$\therefore R = 3\sqrt{3}$

$\therefore 27 = F^2 + (F + 3)^2 + 2 \times F \times (F + 3) \times \frac{-F}{F + 3}$

$\therefore 27 = F^2 + F^2 + 6F + 9 - 2F^2 \quad \therefore F = 3$

\therefore The two forces are 3 newton and 6 newton

$\cos \alpha = \frac{-3}{6} = -\frac{1}{2} \quad \therefore \alpha = 120^\circ$

18

Let the two forces be F_1 and F_2

• In the first case : $10 = F_1^2 + F_2^2$ (1)

• In the second case :

$13 = F_1^2 + F_2^2 + 2F_1 \times F_2 \times \frac{1}{2}$

and from (1) : $\therefore 13 = 10 + F_1 F_2 \quad \therefore F_1 F_2 = 3$

$\therefore F_2 = \frac{3}{F_1}$

substituting in (1) :

$\therefore 10 = F_1^2 + \frac{9}{F_1^2} \quad \therefore 10F_1^2 = F_1^4 + 9$

$\therefore F_1^4 - 10F_1^2 + 9 = 0 \quad \therefore (F_1^2 - 9)(F_1^2 - 1) = 0$

$\therefore F_1^2 = 9 \quad \therefore F_1 = 3 \text{ or } \therefore F_1^2 = 1 \quad \therefore F_1 = 1$

\therefore The two forces are 1 and 3 newton

19

$R_1 = 2F \cos \frac{\alpha}{2} = 12$

$\therefore F \cos \frac{\alpha}{2} = 6$ (1)

$\therefore R_2 = 2F \cos \left(\frac{180^\circ - \alpha}{2} \right) = 6$

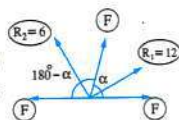
$\therefore F \sin \frac{\alpha}{2} = 3$ (2)

squaring the two equations and adding them

$\therefore F^2 \cos^2 \frac{\alpha}{2} + F^2 \sin^2 \frac{\alpha}{2} = 6^2 + 3^2$

$\therefore F^2 \left(\cos^2 \frac{\alpha}{2} + \sin^2 \frac{\alpha}{2} \right) = 45$

$\therefore F = \sqrt{45} = 3\sqrt{5} \text{ kg.wt.}$



20

• In the first case : $R^2 = F_1^2 + F_2^2 + 2F_1 F_2 \cos 120^\circ$

$\therefore R^2 = F_1^2 + F_2^2 - F_1 F_2$ (1)

• In the second case :

The measure of the angle between the two forces = 60°

$\therefore 3R^2 = F_1^2 + F_2^2 + F_1 F_2$ (2)

Substituting from (1) in (2)

$\therefore 3(F_1^2 + F_2^2 - F_1 F_2) = F_1^2 + F_2^2 + F_1 F_2$

$\therefore 2F_1^2 + 2F_2^2 - 4F_1 F_2 = 0$

$\therefore F_1^2 - 2F_1 F_2 + F_2^2 = 0 \quad \therefore (F_1 - F_2)^2 = 0$

$\therefore F_1 = F_2$

$\therefore \vec{R}_1$ in the first case makes an angle of measure 60° with \vec{F}_1

\vec{R}_2 in the second case makes an angle of measure 30° with \vec{F}_1 from the other side.

\therefore The measure of the angle between the two resultants = 90°

21

$$\vec{F}_1 = (4, 0^\circ), \vec{F}_2 = (F, \alpha), \vec{R} = (10, 60^\circ)$$

$$\vec{R} = \vec{F}_1 + \vec{F}_2$$

$$\therefore (10 \cos 60^\circ, 10 \sin 60^\circ)$$

$$= (4 \cos 0^\circ, 4 \sin 0^\circ) + (F \cos \alpha, F \sin \alpha)$$

$$\therefore (5, 5\sqrt{3}) = (4, 0) + (F \cos \alpha, F \sin \alpha)$$

$$\therefore F \cos \alpha + 4 = 5$$

$$\therefore F \cos \alpha = 1 \quad (1)$$

$$+ F \sin \alpha = 5\sqrt{3}$$

(2)

dividing (2) by (1):

$$\tan \alpha = 5\sqrt{3}$$

$$\therefore \sin \alpha = \frac{5\sqrt{3}}{\sqrt{76}}$$

$$\text{from (2)}: \therefore F \times \frac{5\sqrt{3}}{\sqrt{76}} = 5\sqrt{3}$$

$$\therefore F = \sqrt{76} = 2\sqrt{19} \text{ newton}$$

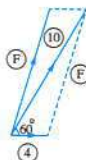
Another solution:

Applying cosine rule:

$$F^2 = (10)^2 + (4)^2 - 2 \times 4 \times 10 \times \cos 60^\circ$$

$$= 76$$

$$\therefore F = 2\sqrt{19} \text{ newton}$$



24

Let the two forces be F_1 and F_2 where $F_1 < F_2$

$$\therefore F_1 + F_2 = 40 \text{ then } F_1 = 40 - F_2 \quad (1)$$

\therefore The resultant = 20 kg.wt.

$$\therefore 400 = F_1^2 + F_2^2 + 2 F_1 F_2 \cos \alpha \quad (2)$$

\therefore The resultant is perpendicular to the smaller force

$$\therefore F_1 + F_2 \cos \alpha = 0$$

$$\therefore \cos \alpha = \frac{-F_1}{F_2} \text{ substituting in (2):}$$

$$\therefore 400 = F_1^2 + F_2^2 - 2 F_1 F_2 \times \frac{F_1}{F_2}$$

$$\therefore 400 = F_1^2 + F_2^2 - 2 F_1^2$$

$$\therefore 400 = F_2^2 - F_1^2$$

$$\text{Substituting from (1): } \therefore 400 = F_2^2 - (40 - F_2)^2$$

$$\therefore 400 = F_2^2 - 1600 + 80 F_2 - F_2^2$$

$$\therefore 2000 = 80 F_2$$

$$\therefore F_2 = 25 \text{ kg.wt.}$$

Substituting in (1):

$$\therefore F_1 = 15 \text{ kg.wt.}, \cos \alpha = \frac{-15}{25} = \frac{-3}{5}$$

25

$$R_1 = 2 F \cos 60^\circ = F \quad R_2 = 2 (2 F) \cos 30^\circ = 2\sqrt{3} F$$

$$\therefore R_2 - R_1 = 11$$

$$\therefore 2\sqrt{3} F - F = 11$$

$$\therefore F = \frac{11}{2\sqrt{3} - 1}$$

$$\therefore F = 1 + 2\sqrt{3}$$

26

First case:

$$(\sqrt{5} F (m+1))^2 = F^2 + (2 F)^2 + 2 \times F \times 2 F \cos \alpha$$

$$\therefore 5 F^2 (m^2 + 2m + 1) - 5 F^2 = 4 F^2 \cos \alpha$$

$$\therefore 5 F^2 m (m+2) = 4 F^2 \cos \alpha \quad (1)$$

Second case:

$$[\sqrt{5} F (m-1)]^2 = F^2 + (2 F)^2 + 2 \times F \times 2 F \cos (90^\circ - \alpha)$$

$$5 F^2 (m^2 - 2m + 1) - 5 F^2 = 4 F^2 \sin \alpha$$

$$\therefore 5 F^2 m (m-2) = 4 F^2 \sin \alpha \quad (2)$$

by dividing (2) by (1):

$$\frac{5 F^2 m (m-2)}{5 F^2 m (m+2)} = \frac{4 F^2 \sin \alpha}{4 F^2 \cos \alpha} \quad \therefore \tan \alpha = \frac{m-2}{m+2}$$

22

$$\text{Let } F_1 > F_2 \quad \therefore F_1 - F_2 = 15 \quad \therefore F_1 = 15 + F_2 \quad (1)$$

$$(35)^2 = F_1^2 + F_2^2 + 2 F_1 F_2 \times -\frac{1}{2}$$

substituting from (1)

$$\therefore 1225 = 225 + F_2^2 + 30 F_2 + F_2^2 - 15 F_2 - F_2^2$$

$$\therefore F_2^2 + 15 F_2 - 1000 = 0 \quad \therefore (F_2 + 40)(F_2 - 25) = 0$$

$$\therefore F_2 = 25$$

\therefore The two forces are 40 and 25 newton

23

$$F_1 + F_2 = 4 \quad \therefore F_1 = 4 - F_2 \quad (1)$$

$$13 = (F_1)^2 + (F_2)^2 + 2 F_1 F_2 \cos 60^\circ$$

Substituting from (1):

$$\therefore 13 = (4 - F_2)^2 + F_2^2 + (4 - F_2) \times F_2$$

$$\therefore 13 = 16 - 8 F_2 + F_2^2 + F_2^2 + 4 F_2 - F_2^2$$

$$\therefore F_2^2 - 4 F_2 + 3 = 0 \quad \therefore (F_2 - 1)(F_2 - 3) = 0$$

$$\therefore F_2 = 1 \text{ or } 3$$

\therefore The two forces are 1 and 3 newton

Third Higher skills

1

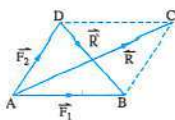
- (1) d (2) d (3) b
 (4) d (5) a (6) b
 (7) c (8) d (9) a
 (10) d (11) c

Instructions to solve 1:

- (1) \therefore The maximum value $= F_1 + F_2$
 , the minimum value $= F_1 - F_2$ where $F_1 > F_2$
 $\therefore \frac{F_1 + F_2}{F_1 - F_2} = \frac{7}{3} \quad \therefore 3F_1 + 3F_2 = 7F_1 - 7F_2$
 $\therefore 4F_1 = 10F_2 \quad \therefore \frac{F_1}{F_2} = \frac{10}{4} = \frac{5}{2}$
 \therefore The ratio between the two forces $= 5 : 2$
- (2) $\therefore F_1 : F_2 : R = 4 : 3 : \sqrt{13}$
 $\therefore F_1 = 4 \text{ m}, F_2 = 3 \text{ m}, R = \sqrt{13} \text{ m}$
 $\therefore (\sqrt{13} \text{ m})^2 = (4 \text{ m})^2 + (3 \text{ m})^2 + 2(4 \text{ m})(3 \text{ m}) \cos \alpha$
 $\therefore \cos \alpha = \frac{13 \text{ m}^2 - 16 \text{ m}^2 - 9 \text{ m}^2}{24 \text{ m}^2} = -\frac{1}{2}$
 \therefore The measure of the angle between the two forces $= \cos^{-1} \left(-\frac{1}{2} \right) = 120^\circ$
- (3) \therefore The resultant $\perp F_1$
 $\therefore F_1 + F_2 \cos \alpha = \text{zero} \quad \therefore \cos \alpha = -\frac{F_1}{F_2}$
 \therefore The measure of the angle between the two forces $= \cos^{-1} \left(-\frac{F_1}{F_2} \right)$
- (4) \therefore The measure of the angle between the two forces $= 90^\circ$
 \therefore the two forces are unequal
 \therefore The force inclined toward the greater force
 \therefore The measure of the angle θ between the greater force and the resultant must be less than 45°

(5) Representing the two

forces \vec{F}_1, \vec{F}_2 as two adjacent sides in parallelogram as in the opposite figure



$$\therefore \vec{F}_1 + \vec{F}_2 = \vec{AB} + \vec{AD} = \vec{AC} = \vec{R} \quad (1)$$

$$\therefore \vec{F}_1 + (-\vec{F}_2) = \vec{AB} + (-\vec{AD}) = \vec{AB} - \vec{AD} = \vec{DB} = \vec{R} \quad (2)$$

$\therefore \vec{R}$ and \vec{R} are represented by diagonals of parallelogram

\therefore In case of $\vec{R} \perp \vec{R}$, then ABCD is rhombus

$$\therefore AB = AD \quad \therefore F_1 = F_2$$

$$(6) R^2 = F^2 + (4)^2 + 2(F)(4) \cos 120^\circ$$

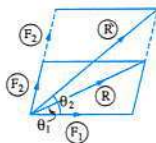
$$\therefore R^2 = F^2 - 4F + 16$$

$$\therefore R^2 = (F - 2)^2 + 12$$

$$\therefore R = \sqrt{(F - 2)^2 + 12}$$

\therefore The smallest value of the resultant is when $F = 2$

- (7) As F_2 increases as the resultant lean towards the greater force which is doubled and hence the angle between the resultant and the second force increases.



$$i.e. \theta_2 > \theta_1$$

The opposite figure show the idea.

$$(8) \therefore R^2 = F^2 + 3F^2 + 2(F)(\sqrt{3}F) \cos \alpha$$

$$\therefore \cos \alpha = -\frac{\sqrt{3}}{2} \quad \therefore \alpha = 150^\circ$$

$$\therefore \tan \theta_1 = \frac{\sqrt{3}F \sin 150^\circ}{F + \sqrt{3}F \cos 150^\circ} = -\sqrt{3}$$

$$\therefore \theta_1 = 120^\circ \text{ and so } \theta_2 = 150^\circ - 120^\circ = 30^\circ$$

$$\therefore \theta_1 = 4\theta_2$$

$$(9) \therefore 3 \leq F_1 \leq 12 \quad \therefore 9 \leq F_1^2 \leq 144 \quad (1)$$

$$\therefore 4 \leq F_2 \leq 16 \quad \therefore 16 \leq F_2^2 \leq 256 \quad (2)$$

By adding (1), (2):

$$\therefore 25 \leq F_1^2 + F_2^2 \leq 400 \quad \therefore 5 \leq R \leq 20$$

$$(10) \therefore 1 \leq F_1 \leq 9 \quad \therefore 3 \leq F_2 \leq 7$$

$$\therefore 4 \leq F_1 + F_2 \leq 16 \quad (1)$$

$$\therefore 1 \leq F_1 \leq 9 \quad \therefore -7 \leq -F_2 \leq -3$$

$$\therefore -6 \leq F_1 - F_2 \leq 6 \quad \therefore 0 \leq |F_1 - F_2| \leq 6 \quad (2)$$

From (1), (2): $0 \leq R \leq 16$

$$(11) \therefore 5 \leq F_1 \leq 20, \quad 12 \leq F_2 \leq 21$$

$$\therefore 17 \leq F_1 + F_2 \leq 41$$

$$\text{When } \theta = \frac{\pi}{2}$$

$$\therefore (5)^2 + (12)^2 \leq F_1^2 + F_2^2 \leq (20)^2 + (21)^2$$

$$\therefore 169 \leq F_1^2 + F_2^2 \leq 841$$

$$\therefore 13 \leq \sqrt{F_1^2 + F_2^2} \leq 29$$

$$\therefore 13 \leq R \leq 41 \text{ when } 0 \leq \theta \leq \frac{\pi}{2}$$

2

In the first case :

Let the two forces be F and $2F$ and the measure of the angle between them $= \alpha$

$$\therefore \tan \theta = \frac{2F \sin \alpha}{F + 2F \cos \alpha}, \therefore \tan \theta = \frac{2 \sin \alpha}{1 + 2 \cos \alpha} \quad (1)$$

In the second case :

The small force $= F + 4$ and the great force $= 4F$

$$\therefore \tan \theta = \frac{4F \sin \alpha}{F + 4 + 4F \cos \alpha} \quad (2)$$

From (1) and (2) :

$$\therefore \frac{2 \sin \alpha}{1 + 2 \cos \alpha} = \frac{4F \sin \alpha}{F + 4 + 4F \cos \alpha}$$

$$\therefore \frac{1}{1 + 2 \cos \alpha} = \frac{2F}{F + 4 + 4F \cos \alpha}$$

$$\therefore F + 4 + 4F \cos \alpha = 2F + 4F \cos \alpha$$

$$\therefore F + 4 = 2F \quad \therefore F = 4 \text{ kg.wt.}$$

\therefore In the first case :

The magnitude of the first force is 4 kg.wt.

\therefore the magnitude of the second force is 8 kg.wt.

$$\therefore R_1 = \sqrt{16 + 64 + 2 \times 4 \times 8 \cos \alpha} = 4\sqrt{5 + 4 \cos \alpha}$$

\therefore in the second case :

The magnitude of the first force is 8 kg.wt.

\therefore the magnitude of the second force is 16 kg.wt.

$$\therefore R_2 = \sqrt{64 + 256 + 2 \times 8 \times 16 \cos \alpha} = 8\sqrt{5 + 4 \cos \alpha}$$

$$\therefore \frac{R_1}{R_2} = \frac{4\sqrt{5 + 4 \cos \alpha}}{8\sqrt{5 + 4 \cos \alpha}} = \frac{1}{2}$$

3

In the first case :

$$R^2 = F_1^2 + F_2^2 + 2F_1 F_2 \cos \alpha \quad (1)$$

In the second case :

$$3R^2 = F_1^2 + F_2^2 + 2F_1 F_2 \cos (180^\circ - \alpha)$$

$$\therefore 3R^2 = F_1^2 + F_2^2 - 2F_1 F_2 \cos \alpha \quad (2)$$

multiplying (1) $\times 3$, then subtracting (2) from (1) :

$$\therefore \text{zero} = 2F_1^2 + 2F_2^2 + 8F_1 F_2 \cos \alpha$$

$$\therefore \cos \alpha = \frac{-F_1^2 - F_2^2}{4F_1 F_2} \quad (3)$$

Let θ_1 is inclination angle of R on F_1

$$\therefore \tan \theta_1 = \frac{F_2 \sin \alpha}{F_1 + F_2 \cos \alpha}$$

Let θ_2 is the inclination angle of $R\sqrt{3}$ on F_1

$$\therefore \tan \theta_2 = \frac{F_2 \sin (180^\circ - \alpha)}{F_1 + F_2 \cos (180^\circ - \alpha)}$$

$$\tan \theta_2 = \frac{F_2 \sin \alpha}{F_1 - F_2 \cos \alpha}$$

$$\therefore \theta_1 + \theta_2 = 90^\circ \quad \therefore \tan \theta_1 = \cot \theta_2$$

$$\therefore \frac{F_2 \sin \alpha}{F_1 + F_2 \cos \alpha} = \frac{F_1 - F_2 \cos \alpha}{F_2 \sin \alpha}$$

$$\therefore F_2^2 \sin^2 \alpha = F_1^2 - F_2^2 \cos^2 \alpha$$

$$\therefore F_2^2 (\sin^2 \alpha + \cos^2 \alpha) = F_1^2$$

$$\therefore F_2^2 = F_1^2 \quad \therefore F_2 = F_1$$

$$\text{Substituting in (3)} : \therefore \cos \alpha = \frac{-2F_1^2}{4F_1^2} = -\frac{1}{2}$$

$$\therefore \alpha = 120^\circ$$

Another solution :

$$\therefore AD = AK = BC = BJ$$

In $\triangle ACJ$:

$$\therefore m(\angle CAJ) = 90^\circ \text{ (given)}$$

and \overline{AB} is a median from the vertex of the right angle

$$\therefore AB = \frac{1}{2}JC = BJ = BC \quad \therefore F_1 = F_2$$

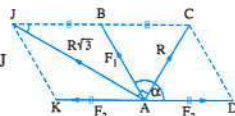
$$\tan(\angle CJA) = \frac{R}{R\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\therefore m(\angle CJA) = 30^\circ \quad \therefore m(\angle ACB) = 60^\circ$$

$\therefore \triangle ABC$ is equilateral triangle

$$\therefore m(\angle ACB) = m(\angle CAB) = m(\angle CAD) = 60^\circ$$

$$\therefore \alpha = 120^\circ$$



Exercise 2

First Multiple choice questions

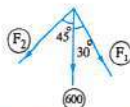
- (1) c (2) d (3) d (4) d (5) c
 (6) a (7) c (8) b (9) b (10) b
 (11) a (12) c (13) b (14) a (15) d
 (16) c (17) c (18) c (19) c (20) a
 (21) a (22) b (23) c (24) b (25) b
 (26) b (27) c (28) c (29) c (30) d

Second Essay questions

1

$$F_1 = \frac{600 \sin 45^\circ}{\sin 75^\circ} \approx 439.23 \text{ gm.wt.}$$

$$F_2 = \frac{600 \sin 30^\circ}{\sin 75^\circ} \approx 310.68 \text{ gm.wt.}$$

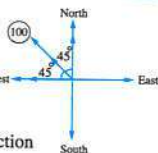


2

The component in the

$$\text{North direction} = 100 \sin 45^\circ = 50\sqrt{2} \text{ gm.wt.}$$

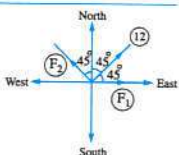
$$\text{The component in the West direction} = 100 \cos 45^\circ = 50\sqrt{2} \text{ gm.wt.}$$



3

$$F_1 = \frac{12 \sin 90^\circ}{\sin 135^\circ} = 12\sqrt{2} \text{ kg.wt.}$$

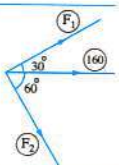
$$F_2 = \frac{12 \sin 45^\circ}{\sin 135^\circ} = 12 \text{ kg.wt.}$$



4

$$F_1 = 160 \cos 30^\circ = 80\sqrt{3} \text{ gm.wt.}$$

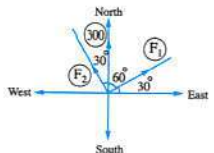
$$F_2 = 160 \sin 30^\circ = 80 \text{ gm.wt.}$$



5

$$F_1 = 300 \cos 60^\circ = 150 \text{ dyne}$$

$$F_2 = 300 \sin 60^\circ = 150\sqrt{3} \text{ dyne}$$



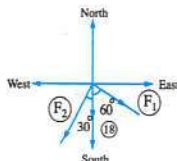
6

$$F_1 = 18 \cos 60^\circ$$

$$= 9 \text{ newton}$$

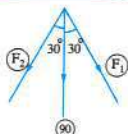
$$F_2 = 18 \sin 60^\circ$$

$$= 9\sqrt{3} \text{ newton}$$



7

$$F_1 = F_2 = \frac{90 \sin 30^\circ}{\sin 60^\circ} = 30\sqrt{3} \text{ newton}$$

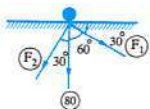


8

$$F_1 = 80 \cos 60^\circ$$

$$= 40 \text{ newton}$$

$$F_2 = 80 \cos 30^\circ = 40\sqrt{3} \text{ newton}$$



9

$$\therefore \tan \alpha = \frac{1}{\sqrt{3}}$$

$$\therefore \alpha = 150^\circ$$

$$\therefore 30 = \frac{R \sin 90^\circ}{\sin 150^\circ}$$

$$\therefore R = \frac{30 \sin 150^\circ}{\sin 90^\circ} = 15 \text{ newton}$$

$$F_1 = \frac{15 \sin 60^\circ}{\sin 150^\circ} = 15\sqrt{3} \text{ newton}$$



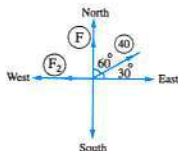
10

$$\therefore F_1 = \frac{F \sin 90^\circ}{\sin 150^\circ}$$

$$\therefore F = \frac{40 \sin 150^\circ}{\sin 90^\circ}$$

$$= 20 \text{ newton}$$

$$F_2 = \frac{20 \sin 60^\circ}{\sin 150^\circ} = 20\sqrt{3} \text{ newton}$$



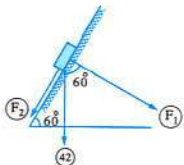
11

$$F_1 = 42 \cos 60^\circ$$

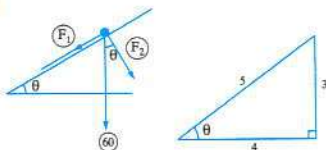
$$= 21 \text{ newton}$$

$$F_2 = 42 \sin 60^\circ$$

$$= 21\sqrt{3} \text{ newton}$$



12



$$F_1 = 60 \sin \theta = 60 \times \frac{3}{5} = 36 \text{ newton}$$

$$F_2 = 60 \cos \theta = 60 \times \frac{4}{5} = 48 \text{ newton}$$

13

$$F_1 = \frac{120 \sin 48^\circ}{\sin (48^\circ + 90^\circ)} \approx 133.27 \text{ gm.wt.}$$

$$F_2 = \frac{120 \sin 90^\circ}{\sin (48^\circ + 90^\circ)} \approx 179.34 \text{ gm.wt.}$$

14

$$(30)^2 = (15\sqrt{3})^2 + (F_2)^2$$

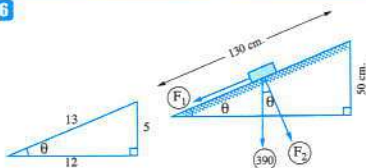
$$F_2 = 15 \text{ newton}$$

15

$$w_1 = w_2 = \frac{20 \sin 85^\circ}{\sin (85^\circ + 85^\circ)} \approx 114.74 \text{ newton}$$

when the angle with the horizontal decreases less than 5° the magnitude of the component of the weight in the direction of the two rods increases till it became unlimited when the rods are horizontal.

16



$$F_1 = 390 \sin \theta = 390 \times \frac{5}{13} = 150 \text{ gm.wt.}$$

$$F_2 = 390 \cos \theta = 390 \times \frac{12}{13} = 360 \text{ gm.wt.}$$

17

T_1 in direction of \overrightarrow{AB}

$$\therefore T_1 = \frac{5000 \times \sin 30^\circ}{\sin 75^\circ} = 2588.2 \text{ N}$$

T_2 in direction of \overrightarrow{AC}

$$\therefore T_2 = \frac{5000 \times \sin 45^\circ}{\sin 75^\circ} = 3660.3 \text{ N}$$

Exercise 3

Important remark :

We will solve the problems of this exercise using the polar angles, but it is possible use the resolution of the forces into two perpendicular directions.

First Multiple choice questions

- (1) c (2) b (3) c (4) d (5) d
 (6) a (7) b (8) c (9) a (10) c
 (11) d (12) c (13) d (14) d
 (15) First : b Second : c (16) b (17) c
 (18) d (19) a (20) a (21) c (22) b
 (23) a (24) b

Second Essay questions

1

$$(1) X = 27 \cos 0^\circ + 18 \cos 90^\circ + 12\sqrt{2} \cos 135^\circ + 15\sqrt{2} \cos 225^\circ + 9 \cos 270^\circ$$

$$= 27 \times 1 + 18 \times 0 + 12\sqrt{2} \times \frac{-1}{\sqrt{2}} + 15 \times \frac{-1}{\sqrt{2}} + 9 \times 0 = \text{zero}$$

$$Y = 27 \sin 0^\circ + 18 \sin 90^\circ + 12\sqrt{2} \sin 135^\circ + 15\sqrt{2} \sin 225^\circ + 9 \sin 270^\circ$$

$$= 27 \times 0 + 18 \times 1 + 12\sqrt{2} \times \frac{1}{\sqrt{2}} + 15\sqrt{2} \times \frac{-1}{\sqrt{2}} + 9 \times -1 = 6$$

$$\therefore \vec{R} = 6 \hat{j}$$

$$\therefore R = 6 \text{ newton}$$

\therefore The magnitude of $\vec{R} = 6$ newton and acts in the direction of \overrightarrow{OY}

$$(2) X = 4 \cos 60^\circ + 3\sqrt{3} \cos 270^\circ + 2\sqrt{3} \cos 330^\circ$$

$$= 4 \times \frac{1}{2} + 3\sqrt{3} \times 0 + 2\sqrt{3} \times \frac{\sqrt{3}}{2} = 5$$

$$Y = 4 \sin 60^\circ + 3\sqrt{3} \sin 270^\circ + 2\sqrt{3} \sin 330^\circ$$

$$= 4 \times \frac{\sqrt{3}}{2} + 3\sqrt{3} \times -1 + 2\sqrt{3} \times \frac{-1}{2} = -2\sqrt{3}$$

$$\therefore \vec{R} = 5 \hat{i} - 2\sqrt{3} \hat{j}$$

$$\therefore R = \sqrt{(5)^2 + (-2\sqrt{3})^2} = \sqrt{37} \text{ newton}$$

$$\tan \theta = \frac{-2\sqrt{3}}{5}$$

$$\therefore \theta = 325^\circ 17'$$

\therefore The magnitude of $\vec{R} = \sqrt{37}$ newton and makes an angle of measure $325^\circ 17'$ with \overrightarrow{OX}

2

Consider \vec{OX} is the direction of the first force.

$$X = 1 \times \cos 0^\circ + 2 \cos 60^\circ + \sqrt{3} \cos 90^\circ$$

$$= 1 \times 1 + 2 \times \frac{1}{2} + \sqrt{3} \times 0 = 2$$

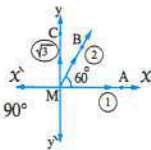
$$Y = 1 \times \sin 0^\circ + 2 \times \sin 60^\circ + \sqrt{3} \sin 90^\circ$$

$$= 1 \times 0 + 2 \times \frac{\sqrt{3}}{2} + \sqrt{3} \times 1 = 2\sqrt{3}$$

$$\therefore \vec{R} = 2\hat{i} + 2\sqrt{3}\hat{j}, R = \sqrt{(2)^2 + (2\sqrt{3})^2} = 4 \text{ newton}$$

$$\therefore \tan \theta = \frac{2\sqrt{3}}{2} = \sqrt{3} \quad \therefore \theta = 60^\circ$$

\therefore The magnitude of $\vec{R} = 4$ newton and its direction is MB



3

Suppose \vec{OX} is the direction of the first force

$$X = 8 \cos 0^\circ + 4\sqrt{3} \cos 30^\circ$$

$$+ 6\sqrt{3} \cos 150^\circ + 14 \cos 240^\circ$$

$$= 8 \times 1 + 4\sqrt{3} \times \frac{\sqrt{3}}{2} + 6\sqrt{3} \times \left(-\frac{\sqrt{3}}{2}\right) + 14 \times \left(-\frac{1}{2}\right)$$

$$= 8 + 6 - 9 - 7 = -2$$

$$Y = 8 \sin 0^\circ + 4\sqrt{3} \sin 30^\circ$$

$$+ 6\sqrt{3} \sin 150^\circ + 14 \sin 240^\circ$$

$$= 8 \times 0 + 4\sqrt{3} \times \frac{1}{2} + 6\sqrt{3} \times \frac{1}{2} + 14 \times \left(-\frac{\sqrt{3}}{2}\right) = -2\sqrt{3}$$

$$\therefore \vec{R} = -2\hat{i} - 2\sqrt{3}\hat{j}$$

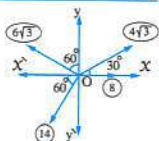
$$\therefore R = \sqrt{(-2)^2 + (-2\sqrt{3})^2} = 4 \text{ newton}$$

$$\therefore \tan \theta = \frac{-2\sqrt{3}}{-2} = \sqrt{3}$$

$$\therefore X \text{ and } Y \text{ are negative} \quad \therefore \theta = 240^\circ$$

\therefore The magnitude of the resultant is 4 newton and makes an angle of measure 240° with \vec{OX}

i.e. In direction of 4th force



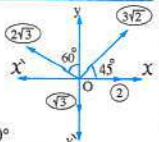
4

Consider \vec{OX} be the direction of first force

$$X = 2 \cos 0^\circ + 3\sqrt{2} \cos 45^\circ$$

$$+ 2\sqrt{3} \cos 150^\circ + \sqrt{3} \cos 270^\circ$$

$$= 2 \times 1 + 3\sqrt{2} \times \frac{1}{\sqrt{2}} + 2\sqrt{3} \times \left(-\frac{\sqrt{3}}{2}\right) + \sqrt{3} \times 0 = 2$$



$$Y = 2 \sin 0^\circ + 3\sqrt{2} \sin 45^\circ + 2\sqrt{3} \sin 150^\circ + \sqrt{3} \sin 270^\circ$$

$$= 2 \times 0 + 3\sqrt{2} \times \frac{1}{\sqrt{2}} + 2\sqrt{3} \times \frac{1}{2} + \sqrt{3} \times -1 = 3$$

$$\therefore \vec{R} = 2\hat{i} + 3\hat{j}, R = \sqrt{(2)^2 + (3)^2} = \sqrt{13} \text{ newton}$$

$$\therefore \tan \theta = \frac{3}{2} \quad \therefore \theta \approx 56^\circ 19'$$

\therefore The magnitude of $\vec{R} = \sqrt{13}$ newton and makes an angle of measure $56^\circ 19'$ with \vec{OX}

i.e. Between 2nd and 3rd forces and makes an angle of measure $11^\circ 19'$ with 2nd force

5

$$X = 9 \cos 0^\circ + 6 \cos 90^\circ$$

$$+ 4\sqrt{2} \cos 135^\circ$$

$$+ 5\sqrt{2} \cos 225^\circ$$

$$+ 5 \cos 270^\circ$$

$$= 9 \times 1 + 6 \times 0 + 4\sqrt{2} \times \left(-\frac{1}{\sqrt{2}}\right) + 5\sqrt{2} \times \left(-\frac{1}{\sqrt{2}}\right) + 5 \times 0 = \text{zero}$$

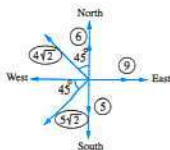
$$Y = 9 \sin 0^\circ + 6 \sin 90^\circ + 4\sqrt{2} \sin 135^\circ + 5\sqrt{2} \sin 225^\circ$$

$$+ 5 \sin 270^\circ$$

$$= 9 \times 0 + 6 \times 1 + 4\sqrt{2} \times \frac{1}{\sqrt{2}} + 5\sqrt{2} \times \left(-\frac{1}{\sqrt{2}}\right) + 5 \times -1 = 0$$

$$\therefore \vec{R} = \vec{0}$$

\therefore The forces are in equilibrium.



6

$$X = 60 \cos 90^\circ + 88 \cos 210^\circ$$

$$+ 60 \cos 330^\circ$$

$$= 60 \times 0 + 88 \times \left(-\frac{\sqrt{3}}{2}\right) + 60 \times \frac{\sqrt{3}}{2} = -14\sqrt{3}$$

$$Y = 60 \sin 90^\circ + 88 \sin 210^\circ + 60 \sin 330^\circ$$

$$= 60 \times 1 + 88 \times \left(-\frac{1}{2}\right) + 60 \times \left(-\frac{1}{2}\right) = -14$$

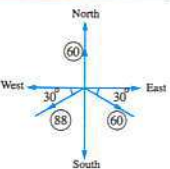
$$\vec{R} = -14\sqrt{3}\hat{i} - 14\hat{j}$$

$$\therefore R = \sqrt{(-14\sqrt{3})^2 + (-14)^2} = 28 \text{ gm.wt.}$$

$$\tan \theta = \frac{-14}{-14\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\therefore X \text{ and } Y \text{ are both negative} \quad \therefore \theta = 210^\circ$$

\therefore The magnitude of $\vec{R} = 28$ gm.wt. and acts in the direction 30° South of the West.



7

$$\therefore X = 4 \cos 0^\circ + 2 \cos 60^\circ$$

$$+ 5 \cos 120^\circ$$

$$+ 3\sqrt{3} \cos 210^\circ$$

$$= 4 \times 1 + 2 \times \frac{1}{2} + 5 \times -\frac{1}{2}$$

$$+ 3\sqrt{3} \times -\frac{\sqrt{3}}{2} = -2$$

$$\therefore Y = 4 \sin 0^\circ + 2 \sin 60^\circ + 5 \sin 120^\circ + 3\sqrt{3} \sin 210^\circ$$

$$= 4 \times 0 + 2 \times \frac{\sqrt{3}}{2} + 5 \times \frac{\sqrt{3}}{2} + 3\sqrt{3} \times -\frac{1}{2} = 2\sqrt{3}$$

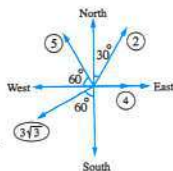
$$\vec{R} = -2\hat{i} + 2\sqrt{3}\hat{j}$$

$$\therefore R = \sqrt{(-2)^2 + (2\sqrt{3})^2} = 4 \text{ newton}$$

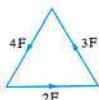
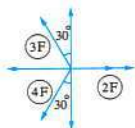
$$\tan \theta = \frac{2\sqrt{3}}{-2} = -\sqrt{3}$$

$$\therefore X < 0, Y > 0$$

$$\therefore \theta = 120^\circ$$



8



Suppose \vec{OX} in the direction of the 1st force.

$$X = 2F \cos 0^\circ + 3F \cos 120^\circ + 4F \cos 240^\circ$$

$$= 2F \times 1 + 3F \times -\frac{1}{2} + 4F \times -\frac{1}{2} = -\frac{3}{2}F$$

$$Y = 2F \sin 0^\circ + 3F \sin 120^\circ + 4F \sin 240^\circ$$

$$= 2F \times 0 + 3F \times \frac{\sqrt{3}}{2} + 4F \times -\frac{\sqrt{3}}{2} = -\frac{\sqrt{3}}{2}F$$

$$\therefore \vec{R} = -\frac{3}{2}F\hat{i} - \frac{\sqrt{3}}{2}F\hat{j}$$

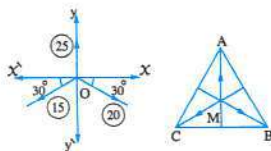
$$\therefore R = \sqrt{\left(-\frac{3}{2}F\right)^2 + \left(-\frac{\sqrt{3}}{2}F\right)^2} = \sqrt{3}F \text{ newton}$$

$$\tan \theta = \frac{-\frac{\sqrt{3}}{2}}{-\frac{3}{2}} = \frac{\sqrt{3}}{3} \therefore \theta = 210^\circ$$

\therefore The magnitude of the resultant is $\sqrt{3}F$ newton and its direction makes an angle of measure 210° with \vec{OX}

i.e. Between the forces $3F$, $4F$ and perpendicular to the force $3F$

9



Let \vec{OY} is the direction of the 3rd force

$$\therefore X = 25 \cos 90^\circ + 15 \cos 210^\circ + 20 \cos 330^\circ$$

$$= 25 \times 0 + 15 \times -\frac{\sqrt{3}}{2} + 20 \times \frac{\sqrt{3}}{2} = \frac{5}{2}\sqrt{3}$$

$$Y = 25 \sin 90^\circ + 15 \sin 210^\circ + 20 \sin 330^\circ$$

$$= 25 \times 1 + 15 \times -\frac{1}{2} + 20 \times -\frac{1}{2} = \frac{15}{2}$$

$$\therefore \vec{R} = \frac{5}{2}\sqrt{3}\hat{i} + \frac{15}{2}\hat{j}$$

$$\therefore R = \sqrt{\left(\frac{5}{2}\sqrt{3}\right)^2 + \left(\frac{15}{2}\right)^2} = 5\sqrt{3} \text{ newton}$$

$$\tan \theta = \frac{7.5}{2.5\sqrt{3}} = \sqrt{3} \therefore \theta = 60^\circ$$

\therefore The resultant = $5\sqrt{3}$ newton in magnitude and makes an angle of measure 60° with \vec{OX}

i.e. Between \vec{MA} , \vec{MB} making an angle of measure 30° with \vec{MA}

10

Let $\vec{YY'}$ is the axis of symmetry

of ΔABC and the point A

Coincides the origin O

$$X = 6\sqrt{3} \cos 0^\circ + 4 \cos 30^\circ + 4 \cos 330^\circ$$

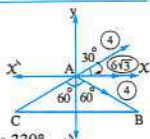
$$= 6\sqrt{3} \times 1 + 4 \times \frac{\sqrt{3}}{2} + 4 \times \frac{\sqrt{3}}{2} = 10\sqrt{3}$$

$$Y = 6\sqrt{3} \sin 0^\circ + 4 \sin 30^\circ + 4 \sin 330^\circ$$

$$= 6\sqrt{3} \times 0 + 4 \times \frac{1}{2} + 4 \times -\frac{1}{2} = \text{zero}$$

$$\therefore \vec{R} = 10\sqrt{3}\hat{i}$$

$$\therefore R = 10\sqrt{3} \text{ newton and acts due to } \vec{OX} \text{ i.e. } \vec{CB}$$



11

Let $\vec{YY'}$ is the symmetric

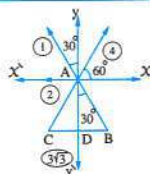
axis of ΔABC , point A

coincides on the origin O

$$\therefore X = 4 \cos 60^\circ$$

$$+ 1 \times \cos 120^\circ + 2 \cos 180^\circ + 3\sqrt{3} \cos 270^\circ$$

$$= 4 \times \frac{1}{2} + 1 \times \left(-\frac{1}{2}\right) + 2 \times -1 + 3\sqrt{3} \times 0 = -\frac{1}{2}$$



i.e. Resultant direction is between \overrightarrow{AC} and \overrightarrow{AD} making an angle of measure 40° with \overrightarrow{AB}

16

Let \overrightarrow{AB} in the direction of \overrightarrow{OX}

$$\begin{aligned} \therefore X &= 2 \cos 0^\circ + 4\sqrt{3} \cos 30^\circ \\ &+ 8 \cos 60^\circ + 2\sqrt{3} \cos 90^\circ \\ &+ 4 \cos 120^\circ \\ &= 2 \times 1 + 4\sqrt{3} \times \frac{\sqrt{3}}{2} + 8 \times \frac{1}{2} \\ &+ 2\sqrt{3} \times 0 + 4 \times \frac{-1}{2} = 10 \end{aligned}$$

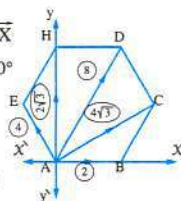
$$\begin{aligned} \therefore Y &= 2 \sin 0^\circ + 4\sqrt{3} \sin 30^\circ + 8 \sin 60^\circ \\ &+ 2\sqrt{3} \sin 90^\circ + 4 \sin 120^\circ \\ &= 2 \times 0 + 4\sqrt{3} \times \frac{1}{2} + 8 \times \frac{\sqrt{3}}{2} + 2\sqrt{3} \times 1 + 4 \times \frac{\sqrt{3}}{2} \\ &= 10\sqrt{3} \end{aligned}$$

$$\therefore \vec{R} = 10\hat{i} + 10\sqrt{3}\hat{j}$$

$$\therefore R = \sqrt{(10)^2 + (10\sqrt{3})^2} = 20 \text{ kg.wt.}$$

$$\therefore \tan \theta = \frac{10\sqrt{3}}{10} = \sqrt{3} \quad \therefore \theta = 60^\circ$$

\therefore The magnitude of $\vec{R} = 20$ kg.wt. and makes an angle of measure 60° with \overrightarrow{AB}



17

Let \overrightarrow{OA} in the direction of \overrightarrow{OX}

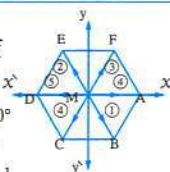
$$\begin{aligned} \therefore X &= 4 \cos 0^\circ + 3 \cos 60^\circ \\ &+ 2 \cos 120^\circ + 5 \cos 180^\circ \\ &+ 4 \cos 240^\circ + \cos 300^\circ \\ &= 4 \times 1 + 3 \times \frac{1}{2} + 2 \times \left(-\frac{1}{2}\right) \\ &+ 5 \times (-1) + 4 \times \left(-\frac{1}{2}\right) + \frac{1}{2} = -2 \end{aligned}$$

$$\begin{aligned} \therefore Y &= 4 \sin 0^\circ + 3 \sin 60^\circ + 2 \sin 120^\circ + 5 \sin 180^\circ \\ &+ 4 \sin 240^\circ + \sin 300^\circ = 4 \times 0 + 3 \times \frac{\sqrt{3}}{2} + 2 \times \frac{\sqrt{3}}{2} \\ &+ 5 \times 0 + 4 \times \left(-\frac{\sqrt{3}}{2}\right) - \frac{\sqrt{3}}{2} = \text{zero} \end{aligned}$$

$$\therefore \vec{R} = -2\hat{i}$$

\therefore The magnitude of the resultant = 2 gm.wt. and acts due to \overrightarrow{OX}

i.e. In the direction of \overrightarrow{OD}



18

\therefore In $\triangle ABC$ which is right-angled at B

$$\therefore AC = \sqrt{(60)^2 + (80)^2} = 100 \text{ cm.}$$

$$\therefore \sin \theta = \frac{80}{100} = \frac{4}{5}$$

$$\therefore \cos \theta = \frac{60}{100} = \frac{3}{5}, \tan \theta = \frac{4}{3}$$

$$\begin{aligned} \therefore X &= 12 \cos 0^\circ + 10 \cos \theta + 15 \cos (180^\circ - \theta) + 8 \cos 270^\circ \\ &= 12 \times 1 + 10 \times \frac{3}{5} + 15 \times \left(-\frac{3}{5}\right) + 8 \times 0 = 9 \end{aligned}$$

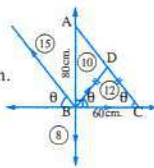
$$\begin{aligned} \therefore Y &= 12 \sin 0^\circ + 10 \sin \theta + 15 \sin (180^\circ - \theta) + 8 \sin 270^\circ \\ &= 12 \times 0 + 10 \times \frac{4}{5} + 15 \times \frac{4}{5} + 8 \times (-1) = 12 \end{aligned}$$

$$\therefore \vec{R} = 9\hat{i} + 12\hat{j}$$

$$\therefore R = \sqrt{(9)^2 + (12)^2} = 15 \text{ newton}, \tan \alpha = \frac{12}{9} = \frac{4}{3}$$

$$\therefore \tan \alpha = \tan \theta, X > 0, Y > 0$$

\therefore The resultant acts in the direction of \overrightarrow{BD}



19

$\therefore \triangle AHB$ is right-angled at B

$$\therefore AH = \sqrt{(5)^2 + (12)^2} = 13 \text{ cm.}$$

$$\therefore \sin (\angle BAH) = \frac{5}{13}$$

$$\therefore \cos (\angle BAH) = \frac{12}{13}$$

$\therefore \overrightarrow{AC}$ is a diagonal of the square ABCD $\therefore \alpha = 45^\circ$

$$\therefore X = 2 \cos 0^\circ + 13 \cos (\angle BAH) + 9 \cos 90^\circ$$

$$\begin{aligned} &+ 4\sqrt{2} \cos 225^\circ \\ &= 2 \times 1 + 13 \times \frac{12}{13} + 9 \times 0 + 4\sqrt{2} \times \frac{-1}{\sqrt{2}} = 10 \end{aligned}$$

$$\therefore Y = 2 \sin 0^\circ + 13 \sin (\angle BAH) + 9 \sin 90^\circ$$

$$\begin{aligned} &+ 4\sqrt{2} \sin 225^\circ \\ &= 2 \times 0 + 13 \times \frac{5}{13} + 9 \times 1 + 4\sqrt{2} \times \frac{-1}{\sqrt{2}} = 10 \end{aligned}$$

$$\therefore \vec{R} = 10\hat{i} + 10\hat{j}$$

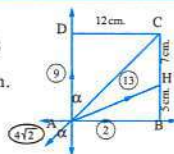
$$\therefore R = \sqrt{(10)^2 + (10)^2} = 10\sqrt{2} \text{ gm.wt.}$$

$$\therefore \tan \theta = \frac{10}{10} = 1$$

$$\therefore X > 0, Y > 0$$

$$\therefore \theta = 45^\circ$$

$\therefore \vec{R}$ acts due to \overrightarrow{AC}

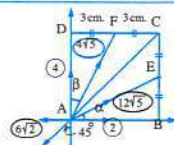


20

$\therefore \overrightarrow{AC}$ is a diagonal in the square

$$\therefore m (\angle CAD) = 45^\circ$$

$\therefore \triangle ABE$ is right-angled at B



$$\therefore AE = \sqrt{(6)^2 + (3)^2} = 3\sqrt{5} \text{ cm.}$$

$$\therefore \sin \alpha = \frac{3}{3\sqrt{5}} = \frac{1}{\sqrt{5}}, \cos \alpha = \frac{6}{3\sqrt{5}} = \frac{2}{\sqrt{5}}$$

in $\triangle AFD$ which is right-angled at D

$$AF = \sqrt{(3)^2 + (6)^2} = 3\sqrt{5} \text{ cm.}$$

$$\therefore \sin \beta = \frac{3}{3\sqrt{5}} = \frac{1}{\sqrt{5}}, \cos \beta = \frac{6}{3\sqrt{5}} = \frac{2}{\sqrt{5}}$$

$$\therefore X = 2 \cos 0^\circ + 12\sqrt{5} \cos \alpha + 4\sqrt{5} \cos (90^\circ - \beta) \\ + 4 \cos 90^\circ + 6\sqrt{2} \cos 225^\circ$$

$$= 2 \times 1 + 12\sqrt{5} \times \frac{2}{\sqrt{5}} + 4\sqrt{5} \times \frac{1}{\sqrt{5}} \\ + 4 \times 0 + 6\sqrt{2} \times \frac{-1}{\sqrt{2}} = 24$$

$$\therefore Y = 2 \sin 0^\circ + 12\sqrt{5} \sin \alpha + 4\sqrt{5} \sin (90^\circ - \alpha) \\ + 4 \sin 90^\circ + 6\sqrt{2} \sin 225^\circ \\ = 2 \times 0 + 12\sqrt{5} \times \frac{1}{\sqrt{5}} + 4\sqrt{5} \times \frac{2}{\sqrt{5}} + 4 \times 1 \\ + 6\sqrt{2} \times \frac{-1}{\sqrt{2}} = 18$$

$$\therefore \vec{R} = 24\hat{i} + 18\hat{j}$$

$$\therefore R = \sqrt{(24)^2 + (18)^2} = 30 \text{ kg.wt.}, \tan \theta = \frac{18}{24} = \frac{3}{4}$$

$$\therefore X > 0, Y > 0 \quad \therefore \theta \approx 36^\circ 52' 12''$$

The magnitude of resultant is 30 kg.wt. and makes an angle of measure $36^\circ 52' 12''$ with \vec{AB}

21

The forces are in equilibrium

$$\therefore X = 0, Y = 0$$

$$\therefore 4 \cos 0^\circ + 4\sqrt{3} \cos (\angle ABE)$$

$$+ F \cos 90^\circ + 10\sqrt{2} \cos 225^\circ \\ = 0$$

$$\therefore 4 \times 1 + 4\sqrt{3} \cos (\angle ABE) + F \times \text{zero} \\ + 10\sqrt{2} \times \frac{-1}{\sqrt{2}} = 0$$

$$\therefore 4 + 4\sqrt{3} \cos (\angle ABE) - 10 = 0$$

$$\therefore \cos (\angle ABE) = \frac{6}{4\sqrt{3}} = \frac{\sqrt{3}}{2}$$

$$\therefore m (\angle ABE) = 30^\circ$$

$$+ 4 \sin 0^\circ + 4\sqrt{3} \sin 30^\circ + F \sin 90^\circ + 10\sqrt{2} \sin 225^\circ \\ = 0$$

$$\therefore 4 \times 0 + 4\sqrt{3} \times \frac{1}{2} + F \times 1 + 10\sqrt{2} \times \frac{-1}{\sqrt{2}} = 0$$

$$\therefore 2\sqrt{3} + F - 10 = 0$$

$$\therefore F = 10 - 2\sqrt{3} = 2(5 - \sqrt{3}) \text{ kg.wt.}$$

22

Let the force of magnitude 5 kg.wt acts in the direction of \vec{OX}

the forces are in equilibrium

$$\therefore X = Y = 0$$

$$\therefore X = 5 \cos 0^\circ + 4 \cos 60^\circ + F \cos 120^\circ \\ + 3 \cos 180^\circ + K \cos 240^\circ + 7 \cos 300^\circ = 0$$

$$\therefore 5 \times 1 + 4 \times \frac{1}{2} + F \times \frac{-1}{2} + 3 \times -1 + K \times \frac{-1}{2} + 7 \times \frac{1}{2} = 0$$

$$\therefore F + K = 15 \quad (1)$$

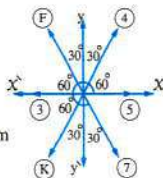
$$\therefore Y = 5 \sin 0^\circ + 4 \sin 60^\circ + F \sin 120^\circ + 3 \sin 180^\circ \\ + K \sin 240^\circ + 7 \sin 300^\circ = 0$$

$$\therefore 5 \times 0 + 4 \times \frac{\sqrt{3}}{2} + F \times \frac{\sqrt{3}}{2} + 3 \times 0 + K \times \frac{-\sqrt{3}}{2} \\ + 7 \times \frac{-\sqrt{3}}{2} = 0$$

$$\therefore F - K = 3 \quad (2)$$

by solving the two equations (1), (2)

$$\therefore F = 9 \text{ kg.wt.}, K = 6 \text{ kg.wt.}$$



23

$$\therefore X = F \cos 0^\circ + 6 \cos 90^\circ$$

$$+ 4\sqrt{2} \cos 135^\circ$$

$$+ 5\sqrt{2} \cos 225^\circ$$

$$+ K \cos 270^\circ$$

$$= F \times 1 + 6 \times 0 + 4\sqrt{2} \times \frac{-1}{\sqrt{2}} \\ + 5\sqrt{2} \times \frac{-1}{\sqrt{2}} + K \times 0 \\ = F - 9$$

$$\therefore Y = F \sin 0^\circ + 6 \sin 90^\circ + 4\sqrt{2} \sin 135^\circ$$

$$+ 5\sqrt{2} \sin 225^\circ + K \sin 270^\circ$$

$$= F \times 0 + 6 \times 1 + 4\sqrt{2} \times \frac{1}{\sqrt{2}} + 5\sqrt{2} \times \frac{-1}{\sqrt{2}} + K \times -1 \\ = 5 - K$$

$$\therefore \vec{R} = (F - 9)\hat{i} + (5 - K)\hat{j}$$

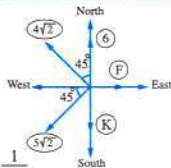
The resultant = 2 newton in the direction of the North

$$\therefore F - 9 = 0$$

$$\therefore F = 9 \text{ newton}$$

$$5 - K = 2$$

$$\therefore K = 3 \text{ newton}$$



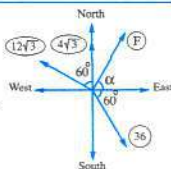
24

Let the measure of the polar angle of \vec{F} be α

$$\therefore X = F \cos \alpha + 4\sqrt{3} \cos 90^\circ$$

$$+ 12\sqrt{3} \cos 150^\circ$$

$$+ 36 \cos 300^\circ$$



$$= F \cos \alpha + 4\sqrt{3} \times 0 + 12\sqrt{3} \times -\frac{\sqrt{3}}{2} + 36 \times \frac{1}{2}$$

$$= F \cos \alpha$$

$$Y = F \sin \alpha + 4\sqrt{3} \sin 90^\circ + 12\sqrt{3} \sin 150^\circ + 36 \sin 300^\circ$$

$$= F \sin \alpha + 4\sqrt{3} \times 1 + 12\sqrt{3} \times \frac{1}{2} + 36 \times -\frac{\sqrt{3}}{2}$$

$$= F \sin \alpha - 8\sqrt{3}$$

$$\therefore \vec{R} = F \cos \alpha \hat{i} + (F \sin \alpha - 8\sqrt{3}) \hat{j}$$

\therefore The magnitude of the resultant = 8 gm.wt.
due to east

$$\therefore F \cos \alpha = 8 \quad (1)$$

$$F \sin \alpha = 8\sqrt{3} \quad (2)$$

$$\text{Dividing (2) by (1): } \therefore \tan \alpha = \frac{8\sqrt{3}}{8} = \sqrt{3}$$

$$\therefore \cos \alpha > 0 \text{ and } \sin \alpha > 0$$

$$\therefore \alpha \text{ lies in the first quadrant } \therefore \alpha = 60^\circ$$

$$\text{substituting in (1): } \therefore F \cos 60^\circ = 8$$

$$\therefore F = 16 \text{ gm.wt. and its direction is } 60^\circ \text{ North of East}$$

25

$$\therefore X = F \cos 0^\circ + 8 \cos 60^\circ$$

$$+ K \cos 90^\circ + 5 \cos 180^\circ$$

$$+ 8\sqrt{3} \cos 270^\circ$$

$$= F \times 1 + 8 \times \frac{1}{2} + K \times 0$$

$$+ 5 \times -1 + 8\sqrt{3} \times 0$$

$$= F - 1$$

$$Y = F \sin 0^\circ + 8 \sin 60^\circ + K \sin 90^\circ + 5 \sin 180^\circ$$

$$+ 8\sqrt{3} \sin 270^\circ$$

$$= F \times 0 + 8 \times \frac{\sqrt{3}}{2} + K \times 1 + 5 \times 0 + 8\sqrt{3} \times -1$$

$$= K - 4\sqrt{3}$$

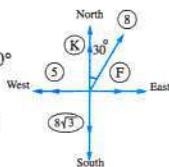
$$\therefore \vec{R} = (F - 1) \hat{i} + (K - 4\sqrt{3}) \hat{j} \quad (1)$$

$$\therefore R = 4 \text{ newton due to } 60^\circ \text{ North of East}$$

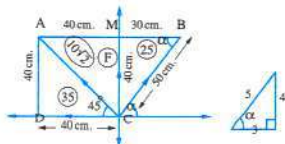
$$\therefore \vec{R} = 4 \cos 60^\circ \hat{i} + 4 \sin 60^\circ \hat{j} = 2\hat{i} + 2\sqrt{3}\hat{j} \quad (2)$$

$$\text{From (1) and (2): } \therefore F - 1 = 2 \quad \therefore F = 3 \text{ newton}$$

$$\therefore K - 4\sqrt{3} = 2\sqrt{3} \quad \therefore K = 6\sqrt{3} \text{ newton}$$



26



$$\therefore X = 25 \cos \alpha + F \cos 90^\circ + 10\sqrt{2} \cos 135^\circ + 35 \cos 180^\circ$$

$$= 25 \times \frac{3}{5} + F \times 0 + 10\sqrt{2} \times \frac{-1}{\sqrt{2}} + 35 \times -1$$

$$= 15 - 10 - 35 = -30$$

$$Y = 25 \sin \alpha + F \sin 90^\circ + 10\sqrt{2} \sin 135^\circ + 35 \sin 180^\circ$$

$$= 25 \times \frac{4}{5} + F \times 1 + 10\sqrt{2} \times \frac{1}{\sqrt{2}} = 20 + F + 10$$

$$= 30 + F$$

$$\therefore R^2 = X^2 + Y^2 \quad \therefore (50)^2 = (-30)^2 + (30 + F)^2$$

$$\therefore 2500 = 900 + 900 + 60F + F^2$$

$$\therefore 2500 = 1800 + 60F + F^2$$

$$\therefore F^2 + 60F - 700 = 0$$

$$\therefore (F + 70)(F - 10) = 0 \quad \therefore F = 10 \text{ gm.wt.}$$

27

$$(1) \therefore X = F \cos 0^\circ + K \cos 120^\circ + 5\sqrt{3} \cos 210^\circ$$

$$+ 7\sqrt{3} \cos 330^\circ = 0$$

$$\therefore F \times 1 + K \times \frac{-1}{2} + 5\sqrt{3} \times \frac{-\sqrt{3}}{2} + 7\sqrt{3} \times \frac{\sqrt{3}}{2} = 0$$

$$\therefore F - \frac{1}{2}K + 3 = 0 \quad (1)$$

$$Y = F \sin 0^\circ + K \sin 120^\circ + 5\sqrt{3} \sin 210^\circ$$

$$+ 7\sqrt{3} \sin 330^\circ = 0$$

$$\therefore F \times 0 + K \times \frac{\sqrt{3}}{2} + 5\sqrt{3} \times \frac{-1}{2} + 7\sqrt{3} \times \frac{-1}{2} = 0$$

$$\therefore \frac{\sqrt{3}}{2}K = 6\sqrt{3} \quad \therefore K = 12 \text{ newton}$$

$$\therefore \text{from (1): } \therefore F = 3 \text{ newton}$$

$$(2) \therefore X = 6\sqrt{3} \cos 0^\circ + F \cos 150^\circ$$

$$+ K \cos 210^\circ = 0$$

$$\therefore 6\sqrt{3} \times 1 + F \times \frac{-\sqrt{3}}{2}$$

$$+ K \times \frac{-\sqrt{3}}{2} = 0$$

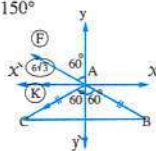
$$\therefore \frac{\sqrt{3}}{2}F + \frac{\sqrt{3}}{2}K = 6\sqrt{3} \quad (1)$$

$$\therefore Y = 6\sqrt{3} \sin 0^\circ + F \sin 150^\circ + K \sin 210^\circ = 0$$

$$\therefore 6\sqrt{3} \times 0 + F \times \frac{1}{2} + K \times \frac{-1}{2} = 0$$

$$\therefore \frac{1}{2}F - \frac{1}{2}K = 0 \quad (2)$$

$$\text{From (1) } \therefore (2) \therefore F = K = 6 \text{ newton}$$



$$(3) \therefore X = F \cos 0^\circ$$

$$+ 4\sqrt{3} \cos 30^\circ \\ + 2\sqrt{3} \cos 90^\circ \\ + K \cos 120^\circ \\ + 12 \cos 240^\circ = 0$$

$$\therefore F \times 1 + 4\sqrt{3} \times \frac{\sqrt{3}}{2}$$

$$+ 2\sqrt{3} \times 0 + K \times \frac{-1}{2} + 12 \times \frac{-1}{2} = 0$$

$$\therefore F - \frac{1}{2}K = 0$$

$$+ Y = F \sin 0^\circ + 4\sqrt{3} \sin 30^\circ + 2\sqrt{3} \sin 90^\circ$$

$$+ K \sin 120^\circ + 12 \sin 240^\circ = 0$$

$$\therefore F \times 0 + 4\sqrt{3} \times \frac{1}{2} + 2\sqrt{3} \times 1 + K \times \frac{\sqrt{3}}{2}$$

$$+ 12 \times \frac{-\sqrt{3}}{2} = 0$$

$$\therefore \frac{\sqrt{3}}{2}K = 2\sqrt{3} \quad \therefore K = 4 \text{ newton}$$

$$\text{From (1)} : \therefore F = 2 \text{ newton}$$

28

$$\therefore X = F \cos 0^\circ + 3\sqrt{2} \cos 45^\circ$$

$$+ 2\sqrt{3} \cos 150^\circ + \sqrt{3} \cos 270^\circ$$

$$= F \times 1 + 3\sqrt{2} \times \frac{1}{\sqrt{2}}$$

$$+ 2\sqrt{3} \times \frac{-\sqrt{3}}{2} + \sqrt{3} \times 0 = F$$

$$+ Y = F \sin 0^\circ + 3\sqrt{2} \sin 45^\circ + 2\sqrt{3} \sin 150^\circ$$

$$+ \sqrt{3} \sin 270^\circ$$

$$= F \times 0 + 3\sqrt{2} \times \frac{1}{\sqrt{2}} + 2\sqrt{3} \times \frac{1}{2} + \sqrt{3} \times -1 = 3$$

$$\therefore \vec{R} = F\hat{i} + 3\hat{j}$$

$$\therefore F^2 = 9$$

$$\therefore \vec{R} = 3\hat{i} + 3\hat{j}$$

$$\therefore \theta = 45^\circ$$

\therefore The angle between the line of action of \vec{R} and the first force is of measure 45°

29

$$\therefore X = 4 \cos 0^\circ + 2\sqrt{3} \cos 30^\circ$$

$$+ F \cos 60^\circ + 2\sqrt{3} \cos 90^\circ$$

$$+ K \cos 120^\circ$$

$$= 4 \times 1 + 2\sqrt{3} \times \frac{\sqrt{3}}{2} + F \times \frac{1}{2}$$

$$+ 2\sqrt{3} \times 0 + K \times \frac{-1}{2} = 7 + \frac{1}{2}F - \frac{1}{2}K$$

$$+ Y = 4 \sin 0^\circ + 2\sqrt{3} \sin 30^\circ + F \sin 60^\circ + 2\sqrt{3} \sin 90^\circ$$

$$+ K \sin 120^\circ$$

$$= 4 \times 0 + 2\sqrt{3} \times \frac{1}{2} + F \times \frac{\sqrt{3}}{2} + 2\sqrt{3} \times 1 + K \times \frac{\sqrt{3}}{2}$$

$$= 3\sqrt{3} + \frac{\sqrt{3}}{2}F + \frac{\sqrt{3}}{2}K$$

$$\therefore \vec{R} = \left(7 + \frac{1}{2}F - \frac{1}{2}K\right)\hat{i} + \left(3\sqrt{3} + \frac{\sqrt{3}}{2}F + \frac{\sqrt{3}}{2}K\right)\hat{j} \quad (1)$$

\therefore The resultant = 20 kg.wt. in the direction \vec{AD}

$$\therefore \vec{R} = 20 \cos 60^\circ \hat{i} + 20 \sin 60^\circ \hat{j} = 10\hat{i} + 10\sqrt{3}\hat{j} \quad (2)$$

From (1) and (2) : $\therefore 7 + \frac{1}{2}F - \frac{1}{2}K = 10$

$$\therefore F - K = 6$$

$$+ 3\sqrt{3} + \frac{\sqrt{3}}{2}F + \frac{\sqrt{3}}{2}K = 10\sqrt{3} \quad \therefore F + K = 14 \quad (4)$$

Adding (3) and (4) : $\therefore 2F = 20$

$\therefore F = 10 \text{ kg.wt.}$, then $K = 4 \text{ kg.wt.}$

30

$\therefore \theta$ is the measure of an acute angle,

$$\sin \theta = \frac{4}{5}$$

$$\therefore \cos \theta = \frac{3}{5}$$

$$\therefore X = 4F \cos (90^\circ - \theta)$$

$$+ 2K \cos (90^\circ + \theta)$$

$$+ 2F \cos (180^\circ + \theta) + K \cos (360^\circ - \theta)$$

$$= 4F \sin \theta - 2K \sin \theta - 2F \cos \theta + K \cos \theta$$

$$= 4F \times \frac{4}{5} - 2K \times \frac{4}{5} - 2F \times \frac{3}{5} + K \times \frac{3}{5}$$

$$= 2F - K$$

$$+ Y = 4F \sin (90^\circ - \theta) + 2K \sin (90^\circ + \theta)$$

$$+ 2F \sin (180^\circ + \theta) + K \sin (360^\circ - \theta)$$

$$= 4F \cos \theta + 2K \cos \theta - 2F \sin \theta - K \sin \theta$$

$$= 4F \times \frac{3}{5} + 2K \times \frac{3}{5} - 2F \times \frac{4}{5} - K \times \frac{4}{5}$$

$$= \frac{4}{5}F + \frac{2}{5}K$$

$$\therefore \vec{R} = (2F - K)\hat{i} + \left(\frac{4}{5}F + \frac{2}{5}K\right)\hat{j} \quad (1)$$

$$\therefore \vec{R} = 8\sqrt{2} \cos 135^\circ \hat{i} + 8\sqrt{2} \sin 135^\circ \hat{j} = -8\hat{i} + 8\hat{j} \quad (2)$$

$$\text{From (1) and (2)} : \therefore 2F - K = -8 \quad (3)$$

$$+ \frac{4}{5}F + \frac{2}{5}K = 8 \quad \text{i.e. } 2F + K = 20 \quad (4)$$

$$\text{Adding (3) and (4)} : \therefore 4F = 12 \quad \therefore F = 3 \text{ newton}$$

and from (3) : $K = 14 \text{ newton}$

31

$$\begin{aligned}\vec{R} &= (5 + a - 14)\hat{i} + (3 + 6 + b)\hat{j} \\ &= (a - 9)\hat{i} + (9 + b)\hat{j} \\ \vec{R} &= (10\sqrt{2}, 135^\circ) \\ \therefore \vec{R} &= 10\sqrt{2} \cos 135^\circ \hat{i} + 10\sqrt{2} \sin 135^\circ \hat{j} \\ &= -10\hat{i} + 10\hat{j} \quad (2) \\ \text{From (1) and (2): } \therefore a - 9 &= -10 \quad \therefore a = -1 \\ 9 + b &= 10 \quad \therefore b = 1\end{aligned}$$

Exercise 4

First Multiple choice questions

- | | | | | |
|--------|--------|--------|--------|--------|
| (1) b | (2) b | (3) b | (4) c | (5) c |
| (6) c | (7) b | (8) c | (9) c | (10) d |
| (11) c | (12) a | (13) c | (14) b | (15) b |
| (16) a | (17) b | (18) a | (19) d | (20) d |
| (21) d | (22) d | (23) b | (24) c | (25) a |
| (26) c | (27) d | (28) b | (29) a | (30) b |
| (31) b | (32) d | (33) a | (34) b | (35) c |
| (36) b | | | | |

Second Essay questions

1

Applying the triangle of forces rule

$$\begin{aligned}\therefore \frac{F_1}{3} &= \frac{F_2}{4} = \frac{75}{5} & F_1 &= \frac{3 \times 75}{5} = 45 \text{ newton} \\ F_2 &= \frac{4 \times 75}{5} = 60 \text{ newton}\end{aligned}$$

2

Applying Lami's rule

$$\begin{aligned}\therefore \frac{60}{\sin 90^\circ} &= \frac{K}{\sin 120^\circ} = \frac{F}{\sin 150^\circ} \\ \therefore F &= 30 \text{ newton} \\ \therefore K &= 30\sqrt{3} \text{ newton}\end{aligned}$$

3

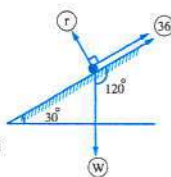
Applying Lami's rule

$$\begin{aligned}\therefore \frac{F}{\sin 150^\circ} &= \frac{r}{\sin 90^\circ} = \frac{12}{\sin 120^\circ} \\ \therefore \frac{F}{\frac{1}{2}} &= \frac{r}{1} = \frac{12}{\frac{\sqrt{3}}{2}} \\ \therefore F &= \frac{12 \times \frac{1}{2}}{\frac{\sqrt{3}}{2}} = 4\sqrt{3} \text{ kg.wt.}, \quad r = \frac{12}{\frac{\sqrt{3}}{2}} = 8\sqrt{3} \text{ kg.wt.}\end{aligned}$$

4

Applying Lami's rule.

$$\begin{aligned}\frac{36}{\sin 150^\circ} &= \frac{W}{\sin 90^\circ} = \frac{r}{\sin 120^\circ} \\ \therefore W &= \frac{36 \sin 90^\circ}{\sin 150^\circ} = 72 \text{ newton} \\ \therefore r &= \frac{36 \sin 120^\circ}{\sin 150^\circ} = 36\sqrt{3} \text{ newton}\end{aligned}$$



5

The three forces are in equilibrium.

The resultant of the two forces \vec{F}_2

and \vec{F}_3 equals in magnitude F_1

$$\begin{aligned}\therefore (8)^2 &= (4\sqrt{3})^2 + (4)^2 + 2 \\ &\times 4\sqrt{3} \times 4 \cos \alpha_1\end{aligned}$$

$$\therefore 64 = 64 + 32\sqrt{3} \cos \alpha_1$$

$$\therefore \cos \alpha_1 = \text{zero}$$

$$\therefore \alpha_1 = 90^\circ$$

The resultant of the two forces \vec{F}_1 and \vec{F}_3 equals F_2 in magnitude.

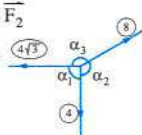
$$\therefore (4\sqrt{3})^2 = (8)^2 + (4)^2 + 2 \times 8 \times 4 \cos \alpha_2$$

$$\therefore 48 = 80 + 64 \cos \alpha_2 \quad \therefore \cos \alpha_2 = -\frac{1}{2}$$

$$\therefore \alpha_2 = 120^\circ$$

$$\therefore \alpha_3 = 360^\circ - (90^\circ + 120^\circ) = 150^\circ$$

The measures of the angles between forces are $90^\circ, 120^\circ, 150^\circ$



6

From the figure

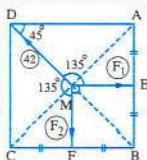
$$\therefore m(\angle EMF) = 90^\circ$$

$$\therefore m(\angle DME) = m(\angle DMF) = 135^\circ$$

Applying Lami's rule

$$\therefore \frac{F_1}{\sin 135^\circ} = \frac{F_2}{\sin 135^\circ} = \frac{42}{\sin 90^\circ}$$

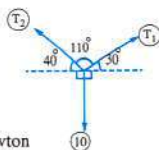
$$\begin{aligned}\therefore \frac{F_1}{\frac{1}{\sqrt{2}}} &= \frac{F_2}{\frac{1}{\sqrt{2}}} = \frac{42}{1} \\ \therefore F_1 &= F_2 = 42 \times \frac{1}{\sqrt{2}} = 21\sqrt{2} \text{ gm.wt.}\end{aligned}$$



7

From the figure and applying lami's rule.

$$\begin{aligned}\therefore \frac{T_1}{\sin 130^\circ} &= \frac{T_2}{\sin 120^\circ} = \frac{10}{\sin 110^\circ} \\ \therefore T_1 &= \frac{10 \times \sin 130^\circ}{\sin 110^\circ} \approx 8.15 \text{ newton} \\ \therefore T_2 &= \frac{10 \times \sin 120^\circ}{\sin 110^\circ} \approx 9.216 \text{ newton}\end{aligned}$$



8

From the figure

$\overline{OD} \perp \overline{AB}$, D is

the midpoint of \overline{AB}

$$\therefore (OD)^2 = (20)^2 - (16)^2 = 144 \quad \therefore OD = 12 \text{ cm.}$$

\therefore D is the midpoint of \overline{AB} , $\overline{DE} \parallel \overline{OB}$

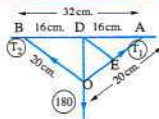
\therefore E is the midpoint of \overline{AO} , $DE = \frac{1}{2} OB$

$\therefore EO = 10 \text{ cm.}, DE = 10 \text{ cm.}$

$\therefore \Delta DOE$ is the triangle of forces

$$\therefore \frac{180}{DO} = \frac{T_1}{OE} = \frac{T_2}{ED} \quad \therefore \frac{180}{12} = \frac{T_1}{10} = \frac{T_2}{10}$$

$$\therefore T_1 = T_2 = \frac{180 \times 10}{12} = 150 \text{ kg.wt.}$$



9

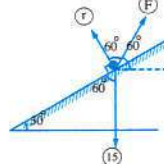
Let the angle between the inclined plane and the horizontal measure θ

$$\therefore \sin \theta = \frac{1}{2} \quad \therefore \theta = 30^\circ$$

Applying lami's rule

$$\therefore \frac{F}{\sin 150^\circ} = \frac{r}{\sin 150^\circ} = \frac{15}{\sin 60^\circ}$$

$$\therefore \frac{F}{\frac{1}{2}} = \frac{r}{\frac{1}{2}} = \frac{15}{\frac{\sqrt{3}}{2}} \quad \therefore F = r = \frac{15 \times \frac{1}{2}}{\frac{\sqrt{3}}{2}} = 5\sqrt{3} \text{ kg.wt.}$$

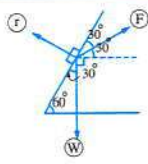


10

Let the measure of the angle between the plane and the horizontal be θ

$$\therefore \cos \theta = \frac{1}{2} \quad \therefore \theta = 60^\circ$$

$$\therefore \frac{F}{\sin 120^\circ} = \frac{r}{\sin 120^\circ} = \frac{W}{\sin 120^\circ} \quad \therefore F = r = W$$



11

$$\therefore (60)^2 + (80)^2 = (100)^2$$

$\therefore \Delta ACB$ is right-angled at C

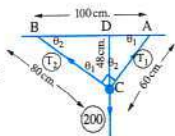
From lami's rule

$$\therefore \frac{200}{\sin 90^\circ} = \frac{T_1}{\sin \theta_1} = \frac{T_2}{\sin \theta_2}$$

$$\therefore \sin \theta_1 = \frac{BC}{AB} = \frac{80}{100} = \frac{4}{5}, \quad \sin \theta_2 = \frac{AC}{AB} = \frac{60}{100} = \frac{3}{5}$$

$$\therefore \frac{200}{1} = \frac{T_1}{\frac{4}{5}} = \frac{T_2}{\frac{3}{5}}$$

$$\therefore T_1 = 200 \times \frac{4}{5} = 160 \text{ gm.wt.}, T_2 = 200 \times \frac{3}{5} = 120 \text{ gm.wt.}$$



12

$$\begin{aligned}AB &= \sqrt{(50)^2 + (120)^2} \\ &= 130 \text{ cm.}\end{aligned}$$

From lami's rule

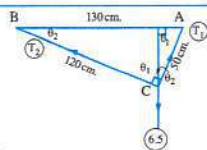
$$\therefore \frac{6.5}{\sin 90^\circ} = \frac{T_1}{\sin \theta_1} = \frac{T_2}{\sin \theta_2}$$

$$\therefore \sin \theta_1 = \frac{BC}{AB} = \frac{120}{130} = \frac{12}{13}, \quad \sin \theta_2 = \frac{AC}{AB} = \frac{50}{130} = \frac{5}{13}$$

$$\therefore \frac{6.5}{1} = \frac{T_1}{\frac{12}{13}} = \frac{T_2}{\frac{5}{13}}$$

$$\therefore T_1 = 6.5 \times \frac{13}{12} = 6 \text{ newton}$$

$$\therefore T_2 = 6.5 \times \frac{5}{13} = 2.5 \text{ newton}$$



13

Suppose that the inclination angles of the two strings to the vertical

be of measures θ_1 and θ_2 from lami's rule.

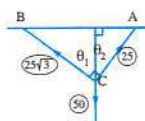
$$\therefore \frac{50}{\sin 90^\circ} = \frac{25}{\sin \theta_1} = \frac{25\sqrt{3}}{\sin \theta_2}$$

$$\therefore \sin \theta_1 = \frac{25 \sin 90^\circ}{50} = \frac{1}{2}$$

$$\therefore \theta_1 = 30^\circ, \quad \sin \theta_2 = \frac{25\sqrt{3} \times \sin 90^\circ}{50} = \frac{\sqrt{3}}{2}$$

$$\therefore \theta_2 = 60^\circ$$

\therefore The two angles of inclination of the strings to the vertical are of measure 30° and 60°

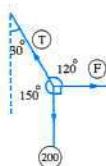


14

From the figure and using lami's rule.

$$\therefore \frac{200}{\sin 120^\circ} = \frac{F}{\sin 150^\circ} = \frac{T}{\sin 90^\circ}$$

$$\therefore \frac{200}{\frac{\sqrt{3}}{2}} = \frac{F}{\frac{1}{2}} = \frac{T}{1}$$



$$\therefore F = \frac{200 \times \frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{200\sqrt{3}}{3} \text{ gm.wt.}$$

$$T = \frac{200 \times 1}{\frac{\sqrt{3}}{2}} = \frac{400\sqrt{3}}{3} \text{ gm.wt.}$$

15

From lami's rule,

$$\begin{aligned} \therefore \frac{60}{\sin(90^\circ + \theta)} &= \frac{F}{\sin(180^\circ - \theta)} = \frac{T}{\sin 90^\circ} \\ \therefore \frac{60}{\cos \theta} &= \frac{F}{\sin \theta} = \frac{T}{1} \\ \therefore \frac{60}{\frac{4}{5}} &= \frac{F}{\frac{3}{5}} = \frac{T}{1} \\ \therefore F &= \frac{60 \times \frac{3}{5}}{\frac{4}{5}} = 45 \text{ gm.wt.}, T = \frac{60}{\frac{4}{5}} = 75 \text{ gm.wt.} \end{aligned}$$

16

Applying lami's rule,

$$\begin{aligned} \therefore \frac{F}{\sin 150^\circ} &= \frac{T}{\sin 120^\circ} = \frac{16}{\sin 90^\circ} \\ \therefore \frac{F}{\frac{1}{2}} &= \frac{T}{\frac{\sqrt{3}}{2}} = 16 \\ \therefore F &= 8 \text{ newton} \\ \therefore T &= 8\sqrt{3} \text{ newton} \end{aligned}$$

17

From the figure and using lami's rule,

$$\begin{aligned} \therefore \frac{600}{\sin 90^\circ} &= \frac{F}{\sin 150^\circ} = \frac{T}{\sin 120^\circ} \\ \therefore \frac{600}{1} &= \frac{F}{\frac{1}{2}} = \frac{T}{\frac{\sqrt{3}}{2}} \\ \therefore F &= 300 \text{ gm.wt.} \quad T = 300\sqrt{3} \text{ gm.wt.} \end{aligned}$$

18

$$(1) \therefore AC = \sqrt{(170)^2 - (80)^2} = 150$$

 $\therefore \triangle ACB$ is the triangle of forces

$$\begin{aligned} \therefore \frac{F}{150} &= \frac{T}{170} = \frac{34}{80} \\ \therefore F &= \frac{34 \times 150}{80} = 63.75 \text{ gm.wt.} \end{aligned}$$

$$T = \frac{34 \times 170}{80} = 72.25 \text{ gm.wt.}$$

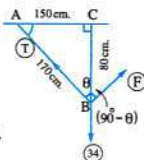
(2) From Lami's rule :

$$\frac{34}{\sin 90^\circ} = \frac{F}{\sin \theta} = \frac{T}{\sin(90^\circ - \theta)}$$

$$\therefore \frac{34}{1} = \frac{F}{\frac{150}{170}} = \frac{T}{\frac{80}{170}}$$

$$\therefore F = 34 \times \frac{150}{170} = 30 \text{ gm.wt.}$$

$$T = 34 \times \frac{80}{170} = 16 \text{ gm.wt.}$$



19

Applying lami's rule,

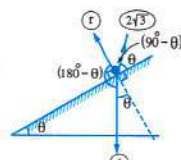
$$\begin{aligned} \frac{r}{\sin(90^\circ + 2\theta)} &= \frac{2\sqrt{3}}{\sin(180^\circ - \theta)} \\ &= \frac{6}{\sin(90^\circ - \theta)} \end{aligned}$$

$$\therefore \frac{r}{\cos 2\theta} = \frac{2\sqrt{3}}{\sin \theta} = \frac{6}{\cos \theta}$$

$$\therefore \frac{2\sqrt{3}}{\sin \theta} = \frac{6}{\cos \theta}$$

$$\therefore \tan \theta = \frac{1}{\sqrt{3}}$$

$$\therefore \frac{r}{\cos 60^\circ} = \frac{2\sqrt{3}}{\sin 30^\circ}$$



$$\therefore \frac{\sin \theta}{\cos \theta} = \frac{2\sqrt{3}}{6} = \frac{\sqrt{3}}{3}$$

$$\therefore \theta = 30^\circ$$

$$\therefore r = 2\sqrt{3} \text{ newton}$$

20

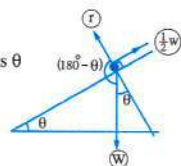
Let the inclination angle of the plane to the horizontal is θ

$$\therefore \frac{W}{\sin 90^\circ} = \frac{\frac{1}{2}W}{\sin(180^\circ - \theta)}$$

$$= \frac{r}{\sin(90^\circ + \theta)}$$

$$\therefore \frac{W}{1} = \frac{\frac{1}{2}W}{\sin \theta} = \frac{r}{\cos \theta}$$

$$\therefore \theta = 30^\circ$$



$$\therefore \sin \theta = \frac{\frac{1}{2}W}{W} = \frac{1}{2}$$

$$\therefore \frac{W}{1} = \frac{r}{\cos 30^\circ}$$

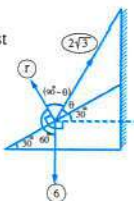
$$\therefore r = W \cos 30^\circ = W \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2} W$$

21

Suppose the string make an angle measures θ with the line of greatest slope of the plane upwards.

$$\begin{aligned} \therefore \frac{2\sqrt{3}}{\sin 150^\circ} &= \frac{6}{\sin(90^\circ - \theta)} \\ &= \frac{r}{\sin(120^\circ + \theta)} \end{aligned}$$

$$\therefore \frac{2\sqrt{3}}{\frac{1}{2}} = \frac{6}{\cos \theta} = \frac{r}{\sin(120^\circ + \theta)}$$



$$\therefore \cos \theta = \frac{6 \times \frac{1}{2}}{2\sqrt{3}} = \frac{\sqrt{3}}{2}$$

$$\therefore \theta = 30^\circ$$

∴ The string makes an angle of measure 30° with the plane.

$$\therefore \frac{2\sqrt{3}}{\frac{1}{2}} = \frac{r}{\sin(120^\circ + 30^\circ)}$$

$$\therefore r = \frac{2\sqrt{3} \sin 150^\circ}{\frac{1}{2}} = \frac{2\sqrt{3} \times \frac{1}{2}}{\frac{1}{2}} = 2\sqrt{3} \text{ kg.wt.}$$

22

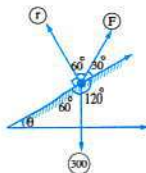
Let the angle between the inclined plane and the horizontal be θ

$$\therefore \tan \theta = \frac{1}{\sqrt{3}} \quad \therefore \theta = 30^\circ$$

$$\therefore \frac{F}{\sin 150^\circ} = \frac{r}{\sin 150^\circ} = \frac{300}{\sin 60^\circ}$$

$$\therefore \frac{F}{\frac{1}{2}} = \frac{r}{\frac{1}{2}} = \frac{300}{\frac{\sqrt{3}}{2}}$$

$$\therefore F = r = \frac{300 \times \frac{1}{2}}{\frac{\sqrt{3}}{2}} = 100\sqrt{3} \text{ gm.wt.}$$



23

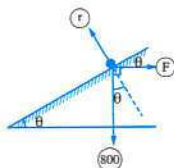
Applying Lami's rule.

$$\therefore \frac{800}{\sin(90^\circ + \theta)} = \frac{r}{\sin 90^\circ} = \frac{F}{\sin(180^\circ - \theta)}$$

$$\therefore \frac{800}{\cos \theta} = \frac{r}{1} = \frac{F}{\sin \theta}$$

$$\therefore r = 800 \times \cos \theta = 800 \times \frac{10}{8} = 1000 \text{ gm.wt.}$$

$$\therefore F = 800 \times \sin \theta = 800 \times \frac{6}{8} = 600 \text{ gm.wt.}$$



24

∴ The ring is smooth.

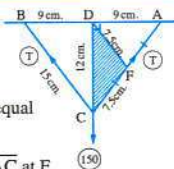
∴ The tensions in the two branches of the string are equal in magnitudes.

Draw $\overline{DF} \parallel \overline{BC}$ and cuts \overline{AC} at F

∴ F is the midpoint of AC

∴ $\triangle CDF$ is the triangle of forces.

$$\therefore \frac{150}{12} = \frac{T}{7.5} = \frac{T}{7.5} \quad \therefore T = \frac{7.5 \times 150}{12} = 93.75 \text{ gm.wt.}$$



25

$$(1) \therefore AB = \sqrt{(130)^2 - (50)^2} = 120 \text{ cm.}$$

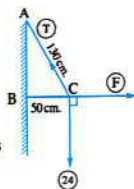
From the figure we get

$\triangle ABC$ is the triangle of forces

$$\therefore \frac{T}{130} = \frac{F}{50} = \frac{24}{120}$$

$$\therefore T = \frac{24 \times 130}{120} = 26 \text{ newton}$$

$$\therefore F = \frac{24 \times 50}{120} = 10 \text{ newton}$$



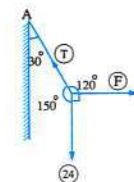
(2) Using Lami's rule :

$$\therefore \frac{T}{\sin 90^\circ} = \frac{F}{\sin 150^\circ} = \frac{24}{\sin 120^\circ}$$

$$\therefore \frac{T}{1} = \frac{F}{\frac{1}{2}} = \frac{24}{\frac{\sqrt{3}}{2}}$$

$$\therefore T = \frac{24}{\frac{\sqrt{3}}{2}} = 16\sqrt{3} \text{ newton}$$

$$F = \frac{24 \times \frac{1}{2}}{\frac{\sqrt{3}}{2}} = 8\sqrt{3} \text{ newton}$$



26

$$AC = \sqrt{(25)^2 - (7)^2} = 24 \text{ cm.}$$

∴ $\triangle ABC$ is the triangle of forces

$$\therefore \frac{72}{AC} = \frac{T_1}{CB} = \frac{T_2}{BA}$$

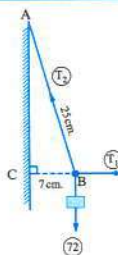
$$\therefore \frac{72}{24} = \frac{T_1}{7} = \frac{T_2}{25}$$

$$\therefore T_1 = \frac{7 \times 72}{24} = 21 \text{ gm.wt.}$$

$$T_2 = \frac{25 \times 72}{24} = 75 \text{ gm.wt.}$$

∴ The tension in the horizontal string = 21 gm.wt.

∴ the tension in each part of the first string 75 and 72 gm.wt.



27

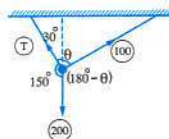
Using Lami's rule

$$\therefore \frac{100}{\sin 150^\circ} = \frac{T}{\sin(180^\circ - \theta)} = \frac{200}{\sin(30^\circ + \theta)}$$

$$\therefore \frac{100}{\frac{1}{2}} = \frac{T}{\sin \theta} = \frac{200}{\sin(30^\circ + \theta)}$$

$$\therefore \sin(30^\circ + \theta) = \frac{200 \times \frac{1}{2}}{100} = 1$$

$$\therefore 30^\circ + \theta = 90^\circ$$



$$\therefore \theta = 60^\circ$$

$$T = \frac{100 \sin 60^\circ}{\frac{1}{2}} = 100\sqrt{3} \text{ gm.wt.}$$

28

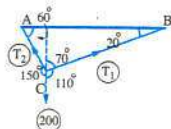
Applying lami's rule :

$$\frac{T_1}{\sin 150^\circ} = \frac{200}{\sin 100^\circ}$$

$$= \frac{T_2}{\sin 110^\circ}$$

$$\therefore T_1 = \frac{200 \sin 150^\circ}{\sin 100^\circ} = 102 \text{ newton}$$

$$T_2 = \frac{200 \sin 110^\circ}{\sin 100^\circ} = 191 \text{ newton.}$$



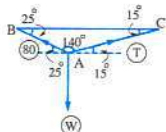
29

From the figure and applying lami's rule :

$$\therefore \frac{T}{\sin 115^\circ} = \frac{W}{\sin 140^\circ}$$

$$= \frac{80}{\sin 105^\circ}$$

$$\therefore T = 75 \text{ newton, } W = 53 \text{ newton.}$$



30

$$T_1 = 20\sqrt{3} \text{ kg.wt.}$$

$$T_2 = 20 \text{ kg.wt.}$$

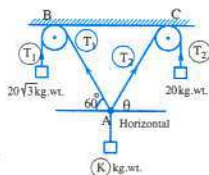
Applying lami's rule :

$$\therefore \frac{20}{\sin 150^\circ} = \frac{20\sqrt{3}}{\sin (90^\circ + \theta)}$$

$$= \frac{K}{\sin (120^\circ - \theta)}$$

$$\therefore \cos \theta = \frac{20\sqrt{3} \times \sin 150^\circ}{20} = \frac{\sqrt{3}}{2} \quad \therefore \theta = 30^\circ$$

$$K = \frac{20 \times \sin 90^\circ}{\sin 150^\circ} = 40 \text{ kg.wt.}$$



Third Higher skills

1

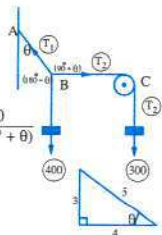
$$T_2 = 300 \text{ gm.wt.}$$

using lami's rule.

$$\therefore \frac{T_1}{\sin 90^\circ} = \frac{300}{\sin (180^\circ - \theta)} = \frac{400}{\sin (90^\circ + \theta)}$$

$$\therefore \frac{T_1}{1} = \frac{300}{\sin \theta} = \frac{400}{\cos \theta}$$

$$\therefore \frac{\sin \theta}{\cos \theta} = \frac{300}{400} = \frac{3}{4}$$



$$\therefore \tan \theta = \frac{3}{4}$$

$$\therefore \theta \approx 36^\circ 52'$$

\therefore The angle of inclination of \overline{AB} to the vertical measures $36^\circ 52'$

$$\therefore \frac{T_1}{1} = \frac{300}{\frac{3}{5}}$$

$$\therefore T_1 = 500 \text{ gm.wt.}$$

2

\therefore The forces whose magnitudes are T_1 , T_2 and K which are meeting at the point C are in equilibrium.

$$\therefore \frac{T_1}{\sin 90^\circ} = \frac{T_2}{\sin 150^\circ} = \frac{K}{\sin 120^\circ} \quad \therefore \frac{T_1}{1} = \frac{T_2}{\frac{1}{2}} = \frac{K}{\frac{\sqrt{3}}{2}}$$

\therefore the forces whose magnitudes are T_2 , T_3 and 20 which are meeting at the point D are in equilibrium

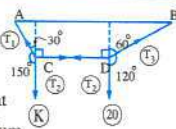
$$\therefore \frac{20}{\sin 150^\circ} = \frac{T_2}{\sin 120^\circ} = \frac{T_3}{\sin 90^\circ} \quad \therefore \frac{20}{\frac{1}{2}} = \frac{T_2}{\frac{\sqrt{3}}{2}} = \frac{T_3}{1}$$

$$\therefore T_2 = \frac{20 \times \frac{\sqrt{3}}{2}}{\frac{1}{2}} = 20\sqrt{3} \text{ gm.wt.}$$

$$T_3 = \frac{20 \times 1}{\frac{1}{2}} = 40 \text{ gm.wt.}$$

$$\text{Substituting in (1)} : \frac{T_1}{1} = \frac{20\sqrt{3}}{\frac{1}{2}} = \frac{K}{\frac{\sqrt{3}}{2}}$$

$$\therefore T_1 = 40\sqrt{3} \text{ gm.wt., } K = \frac{20\sqrt{3} \times \frac{\sqrt{3}}{2}}{\frac{1}{2}} = 60 \text{ gm.wt.}$$



3

Let the length of the string is $2l$ m.

• When fixing the string at the position A , B

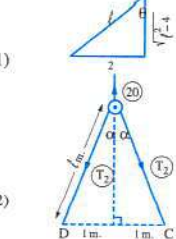
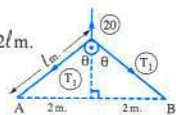
$$\therefore 20 = 2 T_1 \cos \theta$$

$$\therefore T_1 = \frac{10}{\cos \theta} = 10 \times \frac{l}{\sqrt{l^2 - 4}} \quad (1)$$

• When fixing the string at the position C , D

$$\therefore 20 = 2 T_2 \cos \alpha$$

$$\therefore T_2 = \frac{10}{\cos \alpha} = 10 \times \frac{l}{\sqrt{l^2 - 1}} \quad (2)$$



From (1) and (2) :

$$\therefore T_1 > T_2$$

i.e. The tension in the string when fixing at A + B is greater than the tension when fixing at C + D



Exercise 5

First Multiple choice questions

- (1) b (2) b (3) a (4) c (5) b
(6) c (7) c (8) d
(9) First : d Second : a (10) b (11) b
(12) c (13) d

Second Essay questions

1

ΔMAB is the triangle of forces where :

$$MA = 30 + 20 = 50 \text{ cm.}$$

$$BM = 30 \text{ cm.}$$

$$\therefore AB = 40 \text{ cm.}$$

(pythagoras theorem)

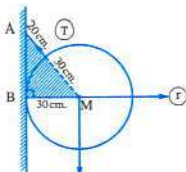
Applying the triangle of forces rule

$$\therefore \frac{r}{BM} = \frac{T}{MA} = \frac{200}{AB}$$

$$\therefore \frac{r}{30} = \frac{T}{50} = \frac{200}{40}$$

$$\therefore r = 200 \times \frac{30}{40} = 150 \text{ gm.wt.}$$

$$T = 200 \times \frac{50}{40} = 250 \text{ gm.wt.}$$



2

\therefore The wall is smooth

$\therefore \hat{r} \perp$ the wall

\therefore The set of forces are in equilibrium.

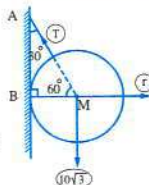
$\therefore \vec{T}$ passes through the point M

Applying lami's rule

$$\therefore \frac{T}{\sin 90^\circ} = \frac{10\sqrt{3}}{\sin 120^\circ} = \frac{r}{\sin 150^\circ}$$

$$\therefore T = 20 \text{ gm.wt.}$$

$$r = 10 \text{ gm.wt.}$$



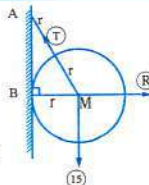
3

\therefore The wall is smooth

$\therefore \vec{R} \perp$ the wall

\therefore The set of forces are in equilibrium.

$\therefore \vec{T}$ passes through the point M



$\therefore \Delta ABM$ is the triangle of forces
where $AM = 2r$, $MB = r$, $AB = \sqrt{3}r$
applying the triangle of forces rule

$$\therefore \frac{T}{2r} = \frac{R}{r} = \frac{15}{\sqrt{3}r}$$

$$\therefore T = 10\sqrt{3} \text{ newton}$$

$$R = 5\sqrt{3} \text{ newton}$$

$$\therefore \text{The pressure on the wall} = 5\sqrt{3} \text{ newton}$$

4

Since the two planes are smooth

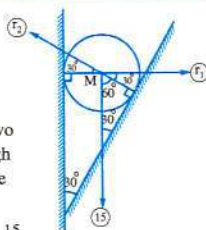
$\therefore r_1$ and r_2 are

perpendicular to the two planes and pass through the center of the sphere
Applying lami's rule

$$\therefore \frac{r_1}{\sin 120^\circ} = \frac{r_2}{\sin 90^\circ} = \frac{15}{\sin 150^\circ}$$

$$\therefore r_1 (\text{The reaction of the vertical plane}) = 15\sqrt{3} \text{ kg.wt.}$$

$$r_2 (\text{The reaction of the inclined plane}) = 30 \text{ kg.wt.}$$



5

\therefore The set of forces are in equilibrium.

\therefore The line of action of the weight passes through the point of meeting of \vec{T}_1 and \vec{T}_2 which is the point C

$$\therefore AC = \frac{1}{2} AB$$

$\therefore \angle ACB$ is right-angle

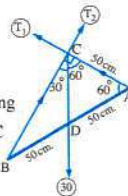
$$\therefore m(\angle B) = 30^\circ \quad m(\angle A) = m(\angle ACD) = 60^\circ$$

\therefore Applying lami's rule

$$\therefore \frac{T_1}{\sin 150^\circ} = \frac{T_2}{\sin 120^\circ} = \frac{30}{\sin 90^\circ}$$

$$\therefore T_1 = 30 \times \frac{1}{2} = 15 \text{ kg.wt.}$$

$$T_2 = 30 \times \frac{\sqrt{3}}{2} = 15\sqrt{3} \text{ kg.wt.}$$



6

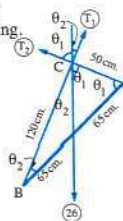
\therefore The two lines of action of tensions are meeting at the point of suspending.

\therefore The line of action of the weight should pass through the same point as shown in the figure

$$\therefore (AC)^2 + (BC)^2 = 16900$$

$$(AB)^2 = 16900$$

$$\therefore m(\angle ACB) = 90^\circ$$



$$\therefore \sin \theta_1 = \frac{12}{13}, \sin \theta_2 = \frac{5}{13}$$

Applying lami's theorem

$$\therefore \frac{T_1}{\sin \theta_1} = \frac{T_2}{\sin \theta_2} = \frac{26}{\sin 90^\circ} \therefore \frac{T_1}{\frac{12}{13}} = \frac{T_2}{\frac{5}{13}} = 26$$

$$\therefore T_1 = 24 \text{ newton} \quad T_2 = 10 \text{ newton}$$

7

The set of forces are in equilibrium.

r passes through the point E

D is the midpoint of \overline{AB}

$\overline{DE} \parallel \overline{AC}$

E is the midpoint of \overline{BC}

$$BC = 60\sqrt{2} \text{ cm.}$$

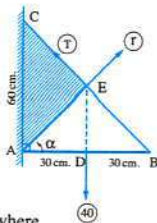
(Pythagoras theorem)

$\triangle AEC$ is the triangle of forces where

$$AE = \frac{1}{2} BC = 30\sqrt{2} \text{ cm.}, EC = 30\sqrt{2} \text{ cm.}$$

$$\therefore AC = 60 \text{ cm.}$$

$$\therefore \frac{r}{30\sqrt{2}} = \frac{T}{30\sqrt{2}} = \frac{40}{60} \therefore r = T = 20\sqrt{2} \text{ newton}$$



8

The set of forces are in equilibrium.

r passes through the point E

$\overline{DE} \parallel \overline{AC}$

E is the midpoint of \overline{BC}

$\overline{AE} \perp \overline{BC}$

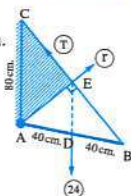
$$\therefore EC = 40\sqrt{3} \quad AC = 80 \text{ cm.}$$

$$\therefore AE = 40 \text{ cm. (Pythagoras theorem)}$$

$\triangle AEC$ is the triangle of forces

$$\therefore \frac{r}{40} = \frac{T}{40\sqrt{3}} = \frac{24}{80} \therefore r = 40 \times \frac{24}{80} = 12 \text{ kg. wt.}$$

$$T = 40\sqrt{3} \times \frac{24}{80} = 12\sqrt{3} \text{ kg. wt.}$$



9

The rod is a tangent to the sphere.

The reaction is perpendicular to the rod

r_1 and r_2 are passing through the centre of the sphere

Applying lami's rule



$$\therefore \frac{r_1}{\sin 150^\circ} = \frac{r_2}{\sin 150^\circ} = \frac{60}{\sin 60^\circ}$$

$$\therefore r_1 = r_2 = 20\sqrt{3} \text{ newton}$$

$$\therefore P_1 = P_2 = 20\sqrt{3} \text{ newton}$$

10

$\triangle MAB$ is the triangle of forces

$$\therefore \frac{50}{MA} = \frac{25}{BM} = \frac{W}{AB}$$

$$\therefore \frac{MA}{MB} = \frac{50}{25} = 2$$

$$\therefore MA = 2 \times 12 = 24$$

The length of the string

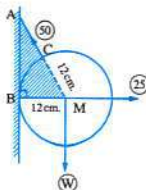
$$\overline{CA} = 24 - 12 = 12 \text{ cm.}$$

From $\triangle MAB$ which is right-angled at B

$$\therefore AB = 12\sqrt{3}$$

$$\therefore \frac{25}{12} = \frac{W}{12\sqrt{3}}$$

$$\therefore W = 25\sqrt{3} \text{ newton}$$



11

The set of forces are in equilibrium.

The line of action of the weight passes through the intersection point of $\overline{T_1}$ and $\overline{T_2}$ which is (C)

The two strings are perpendicular

$\triangle ABC$ is right-angled.

The length of the other string = 64 cm.

$$\therefore \sin \theta_1 = \frac{48}{80} = \frac{3}{5}, \sin \theta_2 = \frac{64}{80} = \frac{4}{5}$$

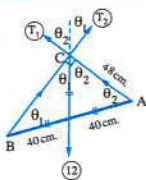
Applying lami's rule

$$\therefore \frac{T_1}{\sin \theta_1} = \frac{T_2}{\sin \theta_2} = \frac{12}{\sin 90^\circ}$$

$$\therefore \frac{T_1}{\frac{3}{5}} = \frac{T_2}{\frac{4}{5}} = 12$$

$$\therefore T_1 = 7.2 \text{ newton}$$

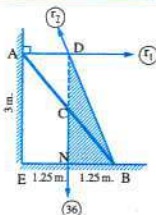
$$T_2 = 9.6 \text{ newton}$$



12

The reaction of the wall (r_1) is perpendicular to it, the weight of the ladder acts vertically downward and they are meeting at D

The reaction of the ground (r_2) should pass through (D)



DN = AE = 3 metres, BN = 1.25 metre

From $\triangle DNB$ which is right angled at N

, then BD = 3.25 metres (Pythagoras)

$\therefore \triangle NBD$ is the triangle of forces, then applying the rule of the triangle of forces.

$$\therefore \frac{r_1}{NB} = \frac{r_2}{BD} = \frac{36}{DN}$$

$$\therefore \frac{r_1}{1.25} = \frac{r_2}{3.25} = \frac{36}{3}$$

$$\therefore r_1 = 15 \text{ kg.wt.}$$

$$\therefore r_2 = 39 \text{ kg.wt.}$$

13

\therefore The set of forces are in equilibrium.

$\therefore \vec{r}$ passes through the point E
BC = 100 cm.

(Pythagoras theorem)

$$\frac{BD}{BA} = \frac{BE}{BC} = \frac{ED}{CA}$$

$$\therefore \frac{2}{3} = \frac{BE}{100} = \frac{ED}{80} \quad \therefore BE = \frac{200}{3} \text{ cm.}$$

$$\therefore ED = \frac{160}{3} \text{ cm.}$$

From $\triangle ADE$: $AE = \frac{20\sqrt{73}}{3} \text{ cm.}$ (Pythagoras)

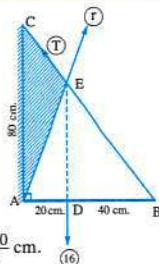
$\therefore \triangle AEC$ is the triangle of forces

Applying the triangle of forces rule.

$$\therefore \frac{r}{\frac{20\sqrt{73}}{3}} = \frac{T}{\frac{100}{3}} = \frac{16}{80}$$

$$\therefore r = \frac{4}{3}\sqrt{73} \text{ kg.wt.}$$

$$T = \frac{20}{3} = 6\frac{2}{3} \text{ kg.wt.}$$



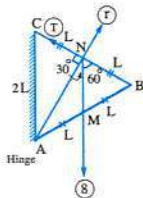
14

\therefore The rod is in equilibrium under the action of three forces meeting at the point N using lami's we get

$$\frac{8}{\sin 90^\circ} = \frac{T}{\sin 150^\circ} = \frac{r}{\sin 120^\circ}$$

$$\therefore \frac{8}{1} = \frac{T}{\frac{1}{2}} = \frac{r}{\frac{\sqrt{3}}{2}}$$

$$\therefore T = 4 \text{ kg.wt.}, r = 4\sqrt{3} \text{ kg.wt.}$$



15

\therefore The set of forces are in equilibrium.

\therefore The weight and the tension are meeting at the point (N)

\therefore The line of action of r should pass through N

$\therefore \overline{DN} \parallel \overline{AC}$, D is the midpoint of \overline{AB}

\therefore N is the midpoint of \overline{BC}

$\therefore \triangle ANC$ is the triangle of forces where NC = 40 cm.
CA = 100 cm.

$$\therefore (AC)^2 = 10000, (AB)^2 + (BC)^2 = 10000$$

$$\therefore m(\angle B) = 90^\circ$$

$\therefore \triangle NBA$ is right-angled.

$$\therefore AN = \sqrt{(40)^2 + (60)^2} = 20\sqrt{13} \text{ cm.}$$

Applying the triangle of forces rule we get

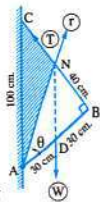
$$\frac{r}{20\sqrt{13}} = \frac{T}{40} = \frac{W}{100} \quad \therefore T = \frac{2}{5}W \text{ kg.wt.}$$

$$r = \frac{\sqrt{13}}{5}W \text{ kg.wt.}$$

Supposing that θ is the measure of the angle between \vec{r} and the rod

\therefore In $\triangle NBA$ which is right-angled at B

$$\therefore \tan \theta = \frac{40}{60} = \frac{2}{3} \quad \therefore \theta \approx 33^\circ 41'$$



16

In $\triangle ADC$:

$$AD = \sqrt{(50)^2 - (30)^2} = 40 \text{ cm.}$$

\therefore the set of forces are in equilibrium.

$\therefore \vec{r}$ passes through N

$\triangle CAD \sim \triangle CMN$ (because of the equality of measures of their corresponding angles)

$$\therefore \frac{CA}{CM} = \frac{AD}{MN} = \frac{CD}{CN} \quad \therefore \frac{30}{15} = \frac{40}{MN} = \frac{50}{CN}$$

$$\therefore MN = 20 \text{ cm.}, CN = 25 \text{ cm.}$$

$$\therefore DN = 50 + 25 = 75 \text{ cm.}$$

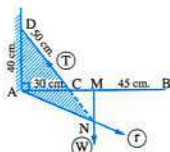
From $\triangle AMN$ we get $AN = 5\sqrt{97} \text{ cm.}$ (Pythagoras)

$\therefore \triangle AND$ is the triangle of forces.

$$\therefore \frac{T}{75} = \frac{r}{5\sqrt{97}} = \frac{W}{40}$$

$$\therefore T = \frac{15}{8}W \text{ kg.wt.}$$

$$r = \frac{\sqrt{97}}{8}W \text{ kg.wt.}$$



17

∴ The set of forces are in equilibrium.

∴ \vec{r} passes through the point E

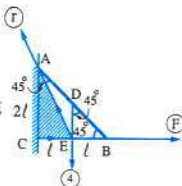
suppose that $AC = 2l$

∴ $BC = 2l$ ∴ $EC = l$

$AE = \sqrt{5}l$ (Pythagoras)

∴ $\triangle AEC$ is the triangle of forces and applying the triangle of forces rule

$$\therefore \frac{4}{2l} = \frac{r}{\sqrt{5}l} = \frac{F}{l} \quad \therefore F = 2 \text{ kg.wt.}, r = 2\sqrt{5} \text{ kg.wt.}$$



18

Let the weight of the rod be $(2W)$

∴ The set of forces are in equilibrium.

∴ \vec{r} passes through the point N

∴ $\triangle ANC$ is the triangle of forces

Applying the rule of the triangle of forces

$$\therefore \frac{W}{l} = \frac{2W}{AC} = \frac{r}{AN} \quad \therefore AC = 2l$$

∴ E is the midpoint of \overline{AB} , $\overline{EN} \parallel \overline{AC}$

∴ N is the midpoint of \overline{BC} ∴ $BC = 2l$

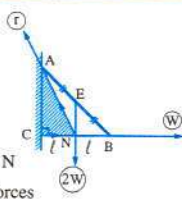
∴ $m(\angle BAC) = 45^\circ$

∴ The rod inclines to the vertical at an angle of measure 45°

From $\triangle ANC$ ∴ $AN = \sqrt{5}l$ (Pythagoras)

$$\therefore \frac{W}{l} = \frac{r}{\sqrt{5}l} \quad \therefore r = \sqrt{5}W$$

∴ The reaction = $\frac{\sqrt{5}}{2} \times$ (The weight of the rod)



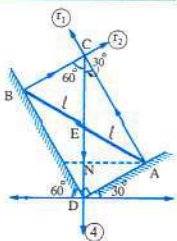
19

∴ The set of forces are in equilibrium.

∴ The line of action of the weight passes through the point of meeting of the two reactions at (C)

∴ The angle between the two planes is right and each of

the two reactions is perpendicular to the plane going out it.



∴ ADBC is a rectangle.

$$\therefore m(\angle ACB) = 90^\circ$$

Applying lami's rule

$$\therefore \frac{4}{\sin 90^\circ} = \frac{r_1}{\sin 120^\circ} = \frac{r_2}{\sin 150^\circ}$$

$$\therefore r_1 = 2\sqrt{3} \text{ newton}, r_2 = 2 \text{ newton}$$

$$\therefore P_1 = 2\sqrt{3} \text{ newton}, P_2 = 2 \text{ newton}$$

∴ $EA = ED$ (properties of the rectangle)

$$\therefore m(\angle ADE) = 60^\circ$$

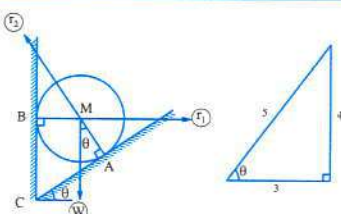
∴ $\triangle AED$ is an equilateral triangle.

$$\therefore m(\angle EAD) = 60^\circ$$

$$\therefore m(\angle EAN) = 30^\circ$$

∴ The rod inclines to the horizontal at an angle of measure 30°

20



∴ \vec{r}_1 is perpendicular to \overline{BC}

\vec{r}_2 is perpendicular to \overline{CA}

They are meeting at M where the weight of the sphere acts

Applying lami's rule we get

$$\frac{r_1}{\sin(180^\circ - \theta)} = \frac{r_2}{\sin 90^\circ} = \frac{W}{\sin(90^\circ + \theta)}$$

$$\therefore \frac{r_1}{\sin \theta} = r_2 = \frac{W}{\cos \theta} \quad \therefore \frac{r_1}{\frac{4}{5}} = r_2 = \frac{W}{\frac{3}{5}}$$

$$\therefore r_1 = \frac{4}{3} W \text{ kg.wt.}, r_2 = \frac{5}{3} W \text{ kg.wt.}$$

∴ The pressure on the wall = $\frac{4}{3} W \text{ kg.wt.}$

∴ the pressure on the inclined plane = $\frac{5}{3} W \text{ kg.wt.}$

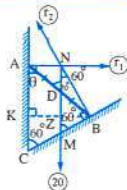
21

∴ The set of forces are in equilibrium and \vec{r}_1 and \vec{r}_2 are meeting at N

∴ The weight of the rod passes through N

∴ $\overline{NM} \parallel \overline{AC}$

$$\therefore m(\angle NMB) = m(\angle C) = 60^\circ$$



$$\therefore \text{In } \triangle NMB : m(\angle B) = 90^\circ, m(\angle M) = 60^\circ$$

$$\therefore m(\angle MNB) = 30^\circ$$

Applying lami's rule we get

$$\therefore \frac{r_1}{\sin 150^\circ} = \frac{r_2}{\sin 90^\circ} = \frac{20}{\sin 120^\circ} \therefore \frac{r_1}{\frac{1}{2}} = \frac{r_2}{1} = \frac{20}{\frac{\sqrt{3}}{2}}$$

$$\therefore r_1 = \frac{20\sqrt{3}}{3} \text{ kg.wt.}, r_2 = \frac{40\sqrt{3}}{3} \text{ kg.wt.}$$

Drawing $BK \perp AC$

$\therefore BZ \perp NM$

Supposing that : $BZ = \ell$

$$\therefore BN = 2\ell, NZ = \sqrt{3}\ell$$

$$\therefore \triangle BZD \equiv \triangle AND$$

$$\therefore ND = \frac{\sqrt{3}}{2}\ell$$

$$AN = BZ = \ell$$

$$\therefore AK = \ell\sqrt{3}, BK = 2\ell$$

$$\therefore \tan(\angle BAK) = \tan \theta = \frac{BK}{AK} = \frac{2\ell}{\ell\sqrt{3}} = \frac{2}{\sqrt{3}}$$

$$\therefore \theta \approx 49^\circ 6'$$

22

$$\therefore m(\angle ACB) = m(\angle DAC) \\ = m(\angle CBD) \\ = 90^\circ$$

\therefore The figure ACBD is a rectangle the rod is in equilibrium under the action of three forces, and D is the meeting point of forces

Assuming that the angle of inclination of the plane on which the end A rests is θ_1 and the angle of inclination of the plane on which the end B rests is θ_2 and using lami's rule

$$\therefore \frac{r}{\sin(90^\circ + \theta_1)} = \frac{4}{\sin(90^\circ + \theta_2)} = \frac{8}{\sin 90^\circ}$$

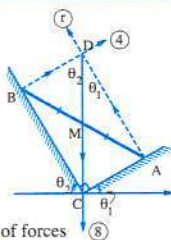
$$\therefore \frac{r}{\cos \theta_1} = \frac{4}{\cos \theta_2} = 8 \therefore \cos \theta_2 = \frac{1}{2}$$

$$\therefore \theta_2 = 60^\circ$$

$$\therefore \theta_1 = 30^\circ$$

$$\therefore r = 8 \cos 30^\circ = 4\sqrt{3} \text{ newton}$$

$$\therefore \text{The pressure on the plane at A} = 4\sqrt{3} \text{ newton}$$



23

\therefore The inclined plane is smooth

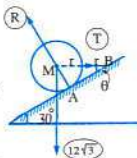
\therefore The reaction of the plane at A is perpendicular to the plane

\therefore It passes through the centre of the sphere (M)

\therefore The set of forces are in equilibrium

\therefore The tension in the string passes through the point M

$$\text{In } \triangle MAB : m(\angle A) = 90^\circ, AM = \frac{1}{2} BM$$



$$\therefore \theta = 30^\circ$$

$\therefore m(\angle MBA)$ = the measure of the angle of inclination of the plane to the horizontal.

\therefore The string MB is horizontal

Applying lami's rule we get

$$\frac{R}{\sin 90^\circ} = \frac{T}{\sin 150^\circ} = \frac{12\sqrt{3}}{\sin 120^\circ} \therefore \frac{R}{1} = \frac{T}{\frac{1}{2}} = \frac{12\sqrt{3}}{\frac{\sqrt{3}}{2}}$$

$$\therefore R = 2T = 24$$

\therefore Tension in the string $T = 12 \text{ kg.wt.}$

The reaction of the plane $R = 24 \text{ kg.wt.}$

24

Let the weight of the rod = $2W$

\therefore the weight of the body = W

\therefore The set of forces are in equilibrium.

$\therefore \hat{r}$ passes through the point of meeting of the weight and the tension in the string (D)

$\therefore N$ is the midpoint of \overline{AB} , $\overline{ND} \parallel \overline{AC}$

$\therefore D$ is the midpoint of \overline{BC}

$$\therefore AB = AC$$

$$\therefore \overline{AD} \perp \overline{BC}$$

Applying lami's rule

$$\therefore \frac{r}{\sin(180^\circ - \theta)} = \frac{2W}{\sin 90^\circ} = \frac{W}{\sin(90^\circ + \theta)}$$

$$\therefore 2W = \frac{W}{\cos \theta} \therefore \cos \theta = \frac{1}{2}$$

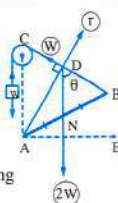
$$\therefore \theta = 60^\circ$$

$$\therefore m(\angle BCA) = 60^\circ, AB = AC$$

$\therefore \triangle ABC$ is an equilateral triangle.

$$\therefore m(\angle BAC) = 60^\circ \therefore m(\angle BAE) = 30^\circ$$

\therefore The measure of the inclination angle of the rod to the horizontal = 30°



25

\therefore The wall is smooth.

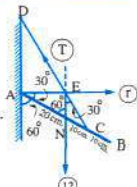
$\therefore \hat{r}$ is perpendicular to the wall

\therefore The set of forces are in equilibrium.

\therefore The line of action of tension

passes through the point of meeting the reaction and the weight (E)

$$EN = \frac{1}{2} AN = 10 \text{ cm.}$$



$\therefore \angle ENA$ is an exterior angle of the isosceles triangle ENC where $EN = NC = 10$ cm.

$\therefore m(\angle CEN) = 30^\circ$

Applying lami's rule

$$\therefore \frac{r}{\sin 150^\circ} = \frac{T}{\sin 90^\circ} = \frac{12}{\sin 120^\circ}$$

$\therefore r = 4\sqrt{3}$ newton, $T = 8\sqrt{3}$ newton

26

$\therefore AC = AD = 4$ metres

$\therefore m(\angle ACD) = 45^\circ$

$\therefore CD = 4\sqrt{2}$ metres

From $\triangle MCN$

$MN = 1$ metre, $NC = \sqrt{2}$ metres

$DN = 4\sqrt{2} - \sqrt{2} = 3\sqrt{2}$ metres

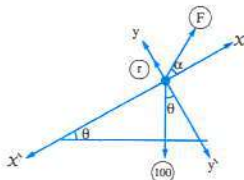
From $\triangle AMN$: $AN = \sqrt{10}$ metres (Pythagoras)

Since $\triangle AND$ is the triangle of forces

$$\therefore \frac{r}{\sqrt{10}} = \frac{T}{3\sqrt{2}} = \frac{8}{4}$$

$\therefore r = 2\sqrt{10}$ kg.wt., $T = 6\sqrt{2}$ kg.wt.

27



\therefore The body is in equilibrium $\therefore X = 0, Y = 0$

$\therefore F \cos \alpha + r \cos 90^\circ + 100 \cos (270^\circ - \theta) = 0$

$\therefore F \times \frac{12}{13} + r \times 0 + 100 \times (-\sin \theta) = 0$

$\therefore \frac{12}{13} F - 100 \times \frac{5}{13} = 0$

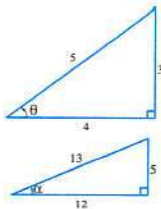
$\therefore F = 65$ newton

$\therefore F \sin \alpha + r \sin 90^\circ + 100 \sin (270^\circ - \theta) = 0$

$\therefore 65 \times \frac{5}{13} + r \times 1 + 100 \times -\cos \theta = 0$

$\therefore 25 + r - 100 \times \frac{4}{5} = 0$

$\therefore r = 55$ newton



28

\therefore The body is in equilibrium

$\therefore X = 0, Y = 0$

$\therefore 50 \cos 0^\circ + 20\sqrt{3} \cos 30^\circ$

$+ r \cos 90^\circ$

$+ W \cos 240^\circ = 0 \quad X$

$\therefore 50 \times 1 + 20\sqrt{3} \times \frac{\sqrt{3}}{2} + r \times 0 + W \times -\cos 60^\circ = 0$

$\therefore 50 + 30 + 0 - \frac{1}{2} W = 0$

$\therefore W = 160$ newton

$\therefore 50 \sin 0^\circ + 20\sqrt{3} \sin 30^\circ + r \sin 90^\circ + W \sin 240^\circ = 0$

$\therefore 50 \times 0 + 20\sqrt{3} \times \frac{1}{2} + r \times 1 - W \sin 60^\circ = 0$

$0 + 10\sqrt{3} + r - 160 \times \frac{\sqrt{3}}{2} = 0$

$\therefore r = 70\sqrt{3}$ newton



29

$\triangle ABC$ in which

$m(\angle B) = 90^\circ, AC + CB = 40$

$\therefore AC = 40 - CB$

$\therefore (AC)^2 = (AB)^2 + (BC)^2$

$\therefore (40 - BC)^2 = (20)^2 + (BC)^2$

$\therefore 1600 - 80BC + (BC)^2 = 400 + (BC)^2$

$\therefore 80BC = 1200$

$\therefore BC = 15$ cm., $AC = 25$ cm.

\therefore The ring is smooth

\therefore The tension in \overline{CA} = the tension in $\overline{CB} = T$

\therefore The ring is in equilibrium $\therefore X = 0, Y = 0$

$\therefore T \cos (90^\circ - \theta) + T \cos 90^\circ + F \cos 180^\circ + 400 \cos 270^\circ = 0$

$\therefore T \sin \theta + T \times 0 + F \times -1 + 400 \times 0 = 0$

$\therefore T \times \frac{20}{25} - F = 0 \quad \therefore 4T - 5F = 0 \quad (1)$

$\therefore T \sin (90^\circ - \theta) + T \sin 90^\circ + F \sin 180^\circ$

$+ 400 \sin 270^\circ = 0$

$\therefore T \cos \theta + T + F \times 0 + 400 \times -1 = 0$

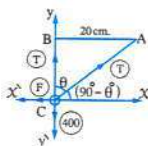
$\therefore T \times \frac{15}{25} + T - 400 = 0 \quad \therefore \frac{8}{5} T = 400$

$\therefore T = 250$ gm.wt.

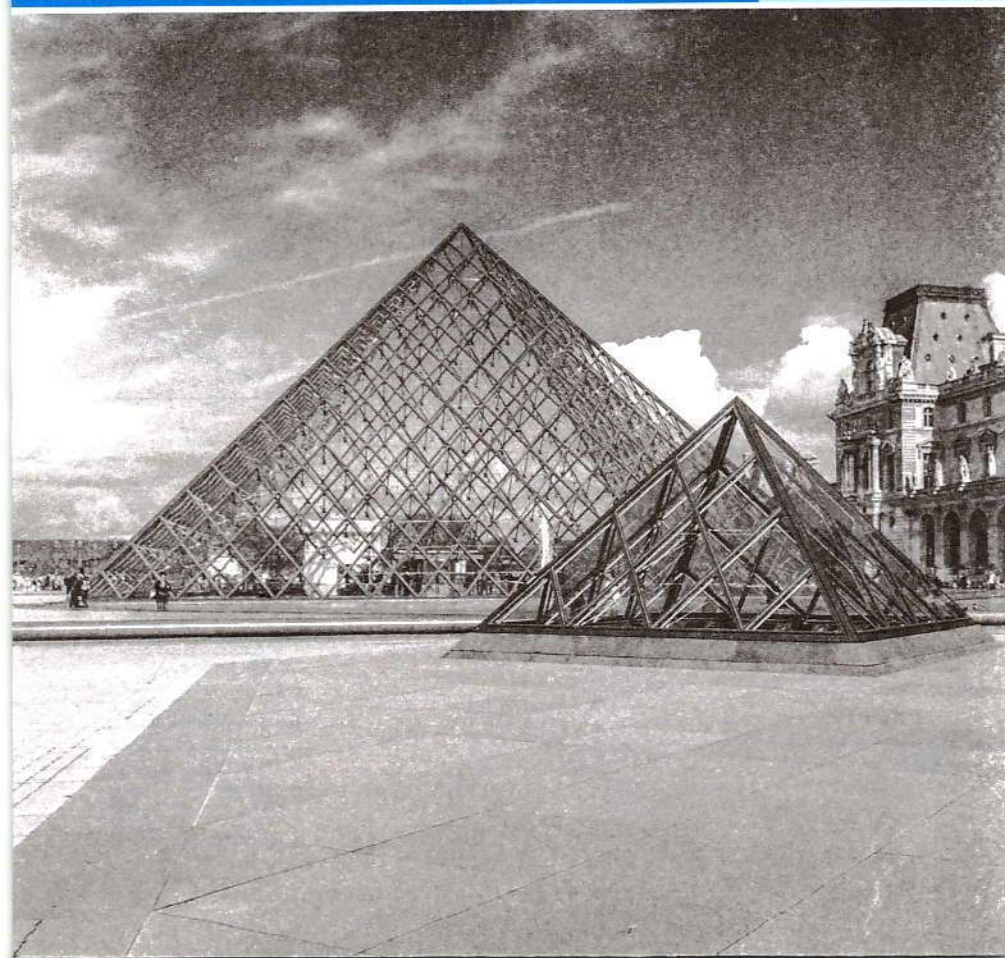
substituting in (1):

$\therefore 1000 - 5F = 0$

$\therefore F = 200$ gm.wt.



Answers of Unit Two



Exercise 6

First

Multiple choice questions

- (1) d (2) a (3) d (4) a (5) d
 (6) d (7) c (8) a (9) d (10) d
 (11) a (12) b (13) a (14) b (15) b
 (16) d (17) c (18) c (19) a (20) c
 (21) c (22) b (23) c (24) d (25) d
 (26) d (27) a (28) d (29) d (30) b
 (31) b (32) a (33) c
 (34) First : c Second : a
 Third : b Fourth : d
 (35) First : a Second : b
 Third : c Fourth : c
 (36) First : b Second : d
 Third : c Fourth : d
 (37) First : a Second : b
 Third : c Fourth : a
 Fifth : b
 (38) First : a Second : d
 Third : c
 (39) First : b Second : d
 Third : d Fourth : c
 Fifth : d
 (40) First : a Second : b
 Third : c Fourth : c
 (41) c

Second

Essay questions

1

- (1) 8 line segments.
 (2) \overline{AB} , \overline{AD} , \overline{AM}
 (3) 5 planes
 (4) The planes : $ABCD$, ABM , ADM

2

- (1) \overline{AB} , \overline{AD} , \overline{AA}
 (2) \overline{AB}

- (3) $ABB\hat{A}$, $ABCD$, $ADD\hat{A}$
 (4) $ABB\hat{A}$, $ABCD$, $AB\hat{C}D$

3

- (1) \overline{AD} , \overline{AA} , \overline{BC} , \overline{BB}
 (2) \overline{AB} , \overline{DC} , \overline{DC}
 (3) \overline{AD} , \overline{BC} , \overline{DD} , \overline{CC}

4

- (1) infinite (2) infinite
 (3) infinite (4) only one plane

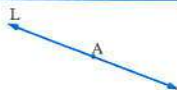
5

- (1) ① skew ② parallel ③ skew
 ④ parallel ⑤ skew ⑥ intersecting
 (2) ① parallel ② intersecting ③ intersecting

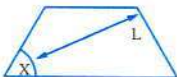
$$(3) \sqrt{(6\sqrt{2})^2 + (6)^2} = 6\sqrt{3} \text{ cm.}$$

6

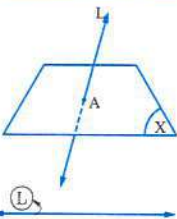
- (1) $A \in L$



- (2) $L \subset X$



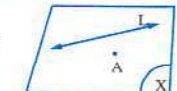
- (3) $L \cap X = \{A\}$



- (4) $L \parallel X$



- (5) $A \in X$, $A \notin L$, $L \subset X$



Exercise 7

First Multiple choice questions

- (1) c (2) b (3) b (4) b (5) b
 (6) d (7) b (8) c (9) d (10) b
 (11) c (12) b (13) b (14) b (15) b
 (16) c (17) c (18) b (19) b (20) c
 (21) d (22) a (23) c (24) d (25) a
 (26) b (27) a (28) a (29) d (30) b
 (31) c (32) c (33) d (34) a (35) b
 (36) d (37) b (38) c (39) a (40) a
 (41) c (42) b (43) c (44) a (45) c
 (46) c (47) d (48) c (49) c (50) d
 (51) b

Second Essay questions

- 1 (1) 5 (2) 6 (3) 5

- (4) 10 (5) 6

- ∴ number of faces + number of vertices
 $= 6 + 6 = 12$
 ∴ number of edges + 2 = 10 + 2 = 12
 ∴ Achieve Euler's rule

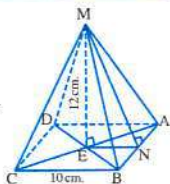
- 2 ∴ ABCD is a square
 of side length 10 cm.

- ∴ NE = 5 cm.

- ∴ the pyramid is regular

- ∴ $\overline{ME} \perp \overline{NE}$

- ∴ The slant height (MN)
 $= \sqrt{5^2 + 12^2} = 13$ cm.



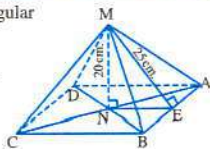
- 3 ∴ The pyramid is regular

- $\overline{MN} \perp \overline{NE}$

- ∴ NE = $\sqrt{25^2 - 20^2}$
 $= 15$ cm.

- ∴ BC = 30 cm.

- ∴ The length of the base side = 30 cm.



- 4 ∴ The pyramid is regular

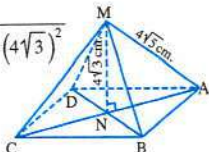
- ∴ $\overline{MN} \perp \overline{AN}$

- ∴ AN = $\sqrt{(4\sqrt{5})^2 - (4\sqrt{3})^2}$
 $= 4\sqrt{2}$ cm.

- ∴ AC = $8\sqrt{2}$ cm.

- (The square diagonal)

- ∴ The square side length = $\frac{8\sqrt{2}}{\sqrt{2}} = 8$ cm.

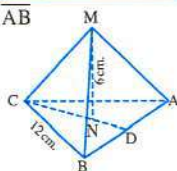


- 5 Let D be the midpoint of \overline{AB}

- ∴ $\triangle ABC$ is equilateral

- ∴ $\overline{CD} \perp \overline{AB}$

- ∴ CD = $\sqrt{(12)^2 - (6)^2}$
 $= 6\sqrt{3}$ cm.



- ∴ N is the intersection point of the medians of $\triangle ABC$

- ∴ NC = $\frac{2}{3} \times 6\sqrt{3} = 4\sqrt{3}$ cm.

- ∴ $\triangle MNC$ is a right-angled at N

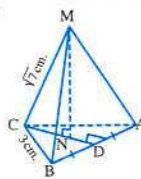
- ∴ MC = $\sqrt{(6)^2 + (4\sqrt{3})^2} = 2\sqrt{21}$ cm.

- 6 Let D be the midpoint of \overline{AB}

- ∴ $\triangle ABC$ is equilateral

- ∴ $\overline{CD} \perp \overline{AB}$

- ∴ CD = $\sqrt{(3)^2 - (1.5)^2}$
 $= \frac{3}{2}\sqrt{3}$ cm.



- ∴ the pyramid is regular

- ∴ $\overline{MN} \perp \overline{NC}$

- ∴ N is the intersection point of the medians
 of $\triangle ABC$ ∴ CN = $\sqrt{3}$ cm.

- ∴ $\triangle MNC$ is right-angled at N

- ∴ MN = $\sqrt{(\sqrt{7})^2 - (\sqrt{3})^2} = 2$ cm.
 ∴ The pyramid height = 2 cm.

- 7 Let D is the midpoint of \overline{AB}

ΔABC is an equilateral

$$\therefore CD = 6\sqrt{3} \text{ cm.}$$

\therefore The pyramid of regular faces.

$$\therefore \overline{MN} \perp \overline{NC}$$

$\therefore N$ is the intersection point of medians of ΔABC

$$CN = 4\sqrt{3} \text{ cm.}$$

$\therefore \Delta MNC$ is right-angled at N

$$\therefore MN = \sqrt{(12)^2 - (4\sqrt{3})^2} = 4\sqrt{6} \text{ cm.}$$

\therefore The height of the pyramid = $4\sqrt{6} \text{ cm.}$

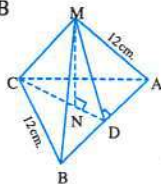
$\therefore D$ is the midpoint of \overline{AB}

in the equilateral ΔABC

$$\therefore \overline{MD} \perp \overline{AD}$$

$$\therefore MD = \sqrt{(12)^2 - (6)^2} = 6\sqrt{3} \text{ cm.}$$

\therefore The slant height = $6\sqrt{3} \text{ cm.}$



- 8 The base side length

$$= \frac{24\sqrt{3}}{6} = 4\sqrt{3} \text{ cm.}$$

\therefore the pyramid is regular

$$\therefore \overline{MN} \perp \overline{AN}$$

$\therefore N$ is the geometrical centre of the base

$$\therefore AN = \text{The base side length} = 4\sqrt{3} \text{ cm.}$$

$$\therefore AM = \sqrt{(8)^2 + (4\sqrt{3})^2} = 4\sqrt{7} \text{ cm.}$$

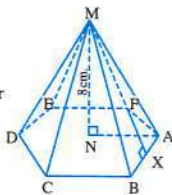
\therefore The length of lateral edge = $4\sqrt{7} \text{ cm.}$

Let X is the midpoint of \overline{AB}

$$\therefore MA = MB \quad \therefore \overline{MX} \perp \overline{AB}$$

$$\therefore MX = \sqrt{(4\sqrt{7})^2 - (2\sqrt{3})^2} = 10 \text{ cm.}$$

\therefore The length of slant height = 10 cm.



- 9 Let X is the midpoint of \overline{AB}

$$\therefore MA = MB$$

$$\therefore \overline{MX} \perp \overline{AB}$$

$$\therefore MX = \sqrt{(130)^2 - (50)^2} = 120 \text{ cm.}$$

\therefore The slant height = 120 cm.

\therefore The height of the lateral face = 120 cm.

\therefore the pyramid is regular

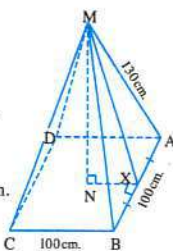
$$\therefore \overline{MN} \perp \overline{NX}$$

$\therefore N$ is the geometrical centre of the base

$$\therefore NX = 50 \text{ cm.}$$

$$\therefore MN = \sqrt{(120)^2 - (50)^2} = 10\sqrt{119} \text{ cm.}$$

\therefore The height = $10\sqrt{119} \text{ cm.}$



- 10 Fig. (1) : (regular quadrilateral pyramid)

\therefore The pyramid is regular

$$\therefore \overline{MN} \perp \overline{XN}$$

$\therefore N$ is the geometrical centre of the base

$$\therefore XN = 5 \text{ cm.}$$

$$\therefore MN = \sqrt{(13)^2 - (5)^2} = 12 \text{ cm.}$$

\therefore The height of the pyramid = 12 cm.

Fig. (2) : (triangular regular faces pyramid)

Let X is the midpoint

of \overline{AB}

$\therefore \Delta ABC$ is equilateral

$$\therefore \overline{CX} \perp \overline{AB}$$

$$\therefore CX = \sqrt{(6)^2 - (3)^2} = 3\sqrt{3} \text{ cm.}$$

\therefore the pyramid of regular faces

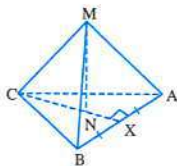
$$\therefore \overline{MN} \perp \overline{CN}$$

$\therefore N$ is the point of intersection of the medians of ΔABC

$$\therefore NC = 2\sqrt{3} \text{ cm.}$$

$$\therefore MN = \sqrt{(6)^2 - (2\sqrt{3})^2} = 2\sqrt{6} \text{ cm.}$$

\therefore The height = $2\sqrt{6} \text{ cm.}$



- 11 ∴ The pyramid is regular

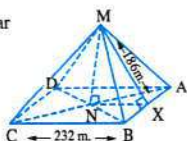
$$\therefore \overline{MN} \perp \overline{XN}$$

∴ N is the geometrical centre of the base

$$\therefore XN = 116 \text{ m.}$$

$$\therefore MN = \sqrt{(186)^2 - (116)^2} \approx 145.4 \text{ m.}$$

$$\therefore \text{The height} \approx 145.4 \text{ m.}$$



- 12 ∴
- $\triangle ABC$
- is right-angled at A

$$\therefore BC = \sqrt{(12.6)^2 + (16.8)^2} = 21 \text{ cm.}$$

∴ \overline{AD} is a median drawn from A

$$\therefore AD = \frac{1}{2} \text{ the length of the hypotenuse} = 10.5 \text{ cm.}$$

∴ The pyramid is right.

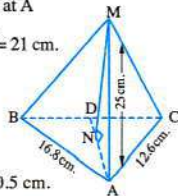
$$\therefore \overline{MN} \perp \overline{NA}$$

∴ N is the intersection point of the medians.

$$\therefore AN = \frac{2}{3} \times 10.5 = 7 \text{ cm.}$$

∴ $\triangle MNA$ is right-angled at N

$$\therefore \text{The height (MN)} = \sqrt{(25)^2 - 7^2} = 24 \text{ cm.}$$



- 13 ∴ The pyramid is right

$$\therefore \overline{MN} \perp \overline{AC}$$

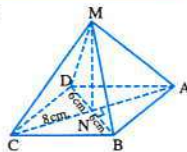
$$\therefore \overline{MN} \perp \overline{BD}$$

$$\therefore MB = MD$$

$$= \sqrt{(10)^2 + (6)^2}$$

$$= 2\sqrt{34} \text{ cm.}$$

$$\therefore MA = MC = \sqrt{(10)^2 + (8)^2} = 2\sqrt{41} \text{ cm.}$$



- 14

∴ The base area of the pyramid

$$= \frac{1}{2} \times 18 \times 18 \times \sin 60^\circ$$

$$= 81\sqrt{3} \text{ cm}^2.$$

$$\therefore \text{The volume of the pyramid} = \frac{1}{3} \times 81\sqrt{3} \times 12 = 324\sqrt{3} \text{ cm}^3.$$

- 15

(1) ∴ The height of the regular pyramid is MN

$$\therefore \overline{MN} \perp \overline{NE}, NE = \frac{1}{2} BC = 5 \text{ cm.}$$

$$\therefore ME = \sqrt{(12)^2 + (5)^2} = 13 \text{ cm.}$$

$$\therefore \text{The slant height} = 13 \text{ cm.}$$

(2) Volume of the pyramid

$$= \frac{1}{3} \text{ base area} \times \text{the height}$$

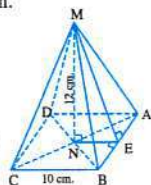
$$= \frac{1}{3} \times (10)^2 \times 12 = 400 \text{ cm}^3.$$

(3) The total area

$$= \text{the lateral area} + \text{base area}$$

$$= \frac{1}{2} \times (4 \times 10) \times 13 + (10 \times 10)$$

$$= 260 + 100 = 360 \text{ cm}^2.$$



- 16

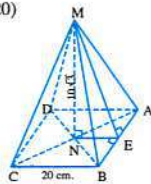
$$\text{The slant height} = \sqrt{(10)^2 + (10\sqrt{3})^2} = 20 \text{ cm.}$$

$$(1) \text{ The lateral area} = \frac{1}{2} \times (4 \times 20) \times 20 = 800 \text{ cm}^2.$$

(2) Volume of the pyramid

$$= \frac{1}{3} \text{ base area} \times \text{height}$$

$$= \frac{1}{3} \times (20)^2 \times 10\sqrt{3} = \frac{4000}{3} \sqrt{3} \text{ cm}^3.$$



- 17

Base of the regular pyramid is a square whose diagonal $24\sqrt{2}$ cm.

$$\therefore \text{The side length} = 24 \text{ cm.}$$

$$\text{The lateral area} = \frac{1}{2} \text{ base perimeter} \times \text{the slant height} = \frac{1}{2} \times (4 \times 24) \times 20 = 960 \text{ cm}^2.$$

$$\therefore \text{The total area} = \text{lateral area} + \text{base area}$$

$$= 960 + (24 \times 24) = 1536 \text{ cm}^2.$$

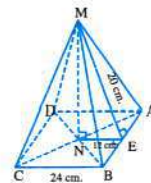
Height of the pyramid

$$= \sqrt{(20)^2 - (12)^2} = 16 \text{ cm.}$$

Volume of the pyramid

$$= \frac{1}{3} \text{ base area} \times \text{height}$$

$$= \frac{1}{3} \times (24)^2 \times 16 = 3072 \text{ cm}^3.$$



18

(1) \therefore MABCD is a right pyramid of a square base

\therefore The pyramid is regular

\therefore the slant height

$$= \sqrt{(4\sqrt{6})^2 - (4\sqrt{2})^2}$$

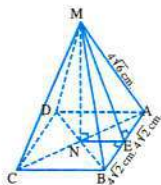
$$= 8 \text{ cm.}$$

The lateral area

$$= \frac{1}{2} \times \text{base perimeter}$$

$$\times \text{slant height} = \frac{1}{2}$$

$$\times (4 \times 8\sqrt{2}) \times 8 = 128\sqrt{2} \text{ cm}^2.$$



(2) \therefore EN = $\frac{1}{2}$ BC = $4\sqrt{2}$ cm.

\therefore In $\triangle MEN$:

Height of the pyramid MN

$$= \sqrt{(8)^2 - (4\sqrt{2})^2} = 4\sqrt{2} \text{ cm.}$$

\therefore Volume of the pyramid

$$= \frac{1}{3} \times \text{base area} \times \text{height}$$

$$= \frac{1}{3} \times (8\sqrt{2})^2 \times 4\sqrt{2} = \frac{512}{3} \sqrt{2} \text{ cm}^3.$$

19

(1) In $\triangle MAE$:

$$ME = \sqrt{(26)^2 - (10)^2} = 24 \text{ cm.}$$

(slant height of the pyramid)

(2) In $\triangle MEN$:

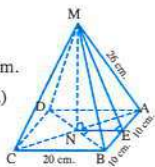
$$MN = \sqrt{(24)^2 - (10)^2}$$

$$= 2\sqrt{119} \text{ cm. (height of the pyramid)}$$

(3) The lateral area = $\frac{1}{2}$ base perimeter \times slant height

$$= \frac{1}{2} \times (4 \times 20) \times 24 = 960 \text{ cm}^2.$$

(4) Volume of the pyramid = $\frac{1}{3}$ base area

$$\times \text{height} = \frac{1}{3} \times (20)^2 \times 2\sqrt{119} = \frac{800}{3} \sqrt{119} \text{ cm}^2.$$


20

\therefore The pyramid is a triangular regular faces pyramid

$$\therefore 2l^2 = 3h^2 \quad \therefore 2 \times (12)^2 = 3 \times h^2$$

$$\therefore h = 4\sqrt{6} \text{ cm.}$$

$$\text{Volume of the pyramid} = \frac{1}{3} \times \left(\frac{1}{2} l^2 \times \frac{\sqrt{3}}{2}\right) \times h$$

$$= \frac{1}{3} \times \frac{1}{2} \times (12)^2 \times \frac{\sqrt{3}}{2} \times 4\sqrt{6} = 144\sqrt{2} \text{ cm}^3.$$

$$\text{Its total area} = l^2 \sqrt{3} = (12)^2 \sqrt{3} = 144\sqrt{3} \text{ cm}^2.$$

21

(1) \therefore MABCD is a right pyramid with a square base

\therefore The pyramid is a regular pyramid

\therefore its slant height

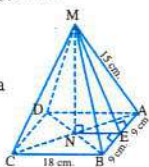
$$= \sqrt{(15)^2 - 9^2} = 12 \text{ cm.}$$

\therefore Its total area = lateral area

$$+ \text{base area} = \frac{1}{2}$$

$$\times (18 \times 4) \times 12 + 18^2$$

$$= 432 + 324 = 756 \text{ cm}^2.$$



(2) The pyramid height = $\sqrt{(12)^2 - 9^2} = 3\sqrt{7}$ cm.

$$\therefore \text{Volume of the pyramid} = \frac{1}{3} \times (18)^2 \times 3\sqrt{7}$$

$$= 324\sqrt{7} \text{ cm}^3.$$

22

$$\text{Base area} = \frac{n}{4} \times x^2 \times \cot \frac{\pi}{n}$$

$$= \frac{5}{4} \times (16)^2 \times \cot \frac{180^\circ}{5} = 440.44 \text{ cm}^2.$$

$$\text{Volume of the pyramid} = \frac{1}{3} \times \text{base area} \times \text{height}$$

$$= \frac{1}{3} \times 440.44 \times 12 \approx 1761.8 \text{ cm}^3.$$

23

The lateral area = $\frac{1}{2}$ base perimeter \times slant height

$$= \frac{1}{2} \times (6 \times 12) \times 10\sqrt{3} = 360\sqrt{3} \text{ cm}^2.$$

$$\therefore \text{base area} = \frac{n}{4} x^2 \cot \frac{\pi}{n}$$

$$= \frac{6}{4} \times (12)^2 \times \cot \frac{180^\circ}{6} = 216\sqrt{3} \text{ cm}^2.$$

\therefore the total area = lateral area + base area

$$= 360\sqrt{3} + 216\sqrt{3} = 576\sqrt{3} \text{ cm}^2.$$

24

$$(1) \text{ Base area} = \frac{1}{2} \times (6)^2 \times \frac{\sqrt{3}}{2} = 9\sqrt{3} \text{ cm}^2.$$

The lateral area = $\frac{1}{2}$ base perimeter \times slant height

$$= \frac{1}{2} \times (3 \times 6) \times 10 = 90 \text{ cm}^2.$$

$$\text{The total area} = (90 + 9\sqrt{3}) \approx 105.6 \text{ cm}^2.$$

$$(2) \text{ Base area} = (12)^2 = 144 \text{ cm}^2.$$

$$\text{The lateral area} = \frac{1}{2} \times (12 \times 4) \times 15 = 360 \text{ cm}^2.$$

The total area = lateral area + base area

$$= 360 + 144 = 504 \text{ cm}^2.$$

(3) Base area = $(20)^2 = 400 \text{ cm}^2$

The slant height = $\sqrt{(24)^2 + (10)^2} = 26 \text{ cm}$.

The lateral area = $\frac{1}{2} \times (20 \times 4) \times 26 = 1040 \text{ cm}^2$.

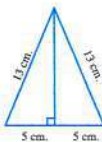
The total area = $1040 + 400 = 1440 \text{ cm}^2$.

(4) Base area = $\frac{6}{4} \times 10^2 \times \cot \frac{\pi}{6}$
 $= 150\sqrt{3} \approx 259.8 \text{ cm}^2$

The slant height = $\sqrt{(13)^2 - 5^2}$
 $= 12 \text{ cm}$.

The lateral area
 $= \frac{1}{2} \times (6 \times 10) \times 12 = 360 \text{ cm}^2$

The total area = $360 + 259.8 = 619.8 \text{ cm}^2$

**25**

(1) The volume = $\frac{1}{3} \times (10)^2 \times 21 = 700 \text{ cm}^3$

(2) The volume = $\frac{1}{3} \times \left(\frac{6}{4} \times 8^2 \times \cot \frac{\pi}{6}\right) \times 14$
 $= 448\sqrt{3} \text{ cm}^3$

(3) Base area = $\frac{1}{2} \times 6^2 \times \frac{\sqrt{3}}{2} = 9\sqrt{3} \text{ cm}^2$

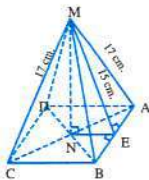
The volume = $\frac{1}{3} \times (9\sqrt{3}) \times 12 = 36\sqrt{3} \text{ cm}^3$

(4) $AE = \sqrt{(17)^2 - (15)^2}$
 $= 8 \text{ cm}$.

\therefore (AB) The side length
 $= 16 \text{ cm}$, $\therefore EN = 8 \text{ cm}$.

\therefore height of the pyramid
 $= \sqrt{(15)^2 - 8^2} = \sqrt{161} \text{ cm}$.

Volume of the pyramid = $\frac{1}{3} \times \text{base area} \times \text{height}$
 $= \frac{1}{3} \times (16)^2 \times \sqrt{161} = \frac{256}{3} \sqrt{161} \approx 1082.8 \text{ cm}^3$

**26**

Volume of the quadrilateral pyramid
 $= \frac{1}{3} \times \left(\frac{1}{2} \times 4 \times 8\right) \times 12 = 64 \text{ cm}^3$

Volume of the cube = $(4)^3 = 64 \text{ cm}^3$

\therefore The two volumes are equal.

27

Base perimeter = $5 + 6 + 7 = 18 \text{ cm}$.

\therefore Half base perimeter (S) = 9 cm .

Base area = $\sqrt{S(S-AB)(S-BC)(S-CA)}$
 $= \sqrt{9 \times (9-5) \times (9-6) \times (9-7)}$
 $= 6\sqrt{6} \text{ cm}^2$

The volume = $\frac{1}{3} \times 6\sqrt{6} \times 15 = 30\sqrt{6} \text{ cm}^3$

28

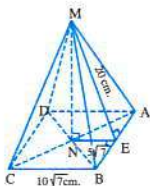
The base of the regular pyramid is a square whose area = 700 cm^2 .

\therefore The side length = $10\sqrt{7} \text{ cm}$.

The height = $\sqrt{(20)^2 - (5\sqrt{7})^2}$
 $= 15 \text{ cm}$.

Volume of the pyramid

$= \frac{1}{3} \times (700) \times 15 = 3500 \text{ cm}^3$

**29**

Base area = 9 cm^2

\therefore The base side length = 3 cm .

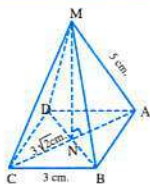
$\therefore AC = 3\sqrt{2}$

$\therefore AN = 1.5\sqrt{2}$

$\therefore \triangle MAN$:

$\therefore MN$ (The height) = $\sqrt{5^2 - (1.5\sqrt{2})^2} = \frac{1}{2}\sqrt{82}$

The volume = $\frac{1}{3} \times 9 \times \frac{1}{2}\sqrt{82} = \frac{3}{2}\sqrt{82} \approx 13.6 \text{ cm}^3$

**30**

Volume of the pyramid

$= \frac{1}{3} \times \text{base area} \times \text{the height}$
 $400 = \frac{1}{3} \times \text{base area} \times 12$

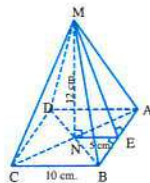
\therefore Base area = 100

\therefore The base side length
 $= 10 \text{ cm}$.

The slant height

$= \sqrt{5^2 + (12)^2} = 13 \text{ cm}$.

\therefore its lateral area = $\frac{1}{2} \times \text{base perimeter} \times \text{slant height}$
 $= \frac{1}{2} \times (4 \times 10) \times 13 = 260 \text{ cm}^2$



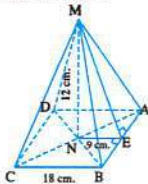
31

Volume of the pyramid = $\frac{1}{3}$ base area \times height
 $1296 = \frac{1}{3} \times (18)^2 \times \text{height}$
 height = 12 cm.

The slant height = $\sqrt{9^2 + (12)^2}$
 = 15 cm.

The lateral area

$$= \frac{1}{2} \times (4 \times 18) \times 15 = 540 \text{ cm}^2$$



32

Total area = lateral area + base area

$$384 = \frac{1}{2} \text{ base perimeter} \times \text{slant height} + \text{base area}$$

$$= \frac{1}{2} \times (4 \times 12) \times \text{slant height} + 12 \times 12$$

Slant height = 10 cm.

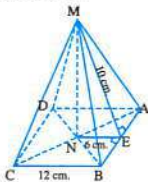
$$\text{Pyramid height} = \sqrt{(10)^2 - (6)^2}$$

$$= 8 \text{ cm.}$$

Volume of the pyramid

$$= \frac{1}{3} \times \text{base area} \times \text{height}$$

$$= \frac{1}{3} \times (12)^2 \times 8 = 384 \text{ cm}^3$$



33

\therefore Length of the diagonal of the squared base
 = $10\sqrt{2}$ cm.

\therefore The side length = 10 cm.

lateral area = $\frac{1}{2} \times \text{base perimeter} \times \text{slant height}$

$$260 = \frac{1}{2} \times (4 \times 10) \times \text{slant height}$$

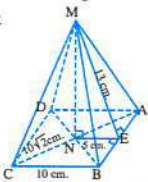
Slant height = 13 cm.

$$\text{Pyramid height} = \sqrt{(13)^2 - 5^2}$$

$$= 12 \text{ cm.}$$

The volume = $\frac{1}{3}$ base area \times height

$$= \frac{1}{3} \times (10)^2 \times 12 = 400 \text{ cm}^3$$



34

Length of the slant height

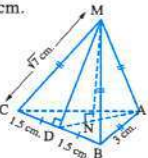
$$MD = \sqrt{(\sqrt{7})^2 - (1.5)^2} = \frac{\sqrt{19}}{2} \text{ cm.}$$

$$\therefore AD = \sqrt{3^2 - (1.5)^2} = \frac{3\sqrt{3}}{2} \text{ cm.}$$

$$\therefore ND = \frac{1}{3} AD = \frac{\sqrt{3}}{2} \text{ cm.}$$

\therefore Height of the Pyramid

$$= MN = \sqrt{\left(\frac{\sqrt{19}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2} = 2 \text{ cm.}$$



\therefore The volume = $\frac{1}{3} \times \text{base area} \times \text{height}$

$$= \frac{1}{3} \times \frac{1}{2} (3)^2 \sin 60^\circ \times 2 = \frac{3\sqrt{3}}{2} \text{ cm}^3$$

\therefore lateral area = $\frac{1}{2} \times \text{base perimeter} \times \text{slant height}$

$$= \frac{1}{2} \times (3 \times 3) \times \frac{\sqrt{19}}{2} = \frac{9}{4} \sqrt{19} \text{ cm}^2$$

35

\therefore The hexagon figure is a regular

$$\therefore NB = BC = 4\sqrt{3} \text{ cm.}$$

\therefore In $\triangle MNB$:

$$MB = \sqrt{8^2 + (4\sqrt{3})^2}$$

$$= 4\sqrt{7} \text{ cm.}$$

In $\triangle MYB$:

$$MY = \sqrt{(4\sqrt{7})^2 - (2\sqrt{3})^2} = 10 \text{ cm.}$$

\therefore The slant height = 10 cm.

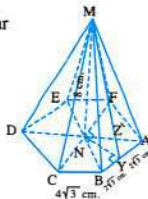
The lateral area = $\frac{1}{2} \times \text{base perimeter} \times \text{slant height}$

$$= \frac{1}{2} \times 24\sqrt{3} \times 10 = 120\sqrt{3} \text{ cm}^2$$

The total area = lateral area + base area

$$= 120\sqrt{3} + \left[\frac{6}{4} \times (4\sqrt{3})^2 \times \cot \frac{\pi}{6} \right]$$

$$= 192\sqrt{3} \text{ cm}^2$$



36

\therefore The base is an equilateral triangle
 passing through its vertices

a circle of radius length 12 cm.

$$\therefore \frac{a}{\sin A} = 2r \text{ (sin law)}$$

$$\therefore \frac{a}{\sin 60^\circ} = 2 \times 12$$

$$\therefore a = 12\sqrt{3}$$

$$\therefore \text{The base side length} = 12\sqrt{3}$$

$$\therefore \text{from } \triangle ABD: \therefore AD = \sqrt{(12\sqrt{3})^2 - (6\sqrt{3})^2} = 18 \text{ cm.}$$

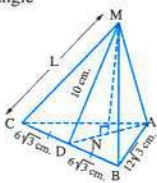
$$\therefore ND = \frac{1}{3} AD = 6 \text{ cm.} \therefore \text{from } \triangle MND:$$

$$\therefore MN = \sqrt{(10)^2 - (6)^2} = 8 \text{ cm.}$$

\therefore volume of the pyramid = $\frac{1}{3} \times \text{base area} \times \text{height}$

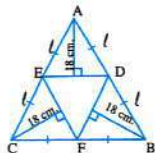
\therefore Volume of the pyramid

$$= \frac{1}{3} \times \frac{1}{2} (12\sqrt{3})^2 \sin 60^\circ \times 8 = 288\sqrt{3} \text{ cm}^3$$



37

Let $BF = l$
 $\therefore DE = \frac{1}{2} BC = l$
 similar;
 $DF = EF = l$



$\therefore DEF$ is an equilateral triangle
 of side length l

\therefore The net for a triangular regular faces pyramid
 and $\frac{18}{l} = \sin 60^\circ$

$\therefore l = 12\sqrt{3}$ cm.

Total area = $4 \times \text{face area} = 4 \times \frac{1}{2} \times (12\sqrt{3})^2 \sin 60^\circ$
 $= 432\sqrt{3}$ cm².

38

The figure after folding it gives a quadrilateral regular pyramid its slant height = $\sqrt{(13)^2 - (5)^2}$
 $= 12$ cm.

Area of one container = $\frac{1}{2} \times (4 \times 10) \times 12 + (10)^2$
 $= 240 + 100 = 340$ cm².

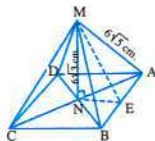
(1) area of 1000 containers = 340000 cm² = 34 m².

(2) the cost = $34 \times 15 = 510$ pounds

39

ΔMNA is right angled at N

$\therefore NA = \sqrt{(6\sqrt{5})^2 - (6\sqrt{3})^2}$
 $= 6\sqrt{2}$ cm.



$\therefore AC = 12\sqrt{2}$

\therefore The length of square diagonal = $12\sqrt{2}$

\therefore The square side length = 12 cm.

$\therefore EN = \frac{1}{2} BC = \frac{1}{2} \times 12 = 6$ cm.

\therefore In ΔMEN :

$\therefore ME = \sqrt{6^2 + (6\sqrt{3})^2} = 12$ cm.

The slant height = 12 cm.

The lateral area = $\frac{1}{2} \times \text{base perimeter} \times \text{slant height}$
 $= \frac{1}{2} \times (4 \times 12) \times 12 = 288$ cm².

The total area = $288 + (12)^2 = 432$ cm².

The volume = $\frac{1}{3} \times (12)^2 \times 6\sqrt{3} = 288\sqrt{3}$ cm³.

40

Volume of the model = $\frac{1}{3} \times \text{base area} \times \text{height}$
 $= \frac{1}{3} \times (11.5)^2 \times 7 \approx 308.58$ cm³.

The mass = density \times volume

$= 3.2 \times 308.58 \approx 987.5$ gm.

41

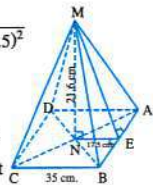
The slant height = $\sqrt{(21.6)^2 + (17.5)^2}$
 ≈ 27.8 m.

Area of the glass

= The lateral area of the pyramid

= $\frac{1}{2} \times \text{base perimeter} \times \text{slant height}$

= $\frac{1}{2} \times (4 \times 35) \times 27.8 \approx 1946$ m².



Third Higher skills

1

$\therefore BN = BC = 2l$

\therefore In ΔMNB :

$MB = \sqrt{(3l)^2 + (2l)^2} = \sqrt{13}l$

\therefore in ΔMBY :

$MY = \sqrt{(\sqrt{13}l)^2 - l^2} = 2\sqrt{3}l$

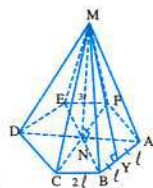
\therefore The slant height = $2\sqrt{3}l$

\therefore The lateral area = $\frac{1}{2} \times \text{base perimeter} \times \text{slant height}$

= $\frac{1}{2} \times (2l \times 6) \times 2\sqrt{3}l = 12\sqrt{3}l^2$ cm².

Base area = $\frac{6}{4} \times (2l)^2 \times \cot \frac{\pi}{6} = 6\sqrt{3}l^2$ cm².

\therefore The lateral area = 2 base area



2

\therefore Length of base diagonal = l

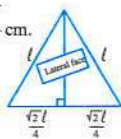
\therefore The side length of the base = $\frac{l\sqrt{2}}{2}$ cm.

The slant height

= $\sqrt{l^2 - \left(\frac{l\sqrt{2}}{4}\right)^2} = \frac{\sqrt{14}}{4}l$

The lateral area = $\frac{1}{2} \times \text{base perimeter} \times \text{slant height}$

= $\frac{1}{2} \times \frac{\sqrt{2}l}{2} \times 4 \times \frac{\sqrt{14}}{4}l = \frac{\sqrt{7}}{2}l^2$



The total area = lateral area + base area

$$= \frac{\sqrt{7}}{2} l^2 + \left(\frac{l}{2}\sqrt{7}\right)^2$$

$$= \left(\frac{\sqrt{7}}{2} + \frac{1}{2}\right) l^2 = \frac{l^2}{2} (1 + \sqrt{7})$$

3

∴ The pyramid is right

∴ Its height meets the base ABC at the centre N which is the point of intersection of the medians, let the radius of the base (r), height of cylinder = height of pyramid = h
base area of the cylinder = πr^2

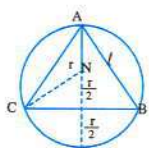
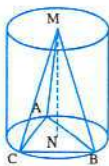
$$\therefore \sin 60^\circ = \frac{1.5r}{l}$$

∴ l (side length of pyramid base) = $r\sqrt{3}$

∴ Base area of the pyramid

$$= \frac{1}{2} (r\sqrt{3})^2 \sin 60^\circ = \frac{3\sqrt{3}}{4} r^2$$

$$\therefore \frac{\text{pyramid volume}}{\text{cylinder volume}} = \frac{\frac{1}{3} \times \frac{3\sqrt{3}}{4} r^2 \times h}{\pi r^2 h} = \frac{\sqrt{3}}{4\pi}$$



4

Let the base side length

= the lateral edge length = l

Base area = l^2

the slant height = $\frac{l}{2}\sqrt{3}$

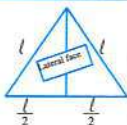
∴ The lateral area = $\frac{1}{2}$ base perimeter \times slant height
 $= \frac{1}{2} \times (4 \times l) \times \frac{l}{2}\sqrt{3} = l^2\sqrt{3}$

∴ Total area = $l^2\sqrt{3} + l^2 = l^2(\sqrt{3} + 1)$

∴ $l^2(\sqrt{3} + 1) = (\sqrt{3} + 1)A$

∴ $l^2 = A$ ∴ $l = \sqrt{A}$

∴ The edge length = \sqrt{A}



Exercise 8

First Multiple choice questions

- (1) d (2) b (3) b (4) c (5) c
 (6) d (7) a (8) b (9) a (10) d
 (11) a (12) d (13) a (14) b (15) b
 (16) c (17) c (18) d (19) c (20) c
 (21) c (22) d (23) b (24) a (25) d

(26) b (27) c (28) b

(29) First : b

Second : c

Third : d

Fourth : b

Fifth : b

(30) d

(31) d

(32) b

(33) a

(34) c

(35) c

(36) d

(37) c

(38) c

(39) b

(40) a

(41) c

(42) c

(43) d

Second Essay questions

1

(1) The volume = $\frac{1}{3} \times \pi \times (9)^2 \times 14$
 $= 378 \pi \text{ cm}^3$

(2) $r = \sqrt{(26)^2 - (24)^2} = 10 \text{ cm}$

The volume = $\frac{1}{3} \times \pi \times (10)^2 \times 24 = 800 \pi \text{ cm}^3$

(3) The height = $\sqrt{(13)^2 - (5)^2} = 12 \text{ cm}$

The volume = $\frac{1}{3} \times \pi \times (5)^2 \times 12 = 100 \pi \text{ cm}^3$

2

(1) The lateral area = $\pi \times (6) \times 12 = 72 \pi \text{ cm}^2$

, the total area = $\pi (6) (6 + 12) = 108 \pi \text{ cm}^2$

(2) Length of the drawer (l) = $\sqrt{(12)^2 + 9^2} = 15 \text{ cm}$

The lateral area = $\pi (9) \times 15 = 135 \pi \text{ cm}^2$

, the total area = $\pi (9) (9 + 15) = 216 \pi \text{ cm}^2$

(3) $r = \sqrt{(15)^2 - (13)^2} = 2\sqrt{14} \text{ cm}$

The lateral area = $\pi (2\sqrt{14}) \times 15 = 30\sqrt{14} \pi \text{ cm}^2$

The total area = $\pi (2\sqrt{14}) (2\sqrt{14} + 15)$
 $= (56 + 30\sqrt{14}) \pi \text{ cm}^2$

3

$r = \sqrt{l^2 - h^2} = \sqrt{(17)^2 - (15)^2} = 8 \text{ cm}$

The lateral area = $\pi r l = \pi (8) \times 17 = 136 \pi \text{ cm}^2$

The total area = $\pi r (l + r) = \pi (8) (17 + 8) = 200 \pi \text{ cm}^2$

The volume = $\frac{1}{3} \pi r^2 h = \frac{1}{3} \times \pi \times 8^2 \times 15 = 320 \pi \text{ cm}^3$

4 ∴ The cone is right

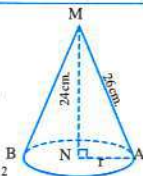
∴ $MN \perp AN$

∴ $r = \sqrt{(26)^2 - (24)^2} = 10 \text{ cm}$

The base circumference

= $2 \pi r = 20 \pi \text{ cm}$

The base area = $\pi r^2 = 100 \pi \text{ cm}^2$



- 5 The opposite figure is a cone
its base circumference = 44 cm.

$$\therefore 2 \times \frac{22}{7} \times r = 44$$

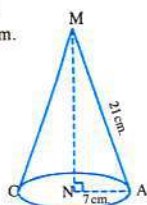
$$\therefore r = 7 \text{ cm.}$$

\therefore the cone is right

$$\therefore \overline{MN} \perp \overline{AN}$$

\therefore The height of the cone

$$= MN = \sqrt{(21)^2 - (7)^2} = 14\sqrt{2} \text{ cm.}$$



- 6 \therefore The area of the sector = $\frac{1}{2} r L$

$$\therefore 20\pi = \frac{1}{2} r \times 8\pi$$

$\therefore r = 5 \text{ cm.}$ and it represent
the drawer of the solid.

\therefore the length of the arc
of the sector

= the circumference of the base

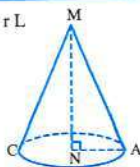
$$\therefore 8\pi = 2\pi r$$

$\therefore r = 4 \text{ cm.}$ (and it represent the radius of the
base of the cone)

\therefore the cone is right

\therefore The height of the solid

$$= MN = \sqrt{(5)^2 - (4)^2} = 3 \text{ cm.}$$



- 7 The net represents
a right circular cone

\therefore the area of the circle

$$= 49\pi \text{ cm}^2$$

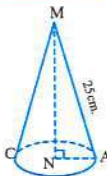
$$\therefore \pi r^2 = 49\pi \quad \therefore r = 7 \text{ cm.}$$

\therefore the cone is right

$$\therefore \overline{MN} \perp \overline{AN}$$

$$\therefore MN = \sqrt{(25)^2 - (7)^2} = 24 \text{ cm.}$$

\therefore The height = 24 cm.

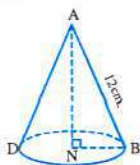


- 8 \therefore The length of the radius
of the circle of the sector
 $r = 12 \text{ cm.}$ represents
the drawer of the cone

\therefore the area of the
sector = $150\pi \text{ cm}^2$

$$\therefore \frac{1}{2} r^2 \theta = 150$$

$$\therefore \frac{1}{2} \times 12 \times \theta = 150$$



$\therefore \theta = 25 \text{ cm.}$ and it represents the circumference
of the base

$$\therefore 25 = 2\pi r$$

\therefore The radius length of the base of

the cone = $\frac{25}{2\pi} \text{ cm.}$

\therefore the cone is right

$$\therefore \overline{AN} \perp \overline{BN}$$

$$\therefore \text{The height of the cone} = \sqrt{(12)^2 - \left(\frac{25}{2\pi}\right)^2} \approx 11.3 \text{ cm.}$$

9

$\therefore r = 5 \text{ cm.}$

length of its drawer (l) = $\sqrt{(5)^2 + (12)^2} = 13 \text{ cm.}$

$$\text{The total area} = \pi (5) (5 + 13) = 90\pi \approx 282.7 \text{ cm}^2$$

10

\therefore The base circumference = 44

$$\therefore 2\pi r = 44$$

$$\therefore r = \frac{22}{\pi}$$

$$\text{Volume of the cone} = \frac{1}{3} \times \pi \times \left(\frac{22}{\pi}\right)^2 \times 25 \approx 1283.8 \text{ cm}^3$$

11

ΔABM is a right-angled at B, $m(\angle AMB) = 30^\circ$

$$\therefore AM = 2 \times 5 = 10 \text{ cm.}$$

$$\therefore l = 10 \text{ cm.}$$

$$\text{The lateral area} = \pi (5) \times 10 = 50\pi \text{ cm}^2$$

$$\text{The total area} = \pi (5) (5 + 10) = 75\pi \text{ cm}^2$$

12

\therefore The lateral area = $\pi r l$

$$\therefore 96\pi = \pi (8) l$$

$$\therefore l = 12 \text{ cm.}$$

$$\therefore h = \sqrt{(12)^2 - 8^2} = 4\sqrt{5} \text{ cm.}$$

$$\text{The volume} = \frac{1}{3} \times \pi r^2 h$$

$$= \frac{1}{3} \times \pi (8)^2 \times 4\sqrt{5} \approx 599.5 \text{ cm}^3$$

13

The First (A) :

$$\text{Its capacity} = \frac{1}{3} \pi \left(\frac{5}{2}\right)^2 \times 11 = \frac{275}{12} \pi \text{ cm}^3$$

The Second (B) :

$$\text{Its capacity} = \frac{1}{3} \pi \left(\frac{11}{2}\right)^2 \times 5 = \frac{605}{12} \pi \text{ cm}^3$$

∴ The capacity of B is the greater

The difference between their capacity

$$= \frac{605}{12} \pi - \frac{275}{12} \pi = \frac{55}{2} \pi \text{ cm}^3.$$

14

$$\cos \theta = \frac{AD}{AB} = \frac{12}{l} = \frac{4}{5}$$

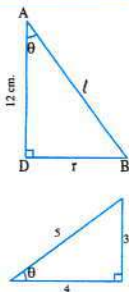
$$\therefore l = 15 \text{ cm.}$$

$$r = \sqrt{(15)^2 - (12)^2} = 9 \text{ cm.}$$

$$\text{The total area} = \pi r (\ell + r)$$

$$= \pi (9) (15 + 9)$$

$$= 216 \pi \text{ cm}^2.$$



15

$$\therefore \text{The volume of the tank} = \frac{1}{3} \pi r^2 h$$

$$\therefore 32 \pi = \frac{1}{3} \pi r^2 \times 6$$

$$r^2 = 16 \quad \therefore r = 4 \text{ m.}$$

$$l = \sqrt{4^2 + 6^2} = 2\sqrt{13} \text{ m.}$$

$$\text{Its total area} = \pi r (\ell + r)$$

$$= \pi \times 4 \times (2\sqrt{13} + 4)$$

$$= (16 + 8\sqrt{13}) \pi \approx 140.9 \text{ m}^2.$$

16

$$\text{Volume of the cone} = \frac{1}{3} \pi r^2 h$$

$$= \frac{1}{3} \pi \times (15)^2 \times 20 = 1500 \pi \text{ cm}^3.$$

$$\approx 4712.4 \text{ cm}^3.$$

$$\therefore \text{the side length of the pyramid base} = 48 \div 4 = 12 \text{ cm.}$$

$$\therefore \text{Volume of the pyramid} = \frac{1}{3} \times \text{base area} \times \text{height}$$

$$= \frac{1}{3} \times (12)^2 \times 40$$

$$= 1920 \text{ cm}^3.$$

$$\therefore \text{Volume of the cone} > \text{volume of the pyramid}$$

17

$$\text{Volume of the cone} = \frac{1}{3} \text{ the base area} \times h$$

$$\pi h^3 = \frac{1}{3} \times \pi r^2 \times h \quad \therefore r^2 = 3 h^2$$

$$\text{The lateral area} = \pi r l = \pi r \sqrt{r^2 + h^2}$$

$$= \pi r \sqrt{3 h^2 + h^2} = 2 \pi r h$$

$$= \text{the circumference of the cylinder base} \times \text{height}$$

$$= \text{The lateral area of the cylinder}$$

18

$$\therefore \text{Volume of the cone} = \frac{1}{3} \times \pi (2)^2 \times 12 = 16 \pi \text{ cm}^3.$$

$$\therefore \text{Volume of the dislodged water in cylinder form} = 16 \pi \text{ cm}^3.$$

$$\therefore 16 \pi = \pi r^2 \times 1 \quad \therefore r^2 = 16$$

$$r = 4 \text{ cm.}$$

$$\text{length of the diameter of the base of the vessel}$$

$$= 4 \times 2 = 8 \text{ cm.}$$

19

$$\text{Volume of the wax} = \text{Volume of cube}$$

$$= (20)^3 = 8000 \text{ cm}^3.$$

$$\therefore 12\% \text{ of wax had been lost during the melting and reforming}$$

$$\therefore \text{The volume of the cone} = 88\% \times 8000$$

$$= \frac{88}{100} \times 8000 = 7040 \text{ cm}^3.$$

$$\therefore \text{volume of the cone} = \frac{1}{3} \pi r^2 h$$

$$\therefore \frac{1}{3} \times \frac{22}{7} \times r^2 \times 21 = 7040$$

$$\therefore r^2 = 320 \quad \therefore r = 8\sqrt{5} \text{ cm.}$$

20

$$\text{Capacity of the cone} = 2.2 \text{ litres}$$

$$= 2.2 \times 1000 = 2200 \text{ cm}^3.$$

$$\text{The volume} = \frac{1}{3} \pi r^2 h$$

$$\therefore \frac{1}{3} \times \frac{22}{7} \times r^2 \times 21 = 2200$$

$$\therefore r^2 = 100 \Rightarrow r = 10 \text{ cm.}$$

21

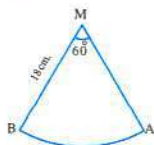
$$\text{The length of the drawer of the cone} = 18 \text{ cm.}$$

$$\therefore \text{the circumference of circle}$$

$$\text{of the cone} = \widehat{AB} = r \times \theta^{\text{rad}}$$

$$= 18 \times \frac{60^\circ \times \pi}{180^\circ}$$

$$\therefore 2 \pi r = 6 \pi \quad \therefore r = 3 \text{ cm.}$$



$$\therefore h = \sqrt{\ell^2 - r^2} = \sqrt{(18)^2 - 3^2}$$

$$= 3\sqrt{35}$$

$$\therefore \text{Volume of the cone} = \frac{1}{3} \pi r^2 h$$

$$= \frac{1}{3} \times \pi \times (3)^2 \times 3\sqrt{35}$$

$$\approx 167.3 \text{ cm}^3$$

22

The length of the drawer of the cone = 20 cm.

∴ the perimeter of the circle of the cone

= the length of $\widehat{AB} = r \theta^{\text{rad}}$

$$= 20 \times \frac{90^\circ \times \pi}{180^\circ} = 10\pi$$

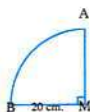
$$\therefore 2\pi r = 10\pi \quad \therefore r = 5 \text{ cm.}$$

$$\therefore h = \sqrt{(20)^2 - (5)^2} = 5\sqrt{15}$$

$$\text{Volume of the cone} = \frac{1}{3} \pi r^2 h$$

$$= \frac{1}{3} \times \pi \times (5)^2 \times 5\sqrt{15}$$

$$= \frac{125\sqrt{15}}{3} \pi \text{ cm}^3$$



23

(1) The formed solid is

a right cone whose

base radius = 6 cm.

and its height = 8 cm.

$$\text{The volume} = \frac{1}{3} \times \pi \times 6^2 \times 8$$

$$= 96 \pi \text{ cm}^3$$

(2) The formed solid is formed from

two cones with common

base whose

$$\text{radius} = \frac{6 \times 8}{10} = 4.8 \text{ cm.}$$

∴ length of the 1st

drawer = 8 cm.

$$\therefore h_1 = \sqrt{8^2 - 4.8^2} = 6.4 \text{ cm.}$$

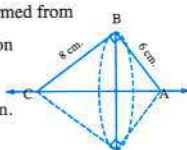
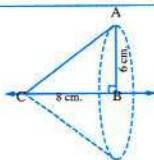
∴ length of the 2nd drawer = 6 cm.

$$\therefore h_2 = \sqrt{6^2 - 4.8^2} = 3.6 \text{ cm.}$$

∴ Volume of resultant solid

$$= \frac{1}{3} \times \pi \times (4.8)^2 \times 6.4 + \frac{1}{3} \times \pi \times (4.8)^2 \times 3.6$$

$$= 76.8 \pi \text{ cm}^3$$



24

(1) About X-axis form a cone

$$r_1 = 3 \text{ length units, } h_1 = 4 \text{ length units}$$

$$\therefore \text{The volume} = \frac{1}{3} \pi (3)^2 \times 4 = 12 \pi \text{ cubic units}$$

(2) About y-axis

$$r_2 = 4 \text{ length units, } h_2 = 3 \text{ length units}$$

$$\therefore \text{The volume} = \frac{1}{3} \pi (4)^2 \times 3 = 16 \pi \text{ cubic units}$$

25

The formed solid as two

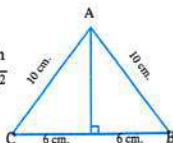
congruent cones with common

base whose radius = $\sqrt{10^2 - 6^2}$

$$= 8 \text{ cm.}$$

∴ height of each = 6 cm.

$$\text{The volume} = 2 \times \frac{1}{3} \pi (8)^2 \times 6 = 256 \pi \text{ cm}^3$$



26

∴ ME is a median drawn from

the right vertex in $\triangle AMB$

∴ AB (The length of drawer of

the cone) = $9 \times 2 = 18 \text{ cm.}$

∴ The lateral area = $\pi r \ell$

$$= \pi \times 6 \times 18 = 108 \pi \text{ cm}^2$$

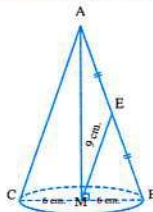
∴ the total area = $\pi r (r + \ell)$

$$= \pi \times 6 \times (6 + 18) = 144 \pi \text{ cm}^2$$

$$\text{The height} = \sqrt{(18)^2 - 6^2} = 12\sqrt{2} \text{ cm.}$$

$$\text{The volume} = \frac{1}{3} \pi r^2 h$$

$$= \frac{1}{3} \pi \times (6)^2 \times 12\sqrt{2} = 144\sqrt{2} \pi \text{ cm}^3$$



27

* First cone : $\ell_1 = 80 \text{ cm.}, r_1 = 50 \text{ cm.}$

$$\therefore \text{The lateral area} = \pi \times 80 \times 50 = 4000 \pi \text{ cm}^2$$

* Second cone : $h_2 = 120 \text{ cm.}, r_2 = 50 \text{ cm.}$

$$\therefore \ell_2 = \sqrt{(120)^2 + (50)^2} = 130 \text{ cm.}$$

$$\therefore \text{The lateral area} = \pi \times 130 \times 50 = 6500 \pi \text{ cm}^2$$

The total area wanted for painting is the sum of two

lateral areas of the two cones = $4000 \pi + 6500 \pi$

$$= 10500 \pi \approx 32987 \text{ cm}^2 \approx 3.3 \text{ m}^2$$

$$\text{The cost} = 3.3 \times 300 = 990 \text{ pounds}$$

28

Volume of the pentagonal pyramid

$$= \frac{1}{3} \times \text{base area} \times \text{height}$$

$$= \frac{1}{3} \left(\frac{5}{4} \times 10^2 \times \cot \frac{\pi}{5} \times 42 \right)$$

$$= 2408.67 \text{ cm}^3$$

Volume of the cone = 90% of volume of pyramid

$$= \frac{90}{100} \times 2408.67 = 2167.8 \text{ cm}^3$$

$$\therefore \frac{1}{3} \pi r^2 h = 2167.8$$

$$\frac{1}{3} \times \pi \times (15)^2 \times h = 2167.8$$

$$h = \frac{2167.8}{\frac{1}{3} \times \pi \times 15^2} \approx 9.2 \text{ cm}$$

29

The volume of the cone = $\frac{1}{3} \pi r^2 h = 100 \text{ cm}^3$

(1) After doubling its height

$$\therefore \text{The volume of the resultant cone}$$

$$= \frac{1}{3} \pi r^2 (2h) = 2 \left[\frac{1}{3} \pi r^2 h \right]$$

$$= 2 \times 100 = 200 \text{ cm}^3$$

(2) After doubling its radius length

$$\therefore \text{The volume of the resultant cone}$$

$$= \frac{1}{3} \pi (2r)^2 (h) = \frac{1}{3} \pi \times 4 r^2 h$$

$$= 4 \left[\frac{1}{3} \pi r^2 h \right] = 4 \times 100 = 400 \text{ cm}^3$$

(3) After doubling its height and radius length

$$\therefore \text{The volume of the resultant cone}$$

$$= \frac{1}{3} \pi (2r)^2 (2h)$$

$$= \frac{1}{3} \pi \times 4 r^2 \times 2 h$$

$$= 8 \left[\frac{1}{3} \pi r^2 h \right] = 8 \times 100 = 800 \text{ cm}^3$$

Third

Higher skills

1

(1) b

(2) c

(3) d

(4) First : c

Second : b

(5) a

(6) d

(7) c

Instructions to solve 1 :

(1) \therefore The volume of hemisphere = The volume of the cone.

$$\therefore \frac{1}{2} \left(\frac{4}{3} \pi r^3 \right) = \frac{1}{3} \pi r^2 h \quad \therefore h = 2r$$

$$(2) \therefore AM = \sqrt{(5k)^2 - (3k)^2} = 4k$$

\therefore the volume of the cone = 96π

$$\therefore \frac{1}{3} \pi \times (3k)^2 \times 4k = 96\pi$$

$$\therefore 12k^3 = 96$$

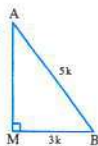
$$\therefore k^3 = 8$$

$$\therefore k = 2$$

$$\therefore r = 6 \text{ cm}, \quad h = 8 \text{ cm}, \quad l = 10 \text{ cm}$$

$$\therefore \text{The total area} = \pi r (r + l) = \pi \times 6 \times (6 + 10)$$

$$= 96\pi \text{ cm}^2$$



(3) \therefore The volume of the cone = 49π

$$\therefore \frac{1}{3} \pi r^2 h = 49\pi \quad \therefore \frac{1}{3} \pi r^2 \times 3 = 49\pi$$

$$\therefore r^2 = 49$$

$$\therefore r = 7 \text{ cm}$$

\therefore The length of the arc of the folded sector

$$= 2\pi r = 2\pi \times 7 = 14\pi \text{ cm}$$

(4) Let the radius of the smallest cone = r_1

and its height = h_1 and its drawer = l_1

Let the radius of the greatest cone = r_2

and its height = h_2 and its drawer = l_2

From the figure we find : $\frac{h_1}{h_2} = \frac{l_1}{l_2} = \frac{r_1}{r_2} = \frac{1}{2}$

First : The volume of the smallest cone
The volume of the greatest cone

$$= \frac{\frac{1}{3} \pi r_1^2 h_1}{\frac{1}{3} \pi r_2^2 h_2} = \left(\frac{r_1}{r_2} \right)^2 \times \frac{h_1}{h_2} = \left(\frac{1}{2} \right)^2 \times \frac{1}{2} = \frac{1}{8}$$

Second : The lateral area of the smallest cone
The lateral area of the greatest cone

$$= \frac{\pi r_1 l_1}{\pi r_2 l_2} = \frac{r_1}{r_2} \times \frac{l_1}{l_2} = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

(5) From the figure we find :

the area of the base of the pyramid

$$= \frac{1}{2} \times (2\sqrt{3}r) \times 3r$$

$$= 3\sqrt{3}r^2$$

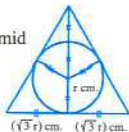
\therefore the area of the base of

$$\text{the cone} = \pi r^2$$

The volume of the regular triangular pyramid

\therefore The volume of the greatest cone can be put inside the pyramid

$$= \frac{\frac{1}{3} \times 3\sqrt{3}r^2 \times h}{\frac{1}{3} \times \pi r^2 \times h} = \frac{3\sqrt{3}}{\pi}$$



(6) From the figure we find :

the area of the base of the pyramid

$$= 3 \times \left(\frac{1}{2} r^2 \sin 120^\circ \right)$$

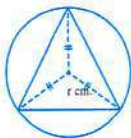
$$= \frac{3\sqrt{3}}{4} r^2$$

∴ the area of the base of the

cone = πr^2

∴ The volume of the regular triangular pyramid
The volume of the smallest cone that contain it

$$= \frac{\frac{1}{3} \times \frac{3\sqrt{3}}{4} r^2 \times h}{\frac{1}{3} \times \pi r^2 \times h} = \frac{3\sqrt{3}}{4\pi}$$



- (7) Let the radius of the base of the cone = r
and its height = h and its volume = v

after increasing we find that the radius of the
base of the cone = $\frac{3}{2}r$, its height = $\frac{3}{2}h$ and its
volume \hat{v}

$$\therefore \frac{\hat{v}}{v} = \frac{\frac{1}{3} \pi \left(\frac{3}{2}r \right)^2 \times \left(\frac{3}{2}h \right)}{\frac{1}{3} \pi r^2 \times h} = \frac{9}{4} \times \frac{3}{2} = \frac{27}{8}$$

$$\therefore \hat{v} = \frac{27}{8} v \quad \therefore \hat{v} = 337.5 \% v$$

2

$$L_1 : 3x - \sqrt{3}y = -6 \text{ dividing by } (-6)$$

$$\therefore \frac{x}{-2} + \frac{y}{2\sqrt{3}} = 1$$

$$L_2 : \sqrt{3}x + y = 2\sqrt{3} \text{ dividing by } (2\sqrt{3})$$

$$\therefore \frac{x}{2} + \frac{y}{2\sqrt{3}} = 1$$

$$\therefore A(0, 2\sqrt{3}), B(2, 0), C(-2, 0)$$

by rotation about X-axis make two cones with the
same base of radius length = $2\sqrt{3}$ unit and the same
height ($h = 2$ units)

∴ The two cones are congruent

The volume of the resultant solid

$$= 2 \times \frac{1}{3} \times \pi \times (2\sqrt{3})^2 \times 2 = 16\pi \text{ cubic unit.}$$

3

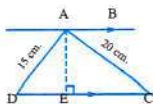
- (1) The formed solid from the rotation is a right
circular cylinder added to it a right circular cone of
height $(7-4) = 3$ cm. and base radius = 3.5 cm.

$$\begin{aligned} \therefore \text{The volume} &= \pi r^2 h_1 + \frac{1}{3} \pi r^2 h_2 \\ &= \pi \times (3.5)^2 \times 4 + \frac{1}{3} \pi \times (3.5)^2 \times 3 \\ &= \frac{245}{4} \pi \approx 192.4 \text{ cm}^3. \end{aligned}$$

- (2) The formed solid is a right circular cone formed
from the rotation of $\triangle CAB$, its radius length of
the base = 6 cm. its height = 8 cm. subtracted
from it a right circular cone formed from the
rotation of the unshaded triangle and the radius
length of its base = 3 cm. and its height = 8 cm.

$$\begin{aligned} \therefore \text{The volume} &= \frac{1}{3} \pi (6)^2 \times 8 - \frac{1}{3} \pi (3)^2 \times 8 \\ &= 72\pi \approx 226.2 \text{ cm}^3. \end{aligned}$$

- (3) The formed solid from the
rotation around AB is a right
circular cylinder its base
radius = AE and height CD



subtracted from it two right
circular cones the base radius
of each of them = AE and
their heights are CE , ED

From the figure we notice that :

$$CD = \sqrt{(15)^2 + (20)^2} = 25 \text{ cm.}$$

$$\therefore AE = \frac{15 \times 20}{25} = 12 \text{ cm.}$$

$$\begin{aligned} \therefore \text{The volume} &= \pi (12)^2 \times 25 - \left(\frac{1}{3} \pi (12)^2 \right. \\ &\quad \times CE + \frac{1}{3} \pi (12)^2 \times ED) \\ &= \pi \times (12)^2 \times 25 - \frac{1}{3} \pi (12)^2 \times (CE + ED) \\ &= \pi \times (12)^2 \times 25 - \frac{1}{3} \pi \times (12)^2 \times 25 \\ &= \pi \times (12)^2 \times 25 \times \frac{2}{3} = 2400\pi \\ &\approx 7539.8 \text{ cm}^3. \end{aligned}$$

Exercise 9

First Multiple choice questions

- | | | | | |
|--------|--------|--------|--------|--------|
| (1) b | (2) c | (3) b | (4) b | (5) c |
| (6) c | (7) b | (8) b | (9) c | (10) c |
| (11) d | (12) a | (13) d | (14) d | (15) c |
| (16) c | (17) c | (18) b | (19) a | (20) c |
| (21) b | (22) d | (23) a | (24) b | (25) b |
| (26) a | (27) d | (28) d | (29) d | (30) b |
| (31) b | (32) a | (33) b | (34) d | (35) d |

- (36) d (37) d (38) c (39) b (40) a
 (41) a (42) b (43) b (44) d (45) b
 (46) c (47) b (48) d (49) a (50) a
 (51) b (52) d (53) d (54) a (55) d
 (56) a (57) d (58) d (59) c (60) a
 (61) d (62) d (63) d (64) c (65) a
 (66) c (67) b (68) b (69) a (70) d
 (71) b

Second Essay questions

1

- (1) $(x-2)^2 + (y-3)^2 = 25$
 (2) $x^2 + y^2 = 9$
 (3) $(x-0)^2 + (y+1)^2 = 12$
 (4) $(x+4)^2 + (y+3)^2 = \frac{9}{4}$

2

- (1) $(x+2)^2 + (y-3)^2 = 4^2$
i.e. $x^2 + y^2 + 4x - 6y - 3 = 0$
 (2) $r = \sqrt{(5-0)^2 + (-12-0)^2} = 13$
 $\therefore (x-5)^2 + (y+12)^2 = (13)^2$
i.e. $x^2 + y^2 - 10x + 24y = 0$
 (3) $r = \sqrt{(7-3)^2 + (-5-2)^2} = \sqrt{65}$
 $\therefore (x-7)^2 + (y+5)^2 = (\sqrt{65})^2$
i.e. $x^2 + y^2 - 14x + 10y + 9 = 0$
 (4) The centre of the circle $M = \left(\frac{6+0}{2}, \frac{-4+2}{2}\right)$
 $= (3, -1)$
 $r = \sqrt{(6-3)^2 + (-4+1)^2} = 3\sqrt{2}$
 $\therefore (x-3)^2 + (y+1)^2 = (3\sqrt{2})^2$
i.e. $x^2 + y^2 - 6x + 2y - 8 = 0$
 (5) $\ell = 3, k = 2$
 \therefore The circle touches X -axis
 $\therefore r = |k| = 2, c = \ell^2 = 9$
 \therefore The equation of the circle is :
 $x^2 + y^2 + 6x + 4y + 9 = 0$
 (6) $\ell = -3, k = 0$
 \therefore The circle touches y -axis
 $\therefore r = |\ell| = 3, c = k^2 = 0$
 \therefore The equation of the circle is : $x^2 + y^2 - 6x = 0$
 (7) $\therefore \ell = -5, k = 5$
 \therefore The circle touches the two coordinate axes
 $\therefore r = |\ell| = |k| = 5, c = 25$

\therefore The equation of the circle is :

$$x^2 + y^2 - 10x + 10y + 25 = 0$$

(8) \therefore The two tangents at A and B are parallel

\therefore AB is a diameter in the circle

$$\therefore \text{The centre of the circle } M = \left(\frac{6+0}{2}, \frac{2-1}{2}\right) = \left(3, \frac{1}{2}\right)$$

$$\therefore r = \sqrt{(6-3)^2 + \left(2-\frac{1}{2}\right)^2} = \frac{3\sqrt{5}}{2}$$

\therefore The equation of the circle is

$$(x-3)^2 + \left(y-\frac{1}{2}\right)^2 = \left(\frac{3\sqrt{5}}{2}\right)^2$$

i.e. $x^2 + y^2 - 6x - y - 2 = 0$

(9) The centre of the circle $M = (5, 0), r = 3$

\therefore The equation of the circle is

$$(x-5)^2 + (y-0)^2 = (3)^2$$

i.e. $x^2 + y^2 - 10x + 16 = 0$

(10) \therefore The circle touches the two coordinate axes, and it lies in the 4th quadrant.

$$\therefore c = \ell^2 = k^2 = r^2 = 36$$

\therefore the centre of the circle $M = (6, -6)$

\therefore The equation of the circle is :

$$(x-6)^2 + (y-6)^2 = (6)^2$$

i.e. $x^2 + y^2 - 12x + 12y + 36 = 0$

3

- (1) The centre of the circle $= (0, 0)$
 $\therefore r = \sqrt{8} = 2\sqrt{2}$ length units
 (2) The centre of the circle $= (-3, 5)$
 $\therefore r = 7$ length units
 (3) The centre of the circle $= (-4, 0)$
 $\therefore r = 3$ length units
 (4) The centre of the circle $= (0, -7)$
 $\therefore r = \sqrt{24} = 2\sqrt{6}$ length units
 (5) $\therefore \ell = -2, k = 3, c = -12$
 \therefore The centre of the circle $= (2, -3)$
 $\therefore r = \sqrt{\ell^2 + k^2 - c} = \sqrt{4 + 9 + 12} = 5$ length units
 (6) $\therefore \ell = -4, k = 0, c = -12$
 \therefore The centre of the circle $= (4, 0)$
 $\therefore r = \sqrt{\ell^2 + k^2 - c} = \sqrt{16 + 0 + 12} = 2\sqrt{7}$ length units

4

$$(1) \because x^2 + y^2 - 4x + 8y = 0$$

$$\therefore \ell = -2, k = 4, c = 0$$

$$\therefore r_1 = \sqrt{\ell^2 + k^2 - c} = \sqrt{4 + 16} = 2\sqrt{5} \text{ length units}$$

$$\because x^2 + y^2 + 12x + 16y = 0$$

$$\therefore \ell = 0, k = 6, c = 16$$

$$\begin{aligned} \therefore r_2 &= \sqrt{\ell^2 + k^2 - c} = \sqrt{0 + 36 - 16} \\ &= 2\sqrt{5} \text{ length units} \end{aligned}$$

$$\therefore r_1 = r_2$$

\therefore The two circles are congruent.

$$(2) \because x^2 + y^2 + 14y = 1$$

$$\therefore \ell = 0, k = 7, c = -1$$

$$\begin{aligned} \therefore r_1 &= \sqrt{\ell^2 + k^2 - c} = \sqrt{0 + 49 + 1} \\ &= 5\sqrt{2} \text{ length units} \end{aligned}$$

$$\because x^2 + y^2 + 10x - 25 = 0$$

$$\therefore \ell = 5, k = 0, c = -25$$

$$\begin{aligned} \therefore r_2 &= \sqrt{\ell^2 + k^2 - c} = \sqrt{25 + 25} \\ &= 5\sqrt{2} \text{ length units} \end{aligned}$$

$$\therefore r_1 = r_2$$

\therefore The two circles are congruent

$$(3) \because x^2 + y^2 - 2x + 4y - 3 = 0$$

$$\therefore \ell = -1, k = 2, c = -3$$

$$\begin{aligned} \therefore r_1 &= \sqrt{\ell^2 + k^2 - c} = \sqrt{1 + 4 + 3} \\ &= 2\sqrt{2} \text{ length units} \end{aligned}$$

$$\because x^2 + y^2 + 6x - 11 = 0$$

$$\therefore \ell = 3, k = 0, c = -11$$

$$\begin{aligned} \therefore r_2 &= \sqrt{\ell^2 + k^2 - c} = \sqrt{9 + 0 + 11} \\ &= 2\sqrt{5} \text{ length units} \end{aligned}$$

$$\therefore r_1 \neq r_2$$

\therefore The two circles are not congruent

5 $\therefore r_1 = r_2 = 2$ length units

\therefore The two circles are congruent

The centre of the circle $C_1 = (0, 0)$

The centre of the circle $C_2 = (5, 2)$

\therefore The equation of the circle C_1 is $x^2 + y^2 = 4$

The equation of the circle C_2

$$\text{is } (x-5)^2 + (y-2)^2 = 4$$

The equation of the circle C_3

$$\text{is } (x+4)^2 + (y-3)^2 = 4$$

6

(1) \because The equation include xy

\therefore The equation does not represent a circle.

(2) \because coefficient of $x^2 =$ coefficient of $y^2 = 1$

and the equation has no terms of xy ,

$$\ell^2 + k^2 - c = (4)^2 + (-8)^2 + 1 = 81 > 0$$

\therefore The equation represents a circle.

(3) \because The coefficient of $x^2 \neq$ the coefficient of y^2

\therefore The equation does not express a circle.

(4) $x^2 + y^2 + \frac{3}{2}y - 4 = 0$

\therefore The coefficient of $x^2 =$ the coefficient of $y^2 = 1$
and the equation is empty from xy ,

$$\ell^2 + k^2 - c = (0)^2 + \left(\frac{3}{2}\right)^2 + 4 = 6\frac{1}{4} > 0$$

\therefore The equation expresses a circle.

(5) $x^2 + y^2 + 2x - 3x + 6y - 4 = 0$

\therefore The equation includes the term $2xy$

\therefore The equation does not express a circle.

(6) \because The coefficient of $x^2 =$ the coefficient of $y^2 = 1$

and the equation is empty from xy

$$\ell^2 + k^2 - c = \left(\frac{1}{2}\right)^2 + (1)^2 - 7 = -\frac{23}{4} < 0$$

\therefore The equation does not represent a circle.

(7) \because The coefficient of $x^2 =$ the coefficient of $y^2 = 1$

and the equation does not include xy

$$\ell^2 + k^2 - c = (1)^2 + (-2)^2 - 5 = 0$$

\therefore The equation does not represent a circle.

(8) $x^2 + y^2 + 4x - 32 = 0$

\therefore The coefficient of $x^2 =$ the coefficient of $y^2 = 1$
and the equation has no term in xy

$$\ell^2 + k^2 - c = (2)^2 + 32 = 36 > 0$$

\therefore The equation represents a circle.

(9) $x^2 - y^2 + x - y - 7 = 0$

\therefore The coefficient of $x^2 \neq$ the coefficient of y^2

\therefore The equation does not express a circle.

7 $r = \sqrt{(2+1)^2 + (-1-3)^2} = 5$ length unit

\therefore The equation of the first circle C_1 is
 $(x-2)^2 + (y+1)^2 = 25$

\therefore the equation of the second circle C_2 is
 $(x+1)^2 + (y-3)^2 = 25$

8 $\therefore C_1 : x^2 + y^2 - 2x + 6y + 1 = 0$

$\therefore M_1 = (1, -3), r_1 = \sqrt{(-1)^2 + (3)^2} - 1$
 $= 3$ length unit

$\therefore C_2 : x^2 + y^2 - 2x + 6y + \frac{15}{4} = 0$

$\therefore M_2 = (1, -3), r_2 = \sqrt{(-1)^2 + (3)^2} - \frac{15}{4}$
 $= 2.5$ length unit

\therefore the two circles are concentric

9 $(x-6)^2 + (y+1)^2 = 25$ substituting by the coordinates of the points A, B, C and D we get

A (9, 3) $\therefore (9-6)^2 + (3+1)^2 = 25 = r^2$

\therefore The point A lies on the circle

B (7, 5) $\therefore (7-6)^2 + (5+1)^2 = 37 > r^2$

\therefore The point B is outside the circle

C (3, 3) $\therefore (3-6)^2 + (3+1)^2 = 25 = r^2$

\therefore The point C lies on the circle

D (2, -3) $\therefore (2-6)^2 + (-3+1)^2 = 20 < r^2$

\therefore The point D lies inside the circle

10 $r = \sqrt{(2+1)^2 + (-1-3)^2} = 5$ and the equation of the circle is $(x-2)^2 + (y+1)^2 = 25$ substituting by the coordinates of the points B, C and D

B (2, 4) $\therefore (2-2)^2 + (4+1)^2 = 25 = r^2$

\therefore The point B lies on the circle

C (-3, 1) $\therefore (-3-2)^2 + (1+1)^2 = 29 > r^2$

\therefore The point C lies outside the circle

D (1, 2) $\therefore (1-2)^2 + (2+1)^2 = 10 < r^2$

\therefore The point D lies inside the circle

11 The centre of the circle is $(-3, 4)$

$\therefore r = 3$ length unit

(1) The length of the perpendicular

$$= \frac{|3 \times -3 - 4 \times 4 + 5|}{\sqrt{3^2 + 4^2}} = 4 > r$$

\therefore The straight line L_1 is outside the circle

(2) The length of the perpendicular

$$= \frac{|6 \times -3 - 8 \times 4 + 23|}{\sqrt{6^2 + (-8)^2}} = 2.7 < r$$

\therefore The straight line L_2 is secant to the circle

(3) The length of the perpendicular

$$= \frac{|3 \times -3 - 4 \times 4 + 10|}{\sqrt{3^2 + (-4)^2}} = 3 = r$$

\therefore The straight line L_3 is a tangent to the circle

12 $\bullet C_1 : (x-5)^2 + (y+2)^2 = 4$

$\therefore M_1 = (5, -2), r_1 = 2$ length unit

$\bullet C_2 : (x+7)^2 + (y-3)^2 = 1$

$\therefore M_2 = (-7, 3), r_2 = 1$ length unit

$\therefore r_1 + r_2 = 3$ length unit

$\therefore M_1M_2 = \sqrt{(5+7)^2 + (-2-3)^2} = 13$ length unit

$\therefore M_1M_2 > r_1 + r_2$

\therefore The two circles are disjoint (distant circles)

13 $\bullet C_1 : x^2 + y^2 - 10x - 8y + 16 = 0$

$\therefore M_1 = (5, 4)$

$\therefore r_1 = \sqrt{(-5)^2 + (-4)^2} - 16 = 5$ length unit

$\bullet C_2 : x^2 + y^2 + 14x + 10y - 26 = 0$

$\therefore M_2 = (-7, -5)$

$\therefore r_2 = \sqrt{7^2 + 5^2} + 26 = 10$ length unit

$\therefore r_1 + r_2 = 15$ length unit.

$\therefore M_1M_2 = \sqrt{(5+7)^2 + (4+5)^2} = 15$ length unit

$\therefore M_1M_2 = r_1 + r_2$

\therefore The two circles are touching externally

14 $\bullet C_1 : (x+2)^2 + (y+11)^2 = K$

$\therefore M_1 = (-2, -11), r_1 = \sqrt{K}$ length unit

$\bullet C_2 : (x-3)^2 + (y-1)^2 = 16$

$$\therefore M_2 = (3, 1), r_2 = 4 \text{ length unit}$$

If the two circles are touching externally

$$\therefore M_1 M_2 = r_1 + r_2$$

$$\therefore \sqrt{(3+2)^2 + (1+1)^2} = 4 + \sqrt{K}$$

$$\therefore 13 = 4 + \sqrt{K} \quad \therefore K = 81$$

If the two circles are touching internally

$$\therefore |r_2 - r_1| = M_1 M_2 \quad \therefore |4 - \sqrt{K}| = 13$$

$$\therefore 4 - \sqrt{K} = 13 \text{ refused or } 4 - \sqrt{K} = -13$$

$$\therefore \sqrt{K} = 4 + 13 = 17 \quad \therefore K = 289$$

15 The point of tangency :

$$x^2 + y^2 - 6x - 4y + 12 = x^2 + y^2 + 2x - 4y - 4$$

$$\therefore -6x - 4y + 12 = 2x - 4y - 4$$

$$\therefore -6x - 2x = -12 - 4$$

$$\therefore -8x = -16$$

$$\therefore x = 2$$

By substitution in the first circle equation by $x = 2$

$$\therefore (2)^2 + y^2 - 6 \times 2 - 4y + 12 = 0$$

$$\therefore y^2 - 4y + 4 = 0 \quad \therefore y = 2$$

\therefore The two circles intersected at one point $(2, 2)$

\therefore The two circles touch each other

, the equation of the circle whose center $(2, 2)$ and passes through the center of the second circle $(-1, 2)$

$$r = \sqrt{(2+1)^2 + (0)^2} = 3$$

$$\therefore \text{The equation : } (x-2)^2 + (y-2)^2 = 9$$

16 $x^2 + y^2 = 1$

$$\therefore (2a \cos \theta, 2a \sin \theta) \in \text{the circle}$$

$$\therefore (2a \cos \theta)^2 + (2a \sin \theta)^2 = 1$$

$$\therefore 4a^2 (\cos^2 \theta + \sin^2 \theta) = 1$$

$$\therefore 4a^2 = 1 \quad \therefore a = \pm \frac{1}{2}$$

17 (1) $\therefore l^2 + k^2 - c > 0$

$$\therefore (-1)^2 + (-2)^2 + h - 2 > 0$$

$$\therefore h > -3 \quad \therefore h \in]-3, \infty[$$

$$(2) \therefore l^2 + k^2 - c > 0$$

$$\therefore (2)^2 + (-3)^2 + h^2 - 4 > 0 \quad \therefore h^2 + 9 > 0$$

$$\therefore \text{For all values of } h \in \mathbb{R} \text{ the expression } h^2 + 9 > 0$$

$$\therefore h \in \mathbb{R}$$

$$(3) \therefore l^2 + k^2 - c > 0$$

$$\therefore (-2h)^2 + (-h)^2 - 10(h-1) > 0$$

$$\therefore 4h^2 + h^2 - 10h + 10 > 0$$

$$\therefore h^2 - 2h + 2 > 0$$

$$\therefore (h-1)^2 + 1 > 0 \text{ this will be satisfied for all values of } h \text{ which belongs to } \mathbb{R}$$

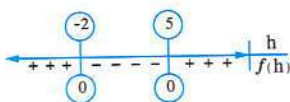
$$(4) \therefore l^2 + k^2 - c > 0$$

$$\therefore (3)^2 + (4)^2 - h^2 + 3h - 15 > 0$$

$$\therefore 9 + 16 - h^2 + 3h - 15 > 0$$

$$\therefore h^2 - 3h - 10 < 0$$

$$\therefore (h+2)(h-5) < 0$$



$$\therefore h \in]-2, 5[$$

$$(5) \therefore l^2 + k^2 - c > 0$$

$$\therefore h^2 + (-3h)^2 + 2h^2 - 12h + 3 > 0$$

$$\therefore h^2 + 9h^2 + 2h^2 - 12h + 3 > 0$$

$$\therefore 12h^2 - 12h + 3 > 0$$

$$\therefore 4h^2 - 4h + 1 > 0 \quad \therefore (2h-1)^2 > 0$$

This will be satisfies for all $h \in \mathbb{R}$

18 (1) $\therefore l^2 + k^2 - c > 0$

$$\therefore (-1)^2 + (2)^2 - 2a + 3 > 0$$

$$\therefore -2a > -8$$

$$\therefore a < 4 \quad \therefore a \in]-\infty, 4[$$

(2) \therefore The circle passes through the origin point

$$\therefore c = 0$$

$$\therefore 2a - 3 = 0$$

$$\therefore a = \frac{3}{2}$$

(3) \therefore The circle touches X-axis

$$\therefore c = l^2$$

$$\therefore 2a - 3 = (-1)^2$$

$$\therefore a = 2$$

(4) \therefore The circle touches y -axis

$$\therefore c = k^2 \quad \therefore 2a - 3 = 2^2 \quad \therefore a = \frac{7}{2}$$

(5) $\therefore M = (1, -2)$

\therefore The circle touches the straight line

$$3x + 4y + 15 = 0$$

$$\therefore r = \frac{|3 \times 1 + 4 \times -2 + 15|}{\sqrt{3^2 + 4^2}} = 2 \text{ length units}$$

$$\therefore \sqrt{l^2 + k^2} - c = r$$

$$\therefore \sqrt{(-1)^2 + (2)^2} - 2a + 3 = 2 \text{ squaring the two sides}$$

$$\therefore 1 + 4 - 2a + 3 = 4 \quad \therefore a = 2$$

(6) $\therefore r = 7$ length unit

$$\therefore \sqrt{(-1)^2 + (2)^2} - 2a + 3 = 7$$

squaring the two sides

$$\therefore 1 + 4 - 2a + 3 = 49 \quad \therefore a = -\frac{41}{2}$$

19

(1) \therefore The circle touches the straight line $X = 2$

$\therefore r = 3$ length units

\therefore The equation of the circle

$$\text{is } (X - 5)^2 + (y - 4)^2 = 9$$

$$\text{i.e. } X^2 + y^2 - 10X - 8y + 32 = 0$$

(2) The equation of the straight line

$$\text{is } \frac{y-7}{X-3} = \frac{7-3}{3+1} \quad \text{i.e. } -X + y - 4 = 0$$

$$\therefore r = \frac{|-1 \times 5 + 1 \times 3 - 4|}{\sqrt{(-1)^2 + (1)^2}} = 3\sqrt{2} \text{ length units}$$

\therefore The equation of the circle

$$\text{is } (X - 5)^2 + (y - 3)^2 = 18$$

$$\text{i.e. } X^2 + y^2 - 10X - 6y + 16 = 0$$

(3) $r = 3$, $M = (4, 5)$

\therefore The equation of the circle

$$\text{is } (X - 4)^2 + (y - 5)^2 = 9$$

$$\text{i.e. } X^2 + y^2 - 8X - 10y + 32 = 0$$

(4) \therefore The circle touches X -axis at $(4, 0)$, $r = 5$

$\therefore M = (4, 5)$ or $(4, -5)$

\therefore The equation of the circle

$$\text{is } (X - 4)^2 + (y - 5)^2 = 25$$

$$\text{i.e. } X^2 + y^2 - 8X - 10y + 16 = 0$$

$$\text{or } (X - 4)^2 + (y + 5)^2 = 25$$

$$\text{i.e. } X^2 + y^2 - 8X + 10y + 16 = 0$$

(5) \therefore The circle touches y -axis

$$\therefore M = (3 \frac{1}{2}, -4) \text{ or } (-3 \frac{1}{2}, -4)$$

\therefore The equation of the circle

$$\text{is } (X - 3 \frac{1}{2})^2 + (y + 4)^2 = 12.25$$

$$\text{i.e. } X^2 + y^2 - 7X + 8y + 16 = 0$$

or the equation of the circle

$$\text{is } (X + 3 \frac{1}{2})^2 + (y + 4)^2 = 12.25$$

$$\text{i.e. } X^2 + y^2 + 7X + 8y + 16 = 0$$

(6) \therefore The circle touches the two coordinate axes

\therefore the point $(-2, -4)$ in the third quadrant.

\therefore The centre $M = (-r, -r)$

\therefore The equation of the circle

$$\text{is } (X + r)^2 + (y + r)^2 = r^2$$

$\therefore (-2, -4) \in$ the circle

$$\therefore (-2 + r)^2 + (-4 + r)^2 = r^2$$

$$\therefore 4 + r^2 - 4r + 16 + r^2 - 8r = r^2$$

$$\therefore r^2 - 12r + 20 = 0$$

$$\therefore (r - 2)(r - 10) = 0$$

$$\therefore r = 2, \text{ then } M = (-2, -2)$$

$$\text{or } r = 10, \text{ then } M = (-10, -10)$$

\therefore There exist two circles, they are

$$(X + 2)^2 + (y + 2)^2 = 4$$

$$\text{i.e. } X^2 + y^2 + 4X + 4y + 4 = 0$$

$$\text{or } (X + 10)^2 + (y + 10)^2 = 100$$

$$\text{i.e. } X^2 + y^2 + 20X + 20y + 100 = 0$$

(7) \therefore The circle touches X -axis at $(-3, 0)$ and touches y -axis.

$\therefore r = 3$ length units

\therefore There exist two circles whose centres are $(-3, 3)$ $(-3, -3)$

The equation of the first equation

$$\text{is } (X + 3)^2 + (y - 3)^2 = 9$$

$$\text{i.e. } X^2 + y^2 + 6X - 6y + 9 = 0$$

The equation of the second equation

$$(X + 3)^2 + (y + 3)^2 = 9$$

$$\text{i.e. } X^2 + y^2 + 6X + 6y + 9 = 0$$

(8) $r = \sqrt{2^2 + (2\sqrt{3})^2} = 4$ length units

∴ The centre of the circle

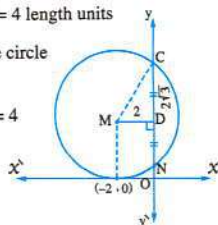
$M = (-2, 4)$

∴ $c = 4 + 16 - 16 = 4$

∴ The equation of the circle

is $x^2 + y^2$

$+ 4x - 8y + 4 = 0$



(9) $r = \sqrt{(1)^2 + (2\sqrt{6})^2} = 5$

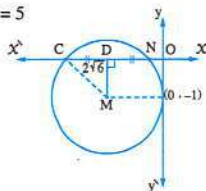
∴ The centre of the circle

M is $(-5, -1)$

∴ $c = k^2 = 1$

∴ The equation of the circle

is $x^2 + y^2 + 10x + 2y + 1 = 0$



(10) ∴ The circle touches X-axis.

$c = l^2$

∴ The equation of the circle

is $x^2 + y^2 + 2lx + 2ky + l^2 = 0$

∴ $(2, 1)$ satisfies the equation.

∴ $4 + 1 + 4l + 2k + l^2 = 0$

∴ $4l + 2k + l^2 = -5$

(1)

∴ $(-5, 2)$ satisfies the equation

∴ $25 + 4 - 10l + 4k + l^2 = 0$

∴ $-10l + 4k + l^2 = -29$

(2)

Multiplying (1) by 2

∴ $8l + 4k + 2l^2 = -10$

(3)

Subtracting (2) from (3):

∴ $l^2 + 18l - 19 = 0$ ∴ $(l - 1)(l + 19) = 0$

∴ $l = 1$ or $l = -19$, then $k = -5$ or $k = -145$

∴ There are two circles

The equation of the first circle

is $x^2 + y^2 + 2x - 10y + 1 = 0$

and the equation of the other

is $x^2 + y^2 - 38x - 290y + 361 = 0$

(11) ∴ The circle touches y-axis.

∴ $c = k^2$

∴ The equation of the circle

is $x^2 + y^2 + 2lx + 2ky + k^2 = 0$

∴ $(-4, 2)$ verifies the equation

∴ $16 + 4 - 8l + 4k + k^2 = 0$

i.e. $-8l + 4k + k^2 = -20$

(1)

∴ $(-1, 2)$ verifies the equation.

∴ $1 + 4 - 2l + 4k + k^2 = 0$

i.e. $-2l + 4k + k^2 = -5$

(2)

Subtracting (1) from (2):

∴ $6l = 15$ ∴ $l = \frac{5}{2}$

and from the equation (2)

∴ $-5 + 4k + k^2 = -5$ ∴ $k^2 + 4k = 0$

∴ $k(k + 4) = 0$

∴ $k = 0$ or $k = -4$

∴ There are two circles

The equation of the first circle

is $x^2 + y^2 + 5x = 0$

The equation of the second circle

is $x^2 + y^2 + 5x - 8y + 16 = 0$

(12) ∴ The centre of the circle lies on X-axis

∴ The equation of the circle

is $x^2 + y^2 + 2lx + c = 0$

∴ $(1, 3)$ verifies the equation

∴ $1 + 9 + 2l + c = 0$, then $2l + c = -10$ (1)

∴ $(2, -4)$ verifies the equation.

∴ $4 + 16 + 4l + c = 0$

, then $4l + c = -20$

(2)

From (1) and (2): ∴ $l = -5$, $c = 0$

∴ The equation of the circle is $x^2 + y^2 - 10x = 0$

(13) $M = (6, 8)$

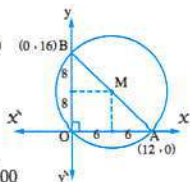
$AB = \sqrt{12^2 + 16^2} = 20$

∴ $r = 10$ length units

∴ The equation of the circle is

$(x - 6)^2 + (y - 8)^2 = 100$

i.e. $x^2 + y^2 - 12x - 16y = 0$



- (14) \therefore The centre of the circle lies on the straight line $y - x = 1$

\therefore The centre $(-\ell, -k)$ verifies the equation.

$$\therefore -k + \ell = 1 \quad (1)$$

\therefore The equation of the circle

$$\text{is } x^2 + y^2 + 2\ell x + 2ky + c = 0$$

$\therefore (-2, 4)$ verifies the equation of the circle.

$$\therefore 4 + 16 - 4\ell + 8k + c = 0, \text{ then}$$

$$-4\ell + 8k + c = -20 \quad (2)$$

$\therefore (6, 8)$ verifies the equation of the circle

$$\therefore 36 + 64 + 12\ell + 16k + c = 0, \text{ then}$$

$$12\ell + 16k + c = -100 \quad (3)$$

Subtracting (2) from (3):

$$\therefore 16\ell + 8k = -80$$

$$\therefore 2\ell + k = -10 \quad (4)$$

Adding (4) and (1):

$$\therefore 3\ell = -9 \quad \therefore \ell = -3$$

From (1): $\therefore k = -4$ and from (2) $c = \text{zero}$

\therefore The equation of the circle

$$\text{is } x^2 + y^2 - 6x - 8y = 0$$

$$(15) \therefore r^2 = \ell^2 + k^2 - c \quad \therefore 85 = \ell^2 + k^2 - c \quad (1)$$

The equation of the circle

$$\text{is } x^2 + y^2 + 2\ell x + 2ky + c = 0$$

$\therefore (-1, 2)$ verifies the equation of the circle.

$$\therefore 1 + 4 - 2\ell + 4k + c = 0$$

$$\text{i.e. } -2\ell + 4k + c = -5 \quad (2)$$

$\therefore (3, 4)$ verifies the equation of the circle.

$$\therefore 9 + 16 + 6\ell + 8k + c = 0$$

$$\text{i.e. } 6\ell + 8k + c = -25 \quad (3)$$

Subtract (2) from (3):

$$8\ell + 4k = -20$$

$$\therefore k = -5 - 2\ell \quad (4) \text{ substituting in (2)}$$

$$\therefore -2\ell + 4(-5 - 2\ell) + c = -5$$

$$\therefore c = 15 + 10\ell \quad (5)$$

Substituting from (4) and (5) in the equation (1)

$$\therefore 85 = \ell^2 + (-5 - 2\ell)^2 - 15 - 10\ell$$

$$\therefore \ell^2 + 25 + 20\ell + 4\ell^2 - 15 - 10\ell - 85 = 0$$

$$\therefore 5\ell^2 + 10\ell - 75 = 0$$

$$\therefore \ell^2 + 2\ell - 15 = 0$$

$$\therefore (\ell - 3)(\ell + 5) = 0 \quad \therefore \ell = 3 \text{ or } \ell = -5$$

$$\therefore k = -11 \text{ or } k = 5, c = 45 \text{ or } c = -35$$

\therefore There are two equations:

$$x^2 + y^2 + 6x - 22y + 45 = 0$$

$$\text{or } x^2 + y^2 - 10x + 10y - 35 = 0$$

- (16) To find the intersection points with x -axis, put $y = 0$ in the circle equation

$$\therefore x^2 + 2x = 0$$

$$\therefore x(x + 2) = 0 \quad \therefore x = 0 \text{ or } x = -2$$

\therefore The intersection points are $(0, 0)$, $(-2, 0)$

$\therefore M$ = the center of the required circle is

$$\text{the mid point of } \overline{AB} = \left(\frac{0-2}{2}, \frac{0}{2} \right)$$

$$\therefore M = (-1, 0)$$

$$\text{and its radius } r = MA = \sqrt{(-1)^2 + (0)^2} = 1$$

$$\therefore \text{The circle equation: } (x + 1)^2 + y^2 = 1$$

$$20 \therefore \ell = \frac{1}{2}, k = -2, c = -2$$

$$\therefore r^2 = \frac{1}{4} + 4 + 2 = 6 \frac{1}{4}$$

$$\begin{aligned} \therefore \text{The area of } \Delta &= \frac{n}{2} r^2 \sin \left(\frac{360^\circ}{n} \right) \\ &= \frac{3}{2} \times 6 \frac{1}{4} \times \sin 120^\circ \\ &= \left(\frac{75\sqrt{3}}{16} \right) \text{ square units} \end{aligned}$$

$$21 \therefore \ell = 3, k = -6, c = 5$$

$$\therefore r^2 = 9 + 36 - 5 = 40$$

\therefore The area of the regular pentagon

$$\begin{aligned} &= \frac{n}{2} r^2 \sin \left(\frac{360^\circ}{n} \right) \\ &= \frac{5}{2} \times 40 \times \sin 72^\circ \approx 95.10565 \text{ square units} \end{aligned}$$

\therefore The square unit represents an area of $(5)^2 = 25 \text{ cm}^2$

$$\begin{aligned} \therefore \text{The area of the required pentagon} \\ = 95.10565 \times 25 \approx 2378 \text{ cm}^2 \end{aligned}$$

22 $l = -5$, $k = 3$, $c = 25$

$$\therefore r^2 = 25 + 9 - 25 = 9$$

\therefore The area of the regular hexagon

$$= \frac{n}{2} r^2 \sin \left(\frac{360^\circ}{n} \right)$$

$$= \frac{6}{2} \times 9 \sin 60^\circ = \left(\frac{27\sqrt{3}}{2} \right) \text{ square units}$$

23 $r^2 = 16$

\therefore The area of the regular polygon

$$= \frac{n}{2} r^2 \sin \left(\frac{360^\circ}{n} \right)$$

$$= \frac{12}{2} \times 16 \times \sin 30^\circ = 48 \text{ square unit}$$

24 The centre of the circle is the point of intersection of the two diameters.

$$3x + y = -2 \quad (1)$$

$$4x - y = 16 \quad (2)$$

From (1) and (2) :

$$\therefore x = 2, y = -8$$

\therefore The centre of the circle $M = (2, -8)$

\therefore The equation of the circle is $(x-2)^2 + (y+8)^2 = 25$

$$\text{i.e. } x^2 + y^2 - 4x + 16y + 43 = 0$$

Substituting by $x = 5$ and $y = -4$

$$\begin{aligned} \therefore \text{The left hand side} &= (5)^2 + (-4)^2 - 4(5) \\ &\quad + 16(-4) + 43 = 0 \\ &= \text{the right hand side} \end{aligned}$$

$\therefore (5, -4) \in \text{the circle.}$

25 $\vec{r} = (1, 5) + k(1, 2) \quad \text{i.e. } \frac{y-5}{x-1} = 2$

$$\therefore 2x - 2 = y - 5$$

$$\therefore 2x - y = -3 \quad (1), x + y = 0 \quad (2)$$

From (1) and (2)

\therefore The point of intersection of the two diameters is $(-1, 1)$

\therefore The centre of the circle $M = (-1, 1)$

$$\therefore x^2 + y^2 - 2x \cos \theta - 2y \sin \theta - 8 = 0$$

$$\therefore l = -\cos \theta, k = -\sin \theta, c = -8$$

$$\begin{aligned} r &= \sqrt{\cos^2 \theta + \sin^2 \theta + 8} = \sqrt{1+8} \\ &= 3 \text{ length units} \end{aligned}$$

\therefore The equation of the required circle is $(x+1)^2 + (y-1)^2 = 9$

26 The intersection points of the two circles

$$x^2 + y^2 - 10x = x^2 + y^2 + 2x - 12$$

$$\therefore -10x - 2x = -12$$

$$\therefore -12x = -12$$

$$\therefore x = 1$$

By substitution in the first circle equation to find the value of y by $x = 1$

$$\therefore (1)^2 + y^2 - 10(1) = 0$$

$$\therefore y^2 = 9$$

$$\therefore y = \pm 3$$

\therefore The intersection points are $(1, 3)$, $(1, -3)$

(1) The centre of the circle is $(0, 0)$:

distance between first point and the centre

$$= \sqrt{(1-0)^2 + (3-0)^2} = \sqrt{10}$$

distance between second point and the centre

$$= \sqrt{(1-0)^2 + (-3-0)^2} = \sqrt{10}$$

\therefore The two points lie on a circle of centre

$(0, 0)$ and radius length $\sqrt{10}$ length unit

\therefore Equation of the circle is : $x^2 + y^2 = 10$

(2) The centre of the circle is $(2, 0)$:

distance between first point and the centre

$$= \sqrt{(1-2)^2 + (3-0)^2} = \sqrt{10}$$

distance between second point and the centre

$$= \sqrt{(1-2)^2 + (-3-0)^2} = \sqrt{10}$$

\therefore The two points lie on a circle of centre

$(2, 0)$ and radius length $\sqrt{10}$ length unit

\therefore Equation of the circle is : $(x-2)^2 + y^2 = 10$

27 $\therefore MA = \sqrt{(-5)^2 + (-4)^2} = \sqrt{41}$ length units

$$MB = \sqrt{(-4)^2 + (-5)^2} = \sqrt{41} \text{ length units}$$

$$MC = \sqrt{(-4)^2 + 5^2} = \sqrt{41} \text{ length units}$$

$$\therefore MA = MB = MC$$

$\therefore A, B$ and C lie on the same circle.

The equation of the circle

$$\text{is } (x+5)^2 + (y+5)^2 = 41$$

28 The equation of the circle

$$\text{is } x^2 + y^2 + 2lx + 2ky + c = 0$$

∴ The points A, B and C lie on the circle.

$$\therefore 9 + 4 + 6l - 4k + c = 0 \quad (1)$$

$$9 + 64 + 6l + 16k + c = 0 \quad (2)$$

$$1 - 2l + c = 0 \quad (3)$$

From (1), (2) and (3)

$$\therefore \text{The centre of the circle } M = (3, 3)$$

∴ The equation of the circle

$$\text{is } x^2 + y^2 - 6x - 6y - 7 = 0$$

$$\therefore \text{The midpoint of } \overline{AB} = \left(\frac{3+3}{2}, \frac{-2+8}{2} \right) \\ = (3, 3) = M$$

∴ \overline{AB} is a diameter in the circle.

29 $AB = \sqrt{(8-0)^2 + (0-6)^2} = 10$

$$BC = \sqrt{0 + (6-0)^2} = 6$$

$$AC = \sqrt{(8-0)^2 + 0} = 8$$

$$\therefore (AB)^2 = (BC)^2 + (AC)^2$$

∴ $\triangle ABC$ is right angled triangle at $\angle C$

∴ \overline{AB} is a diameter in the circle which passes through its vertices

$$\text{The center } M = \left(\frac{8+0}{2}, \frac{0+6}{2} \right) = (4, 3), r = 5$$

$$\therefore \text{The equation : } (x-4)^2 + (y-3)^2 = 25$$

30 $AB = \sqrt{6^2 + (0)^2} = 6, BC = \sqrt{3^2 + (-3\sqrt{3})^2} = 6$

$$CA = \sqrt{(-3)^2 + (-3\sqrt{3})^2} = 6$$

∴ $\triangle ABC$ is an equilateral triangle.

∴ The centre of its circumcircle

$$= \left(\frac{-2+4+1}{3}, \frac{0+0+3\sqrt{3}}{3} \right) = (1, \sqrt{3})$$

$$\therefore r = MA = \sqrt{(-3)^2 + (-3\sqrt{3})^2}$$

$$= 2\sqrt{3} \text{ length units}$$

∴ The equation of the circle

$$\text{is } (x-1)^2 + (y-\sqrt{3})^2 = 12$$

31 $x^2 + y^2 + 2lx + 2ky + c = 0$

$$\therefore (2, -1), (-2, 0), (0, -9)$$

Verify the equation of the circle.

$$\therefore 4 + 1 + 4l - 2k + c = 0$$

$$\text{i.e. } 4l - 2k + c = -5 \quad (1)$$

$$4 - 4l + c = 0 \quad (2), 81 - 18k + c = 0 \quad (3)$$

From (1), (2) and (3):

$$\therefore l = 1, c = 0, k = \frac{9}{2}$$

$$\therefore \text{The equation is } x^2 + y^2 + 2x + 9y = 0$$

$$\therefore \text{the centre of the circle } M = (-1, -4.5)$$

$$r = \sqrt{1 + (4.5)^2} = \frac{\sqrt{85}}{2} \text{ length unit.}$$

32 let the circle which passes through the points A, B and C be

$$x^2 + y^2 + 2lx + 2ky + c = 0$$

∴ The points verify the equation of the circle.

$$\therefore 9 + 6l + c = 0 \quad (1)$$

$$81 + 18k + c = 0 \quad (2)$$

$$1 + 2k + c = 0 \quad (3)$$

From (1), (2) and (3):

$$\therefore l = -3, k = -5, c = 9$$

∴ The equation of the circle

$$\text{is } x^2 + y^2 - 6x - 10y + 9 = 0$$

Substituting by the coordinates of the point D

$$\therefore \text{The left hand side} = (-1)^2 + (2)^2 - 6(-1) - 10(2) + 9 \\ = 0$$

= The right hand side.

∴ The point D lies on the circle.

i.e. The quadrilateral ABCD is cyclic.

33 (1) $x^2 + y^2 - 8x + 6y = 0$

$$(2) x^2 + y^2 - 4x + 4y + 4 = 0$$

$$(3) x^2 + y^2 - 12y + 20 = 0$$

$$(4) x^2 + y^2 - 12x - 4y + 36 = 0$$

$$(5) x^2 + y^2 - 12x - 15y + 36 = 0$$

$$(6) x^2 + y^2 - 4x - 2y - 11 = 0$$

$$(7) x^2 + y^2 - 24x - 8\sqrt{3}y + 144 = 0$$

$$(8) x^2 + y^2 - 8x - 4y + 16 = 0$$

$$(9) x^2 + y^2 - 2x - 8y + 16 = 0$$

$$(10) x^2 + y^2 - 2x - 2y + 1 = 0$$

$$(11) 2x^2 + 2y^2 - 17x + 8 = 0$$

$$(12) x^2 + y^2 - 6x - 2\sqrt{3}y + 9 = 0$$

34 From the geometry of the figure :

$$(4)^2 = 8 \times BD$$

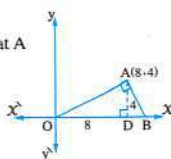
$$\therefore BD = 2$$

\therefore The point B (10, 0)

$\therefore \triangle ABC$ is right angled at A

$\therefore \overline{BO}$ is a diameter

in the circle which passes through the points A, B and O



\therefore The centre of the circle is (5, 0), $r = 5$ units.

\therefore The equation of the circle

$$\text{is } (x-5)^2 + y^2 = 25$$

$$\text{i.e. } x^2 + y^2 - 10x = 0$$

Third Higher skills

1

$$(1) d \quad (2) c \quad (3) b \quad (4) d$$

$$(5) b \quad (6) c \quad (7) b \quad (8) b$$

Instructions to solve 1 :

$$(1) \text{ Put } k-2 = 2-k \quad \therefore 2k = 4 \quad \therefore k = 2$$

\therefore then the equation be $-2x + 6y - 25 = 0$

«equation of straight line»

\therefore at $k \neq 2$, then the coefficient of $x^2 \neq$ the coefficient of y^2

\therefore The equation does not express a circle whatever the value of k

$$(2) \therefore \text{ The equation of the base of the cone is } x^2 + y^2 = 64$$

\therefore The radius length of the base of the cone = 8 length unit.

$$\therefore \text{ The volume of the cone } = \frac{1}{3} \pi r^2 h$$

$$= \frac{1}{3} \pi \times 64 \times 6$$

$$= 128 \pi \text{ cubic unit.}$$

$$(3) \therefore \text{ The centre of the circle } = (7, 5)$$

$r = 4$ length unit.

\therefore The distance between the centre and the y-axis = 7 length unit.

\therefore The least distance between the y-axis and any point on the circle = $7 - 4 = 3$ length unit.

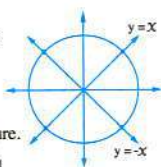
$$(4) \therefore \text{ The centre of the circle that touches the two coordinates axes must lie on the straight line } y = x \text{ or } y = -x$$

\therefore the circle $x^2 + y^2 = 25$

intersects the two lines

$y = x$ and $y = -x$ at four points as in the opposite figure.

\therefore The number of circles = 4



(5) From the opposite figure :

$\therefore \triangle AOC$ is a right-angled triangle

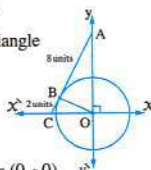
at O and $\overline{OB} \perp \overline{AC}$

$$\therefore (OB)^2 = 8 \times 2 = 16$$

$\therefore OB = 4$ length unit.

\therefore the centre of the circle = (0, 0)

$$\therefore \text{ The equation of the circle is } x^2 + y^2 = 16$$



$$(6) \therefore OB \times OA = OC \times OD$$

$$\therefore 4 \times OA = 8 \times 6$$

$$\therefore OA = 12$$

$$\therefore A = (12, 0)$$

\therefore draw $\overline{ME} \perp \overline{AB}$

$\therefore \overline{MN} \perp \overline{DC}$

$\therefore E$ is midpoint of \overline{AB}

$\therefore E(4, 0)$, N is midpoint of \overline{DC}

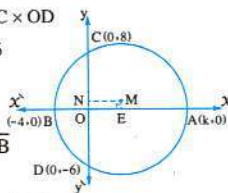
$$\therefore N(0, 1)$$

$$\therefore M = (4, 1)$$

$$\therefore r = MA = \sqrt{(12-4)^2 + (0-1)^2} = \sqrt{65}$$

\therefore The equation of the circle is

$$(x-4)^2 + (y-1)^2 = 65$$



- (7) ∴ The centre of the given

circle (M) = (5, -2)

, ∴ $\overline{MA} \perp \overline{AO}$

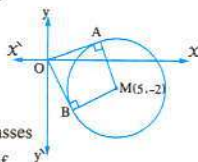
, $\overline{MB} \perp \overline{OB}$

∴ There is a circle passes through the vertices of the figure AMBO and its

diameter \overline{OM} and its centre is the midpoint of \overline{OM}

∴ The centre is $\left(\frac{5+0}{2}, \frac{-2+0}{2}\right) = \left(\frac{5}{2}, -1\right)$

and it is the same circle that passes through the vertices of the ΔAOB



- (8) According to the first circle :

$M_1 = (5, 5)$

, $r_1 = 5\sqrt{2}$

According to the second circle :

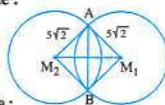
$M_2 = (-3, -1)$, $r_2 = 5\sqrt{2}$

, ∴ the length of $\overline{M_1M_2} = \sqrt{(5+3)^2 + (5+1)^2}$
= 10 length unit

, ∴ $(M_1M_2)^2 = (AM_1)^2 + (AM_2)^2$

∴ The figure $A M_1 B M_2$ is a square

∴ $AB = 10$ length unit.



- 2 ∴ The circle N touches the two coordinates axes in the 3rd quadrant.

∴ N is $(-\ell, -\ell)$ satisfies the equation of the straight line $y = 2x + 1$

∴ $-\ell = -2\ell + 1$

∴ $\ell = 1$

∴ The centre of the circle N is $(-1, -1)$

∴ The equation of the circle N

is $(x+1)^2 + (y+1)^2 = 1$

, then $x^2 + y^2 + 2x + 2y + 1 = 0$

∴ The circle M touches the two coordinate axes in the 2nd quadrant.

∴ M is $(-k, k)$ verifies the equation of the straight line $y = 2x + 1$

∴ $k = -2k + 1$

∴ $k = \frac{1}{3}$

∴ The centre of the circle M is $\left(-\frac{1}{3}, \frac{1}{3}\right)$

∴ The equation of the circle M

is $\left(x + \frac{1}{3}\right)^2 + \left(y - \frac{1}{3}\right)^2 = \left(\frac{1}{3}\right)^2$

i.e. $x^2 + y^2 + \frac{2}{3}x - \frac{2}{3}y + \frac{1}{9} = 0$

- 3 Draw $\overline{MD} \perp \overline{OY}$

let the radius length of the circle M be r length unit

In ΔMND :

$(8+r)^2 = 8^2 + (8-r)^2$

∴ $64 + r^2 + 16r = 64 + 64 + r^2 - 16r$

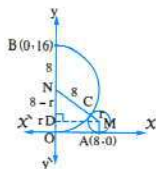
∴ $32r = 64$ ∴ $r = 2$

∴ The centre of the circle M is $(8, 2)$

∴ The equation of the circle M

is $(x-8)^2 + (y-2)^2 = 4$

i.e. $x^2 + y^2 - 16x - 4y + 64 = 0$



Life Applications

- 1

$r = \sqrt{\ell^2 + k^2 - c} = \sqrt{(3)^2 + (-4)^2 - 11}$

= $\sqrt{14}$ length unit

∴ The area of the circle = $\pi r^2 = 14\pi$

∴ The square unit in the plane represents $(5)^2 = 25 \text{ cm}^2$

∴ The area of the square = $14 \times \frac{27}{7} \times 25 = 1100 \text{ cm}^2$

- 2 ∴ The equation of the circle

is $(x-7)^2 + (y+9)^2 = (30)^2$

$BA = \sqrt{(25-7)^2 + (-30+9)^2} \approx 27.66$

∴ $BA < r$

∴ The radar can observe the ship whose position is at (B)

$$3 \therefore l = -2, k = 6, c = -60$$

$$\therefore r^2 = 4 + 36 + 60 = 100$$

\therefore The area of the regular octagon

$$= \frac{n}{2} r^2 \sin \left(\frac{360^\circ}{n} \right) = \frac{8}{2} \times 100 \times \sin 45^\circ \\ = 200\sqrt{2} \text{ square units}$$

4

(1) The pulley A touches the coordinate axes and its radius length = 5 units.

\therefore The centre of its circle M (5, 5)

\therefore Its equation is $(X - 5)^2 + (y - 5)^2 = 25$

$$\text{i.e. } X^2 + y^2 - 10X - 10y + 25 = 0$$

(2) \therefore The equation of the circle of the pulley (B)

$$\text{is } X^2 + y^2 + 14X + 45 = 0$$

$\therefore l = 7, k = 0$

Its centre N = (-7, 0)

\therefore The distance between the two centres.

$$= \sqrt{(5+7)^2 + 5^2} = 13 \text{ length unit.}$$

\therefore Each unit in the coordinates plane represents 6 cm.

\therefore The distance between the two centres of the two pulleys = $13 \times 6 = 78$ cm.

5

\therefore The maximum height between the two edges.

= 10 units, let r_1 is the radius of the great gear and r_2 is the radius of the small gear

$$\therefore 2r_1 + 2r_2 = 10$$

$$\therefore r_1 + r_2 = 5$$

\therefore The centre of the great gear is (5, 4) and the centre of the small gear is (5, 9)

$$r_1^2 = 25 + 16 - 32 = 9$$

$$\therefore r_1 = 3 \text{ length units}$$

$$\therefore r_2 = 2 \text{ length units}$$

\therefore The equation of the circle of the small gear

$$\text{is } (X - 5)^2 + (y - 9)^2 = 4$$

$$\text{i.e. } X^2 + y^2 - 10X - 18y + 102 = 0$$

1

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First

Multiple choice questions

Choose the correct answer from the given ones :

- (1) Two forces of magnitudes 3 , F newton and the measure of the angle between them is 120° , if their resultant is perpendicular to the first force , so the value of F in newton is

(a) 1.5 (b) 3 (c) $3\sqrt{3}$ (d) 6

- (2) If the resultant of two forces acting at a point reached to its maximum value , then the measure of the included angle between the two forces =

(a) zero (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) π

- (3) Two forces of magnitudes 6 N. and 8 N. , if the magnitude of their resultant is 10 N. , then the measure of the angle between the two forces is

(a) 0° (b) 60° (c) 90° (d) 180°

- (4) A forces of magnitude 6 newton acts in direction of north. It is resolved into two perpendicular components , so its component in direction of the East equals newton.

(a) zero (b) 3 (c) $3\sqrt{2}$ (d) 6

- (5) If a body of weight (W) is placed on a smooth plane inclined to horizontal by angle θ , so the component of its weight in direction of the plane equals

(a) W (b) $W \sin \theta$ (c) $W \cos \theta$ (d) $W \tan \theta$

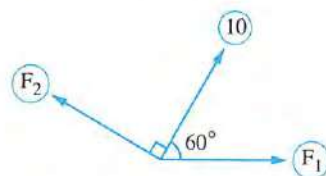
- (6) In the opposite figure :

If the force 10 N is resolved into two components

F_1 and F_2 inclined to the force by 60° and 90° respectively

, then $F_2 = \dots\dots\dots$ N.

(a) $5\sqrt{3}$ (b) 10 (c) $10\sqrt{3}$ (d) 20



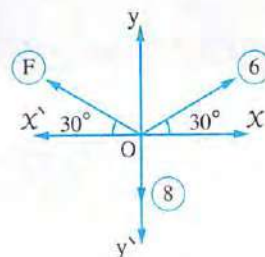
- (7) If $\vec{F}_1 = 3\vec{i} - 2\vec{j}$, $\vec{F}_2 = a\vec{i} - \vec{j}$, $\vec{F}_3 = 4\vec{i} - b\vec{j}$, $\vec{R} = 6\vec{i} - 4\vec{j}$, then (a , b) =

(a) (1 , -1) (b) (-1 , 1) (c) (-1 , -1) (d) (1 , 1)

- (8) If $\vec{F}_1 = 5\vec{i}$, $\vec{F}_2 = 7\vec{i} - 5\vec{j}$, then $\|\vec{R}\| = \dots\dots\dots$ force unit.

(a) 12 (b) 5 (c) 13 (d) $\sqrt{73}$

- (9) If the resultant of the forces in the given figure acts in direction of y-axis, then $F = \dots\dots\dots$ newton.

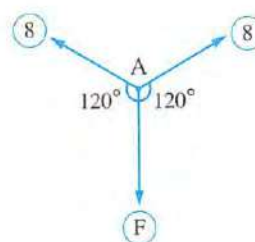


- (a) 2 (b) 6
(c) 8 (d) 14

- (10) If a force of magnitude (F) is in equilibrium with two forces of magnitudes 5 and 3 newton and the measure of the angle between them is 60° , then $F = \dots\dots\dots$ newton.

- (a) $\sqrt{19}$ (b) $\sqrt{34}$ (c) 7 (d) 15

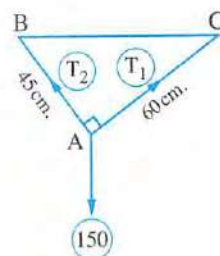
- (11) A is a particle balanced under the effect of the three forces as shown in the opposite figure where \vec{F} is balanced with two forces the magnitude of each is 8 newton and the angle between them is a measure 120° , then $F = \dots\dots\dots$ newton.



- (a) zero (b) 8
(c) 16 (d) $8 \sin 120^\circ$

- (12) In the opposite figure :

A body of weight 150 gm.wt. is in equilibrium by suspending it by two perpendicular strings of lengths 60 cm. and 45 cm. and the other two ends C and B are on a horizontal line, then $T_2 - T_1 = \dots\dots\dots$ gm.wt.



- (a) 120 (b) 90 (c) 60 (d) 30

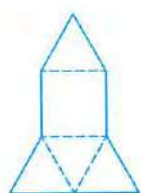
- (13) The two planes coincident if they have $\dots\dots\dots$ in common.

- (a) only one point (b) two points
(c) three collinear points (d) three non-collinear points

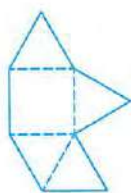
- (14) If the straight line $L \parallel$ the plane X and $A \in X$, then $L \cap X = \dots\dots\dots$

- (a) \emptyset (b) L (c) X (d) $\{A\}$

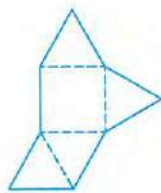
- (15) Which of the following nets does not make a regular quadrilateral pyramid when it folded ?



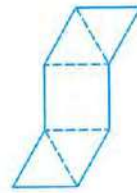
(a)



(b)

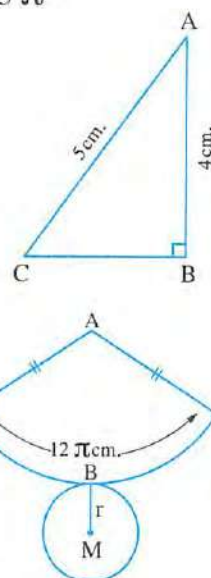


(c)



(d)

- (16) A regular pyramid whose volume is 12 cm^3 and its base area is 4 cm^2 , then its height = cm.
 (a) 3 (b) 6 (c) 9 (d) 2
- (17) A right circular cone, the radius length of its base = 15 cm. and its height = 20 cm., then its lateral area = cm^2 .
 (a) 375π (b) 600π (c) 1500π (d) 1875π
- (18) If the triangle ABC is rotated about \overleftrightarrow{AB} with a full turn, then the volume of the formed solid = cm^3 .
 (a) 6π (b) 8π
 (c) 10π (d) 12π
- (19) The opposite net describes a solid its volume = $96 \pi \text{ cm}^3$, then its total area = cm^2 .
 (a) 16π (b) 64π
 (c) 48π (d) 96π
- (20) The circumference of the circle whose equation is $x^2 + y^2 = 8$ is length units.
 (a) 8π (b) 64π (c) $2\sqrt{2} \pi$ (d) $4\sqrt{2} \pi$
- (21) The equation of the circle whose centre (1, 2) and touches the line : $3x + 4y + 9 = 0$ is
 (a) $x^2 + y^2 - 2x - 4y = 16$ (b) $x^2 + y^2 + 2x + 4y - 11 = 0$
 (c) $x^2 + y^2 + 2x + 4y - 16 = 0$ (d) $x^2 + y^2 - 2x - 4y = 11$



Second Essay questions

Answer the following questions :

- 1 ABCDEF is a uniform hexagon, the forces of magnitudes 8, $6\sqrt{3}$, 5 and $4\sqrt{3}$ newton act on \overrightarrow{AB} , \overrightarrow{AC} , \overrightarrow{AD} , \overrightarrow{AE} respectively, find the magnitude and the direction of their resultant.
- 2 A homogeneous smooth sphere its radius length is 10 cm., its weight = 30 gm.wt. is in equilibrium by a string of length 10 cm. attached to a point of its surface and the other end of the string is fixed at the point in vertical smooth wall, find the tension in the string and the reaction of the wall on the sphere.
- 3 A regular quadrilateral pyramid, the side length of its base 18 cm. if its volume is 1296 cm^3 , then find the slant height and lateral surface area.

**First Multiple choice questions****Choose the correct answer from the given ones :**

- (1) The magnitude of maximum resultant force of the two forces F_1 , F_2 is 15 N. and the minimum resultant one is 5 N. , then $F_1 > F_2$, then $F_1^2 - F_2^2 = \dots\dots\dots$ N.
 (a) 20 (b) 75 (c) 15 (d) 10
- (2) The magnitude of resultant of the two forces $F_1 = 8$ N. , $F_2 = 9$ N. act on the same direction = $\dots\dots\dots$ N.
 (a) 72 (b) 17 (c) 1 (d) - 17
- (3) The magnitude of resultant of the two forces $F_1 = 9$ N. , $F_2 = 19$ N. act on the opposite direction = $\dots\dots\dots$ N.
 (a) 10 (b) 153 (c) 1 (d) 26
- (4) If magnitude of the resultant of two equal forces is the double of each , then the angle between them $\dots\dots\dots$
 (a) 0° (b) 90° (c) 45° (d) 180°
- (5) If $F_1 = 3$ N. , $F_2 = 4$ N. , the angle between them = 60° magnitude of the resultant of two forces = $\dots\dots\dots$ N.
 (a) 7 (b) $\sqrt{37}$ (c) 12 (d) 1
- (6) If the three equilibrium forces were acting on the same point of magnitude 3 X , 4 X , 5 X of newton , then the smallest angle between two of them = $\dots\dots\dots$
 (a) 105° (b) 85° (c) 90° (d) 120°
- (7) The force of magnitude 10 newton and make an angle of tangent = 1 with one of two perpendicular directions , can be resolved into two forces act in the two perpendicular direction of magnitudes $\dots\dots\dots$ N.
 (a) 5 , 5 (b) $5\sqrt{2}$, $5\sqrt{2}$ (c) $5\sqrt{2}$, $10\sqrt{2}$ (d) 5 , 10 N
- (8) The force of magnitude 8 N. can be resolved into two forces act in the two perpendicular direction of magnitudes $\dots\dots\dots$ N.
 (a) 10 , 2 (b) 9 , - 1 (c) 4 , $4\sqrt{3}$ (d) 6 , 8
- (9) If the resultant of two forces acting on a point is zero , then the angle between them = $\dots\dots\dots$
 (a) 180° (b) 0° (c) 45° (d) 90°

- (10) A regular quadrilateral pyramid its volume = 96 cm^3 , its height = 8 cm.
 , its base side length = cm.
 (a) 72 (b) 36 (c) 6 (d) 12
- (11) The right circular cone , which its base radius 21 cm. , its height = 10 cm.
 , its volume = cm^3 .
 (a) 1210π (b) 1470π (c) 1105π (d) 1409π
- (12) A regular quadrilateral pyramid its volume = 160 cm^3 , its height = 10 cm.
 , its slant height = cm.
 (a) $4\sqrt{3}$ (b) $4\sqrt{7}$ (c) 6 (d) 5
- (13) If you put a body of mass X kg. on an inclined smooth plane with 30° to the horizontal
 , the component of its weight on the direction of plane = kg.wt.
 (a) $0.5 X$ (b) $X\sqrt{2}$ (c) $2 X$ (d) $X\sqrt{3}$
- (14) If you put a body of mass y kg. on an inclined smooth plane with 60° to the horizontal , the
 component of its weight on the direction of perpendicular to the plane = kg.wt.
 (a) $y\sqrt{2}$ (b) $0.5 y$ (c) $2 y$ (d) $y\sqrt{3}$
- (15) The wire attached to body of mass 3 kg. on inclined smooth plane with 45° to the
 horizontal and parallel to the plane is in equilibrium case has tension of N.
 (a) $3\sqrt{2}$ (b) 3 (c) $1.5\sqrt{2}$ (d) 1.5
- (16) The equilibrium case means the resultant of forces acting on the body is
 (a) maximum. (b) minimum. (c) zero. (d) horizontal.
- (17) The circumference of the circle whose equation is $X^2 + y^2 = 8$ is length units.
 (a) 8π (b) 64π (c) $2\sqrt{2} \pi$ (d) $4\sqrt{2} \pi$
- (18) If the two straight lines : $X = -3$, $X = 4$ touch the circle M
 , then its circumference = length units where $\left(\pi = \frac{22}{7}\right)$
 (a) 22 (b) 44 (c) 12 (d) 14
- (19) If $(X \ y \ 8) \begin{pmatrix} X \\ y \\ -2 \end{pmatrix} = \square$, then the obtained equation represents a circle with
 diameter length = length units (Where \square is the zero matrix)
 (a) 2 (b) 4 (c) 6 (d) 8

(20) The equation $\begin{vmatrix} x & y i \\ y i & x \end{vmatrix} - 49 = 0$ represents the equation of a circle with

radius length = length units.

- (a) 49 (b) 14 (c) 9 (d) 7

(21) Which of the following equations represent a circle ?

- (a) $x^2 - y^2 + x - y = 6$ (b) $2x^2 + y^2 - x + y = 5$
 (c) $x^2 + y^2 - x = 6$ (d) $x^2 + y^2 - xy = 6$

Second Essay questions

Answer the following questions :

- 1 A regular quadrilateral pyramid , the perimeter of its base is 16 cm. and whose height 9 cm. is put inside a container in the shape of a right circular cylinder , contains water. If the level of water raises $\frac{21}{88}$ cm. , find the radius length of the base of the cylinder given that $(\pi \approx \frac{22}{7})$
- 2 \overline{BC} is uniform rod of length one meter and its weight (W) newton is suspended from its two ends by two perpendicular strings their other end fixed at a point on the ceiling of a room , if the length of one of the two strings equals $50\sqrt{3}$ cm. , find the magnitude of the tension in strings in terms of the weight of the rod (W).
- 3 A body of weight 12 newton is placed on a smooth plane inclined to the horizontal at an angle of measure 60° , it is kept in equilibrium under the effect of a force F newton acts in the direction of the line of the greatest slope of the plane up , find F and the reaction of the plane on the body.

3 Cairo Governorate



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First Multiple choice questions

Choose the correct answer from the given ones :

- (1) Two forces of magnitudes 7 F , 6 F newton act at a point and their resultant is F newton , then the measure of the angle between them =
 (a) zero (b) 60° (c) 120° (d) 180°
- (2) Two forces of magnitudes 5 , 3 newton act a point and the measure of the angle between them is 60° , then the magnitude of their resultant (R) equals newton.
 (a) 2 (b) 7 (c) 8 (d) 5

- (3) Two forces act at a point magnitude of the first equals 4 kg.wt. and acts towards west , magnitude of the second equals $2\sqrt{3}$ kg.wt. and acts in direction 60° east of north , then the magnitude their resultant = kg.wt.

(a) 2 (b) 3 (c) 4 (d) $3\sqrt{3}$

- (4) Two forces of magnitudes F , $\sqrt{3} F$ newton act on a particle , if the magnitude of their resultant is $2 F$ newton , then the measure of the angle between the two forces =

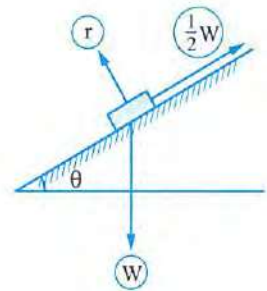
(a) 90° (b) 60° (c) 120° (d) 30°

- (5) In the opposite figure :

If the body is in equilibrium under acting of the shown forces

, then $m (\angle \theta) = \dots\dots\dots^\circ$

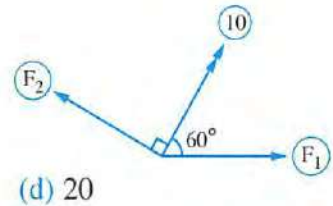
(a) 30 (b) 60
(c) 45 (d) 15



- (6) In the opposite figure :

If the force 10 newton is resolved into two components F_1 and F_2 inclined to the force by 60° and 90° respectively , then $F_2 = \dots\dots\dots$ newton.

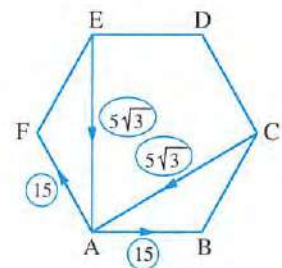
(a) $5\sqrt{3}$ (b) 10 (c) $10\sqrt{3}$ (d) 20



- (7) In the opposite figure :

ABCDEF is a regular hexagon , forces of magnitudes 15 , $5\sqrt{3}$, $5\sqrt{3}$ and 15 act along \overrightarrow{AB} , \overrightarrow{CA} , \overrightarrow{EA} and \overrightarrow{AF} , then the magnitude of the resultant $R = \dots\dots\dots$ newton.

(a) 5 (b) 10
(c) zero (d) 25



- (8) A uniform smooth sphere of weight 1.5 gm.wt. and radius length 25 cm. , is suspended at a point on its surface by a light string of length 25 cm. and the other end of the string is fixed at the point in vertical smooth wall , if the sphere is in equilibrium , then the tension in the string = gm.wt.

(a) $2\sqrt{3}$ (b) 6 (c) $\sqrt{3}$ (d) 3

- (9) Three forces are equal in magnitude and meeting at a point are in equilibrium , then the measure of the angle between any two forces of them is

(a) 60° (b) 120° (c) 90° (d) 150°

- (10) The point which lies on the circle : $x^2 + (y - 5)^2 = 20$ is
- (a) (2 , 3) (b) (3 , -2) (c) (2 , 5) (d) (4 , 3)
- (11) The center of the circle $5x^2 + 5y^2 + 10x = 30y + 20$ is
- (a) (3 , 1) (b) (3 , -1) (c) (1 , -3) (d) (-1 , 3)
- (12) The radius of the circle $(2x - 6)^2 + (2y - 4)^2 = 20$ is
- (a) $\sqrt{20}$ (b) $\sqrt{10}$ (c) $\sqrt{5}$ (d) $\sqrt{2}$
- (13) Two circle whose centers M , N are touching internally if $r_1 = 5$ cm. , $r_2 = 8$ cm. , then MN = cm.
- (a) 13 (b) 3 (c) 4 (d) 15
- (14) The two straight lines are skew if and only if they are
- (a) not contained in the same plane. (b) not intersecting.
(c) not coincident. (d) not parallel.
- (15) If the side length of the base of a regular quadrilateral pyramid is doubled , then its volume
- (a) will doubled. (b) will be three times its first volume.
(c) will be four times its first volume. (d) will not change.
- (16) The length of the drawer of a right circular cone is 17 cm. and its height = 15 cm. , then its total surface area = cm^2
- (a) 200π (b) 136π (c) 320π (d) 400π
- (17) Which of the following statements is not true ?
- (a) Any two different parallel straight lines identify a plane.
(b) Any two intersecting different straight lines have a common point.
(c) The two skew lines aren't contained in one plane.
(d) Any three non collinear points , there is at least one plane passes through them.
- (18) Number of the planes which passes through two given points is
- (a) zero (b) 1 (c) 2 (d) infinite.
- (19) A piece of chocolate in the shape of a right circular cone its volume = $27\pi \text{ cm}^3$ and the circumference of its base = 6π cm. , then its height = cm.
- (a) 9 (b) 6 (c) 12 (d) 15
- (20) Which of the following equations represent equation of a circle ?
- (a) $3x^2 + 2y^2 + 6x - 8y - 10 = 0$ (b) $x^2 + y^2 + 4x + 25 = 0$
(c) $2x^2 + 2y^2 - 12x + 8y - 30 = 0$ (d) $x^2 + y^2 + 2xy + 3 = 0$

(21) In the opposite figure :

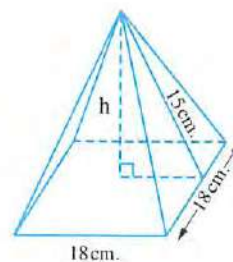
A regular quadrilateral pyramid in which the side length of its base = 18 cm. and the slant height = 15 cm. , then its volume = cm^3

(a) 1296

(b) 1620

(c) 540

(d) 1944

**Second****Essay questions****Answer the following questions :**

- 1 \overline{AB} is a uniform rod of length 65 cm. , weight 130 gm.wt. hanged freely from its two ends in a point (C) with two strings of length 25 cm. , 60 cm. , find the tension in the two strings in the equilibrium position.
- 2 Write the general form of the equation of the circle if the two points A (4 , 2) , B (− 1 , − 3) are the end points of its diameter.
- 3 Five coplanar forces meeting at a point , their magnitudes are 12 , 9 , $5\sqrt{2}$, $7\sqrt{2}$ and 7 kg.wt. act in the directions east , north , western north , western south and south respectively , show that the set of these forces are in equilibrium or not.

4**Giza Governorate**October Directorate
Om EL-Mo'emeneen School**First****Multiple choice questions****Choose the correct answer from the given ones :**

- (1) A force of magnitude $5\sqrt{3}$ newton act in direction 30° east of north , is resolved into two perpendicular components , then the magnitude of its component in direction the east =
 (a) 5 (b) 7.5 (c) $\frac{5\sqrt{3}}{2}$ (d) 15
- (2) When the two forces 6 and 8 newtons are perpendicular , then the sine of inclination of the resultant with the first force equals
 (a) $\frac{3}{5}$ (b) $\frac{4}{5}$ (c) $\frac{3}{4}$ (d) $\frac{4}{3}$
- (3) Three forces are equal in magnitude and meeting at a point are in equilibrium , then the measure of the angle between any two of them is
 (a) 60° (b) 120° (c) 90° (d) 150°

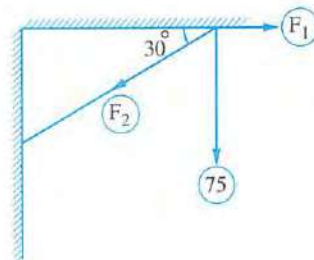
(4) If $\vec{F}_1 = 5\hat{i} - 3\hat{j}$, $\vec{F}_2 = -7\hat{i} + 2\hat{j}$, $\vec{F}_3 = 2\hat{i} + \hat{j}$, then $\vec{R} = \dots\dots\dots$

- (a) $7\hat{i} - 2\hat{j}$ (b) $14\hat{i} - 4\hat{j}$ (c) $-14\hat{i} + 4\hat{j}$ (d) $\vec{0}$

(5) In the opposite figure :

A vertical force of magnitude 75 newton is resolved into two components, one of them is horizontal (F_1) and the other F_2 , then $F_2 = \dots\dots\dots$ newton.

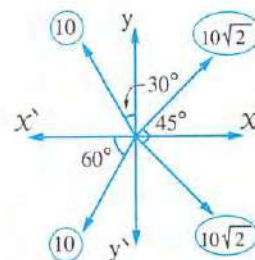
- (a) 75 (b) $75\sqrt{3}$
(c) 150 (d) $150\sqrt{3}$



(6) In the opposite figure :

The resultant of the system of forces "R" = $\dots\dots\dots$ newton.

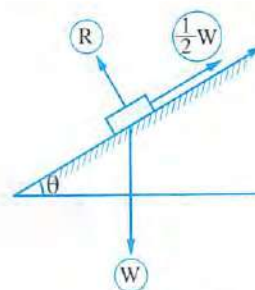
- (a) 20 (b) $10\sqrt{2}$
(c) 10 (d) zero.



(7) In the opposite figure :

If the body is in equilibrium under acting of the shown forces, then $m(\angle \theta) = \dots\dots\dots$

- (a) 30° (b) 60°
(c) 45° (d) 15°



(8) Two forces of magnitudes $5F$, $2F$ and their resultant is $7F$ newton, then the measure of the angle between them = $\dots\dots\dots$

- (a) 180° (b) 60° (c) 20° (d) zero.

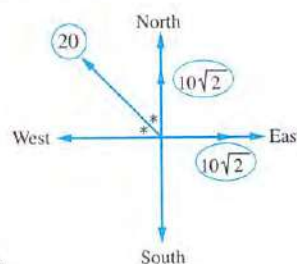
(9) If the resultant of two perpendicular forces, inclined to the greatest one by angle of measure θ , then which of the following values is suitable value of θ ?

- (a) 90° (b) 70° (c) 45° (d) 10°

(10) In the opposite figure :

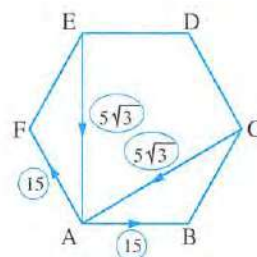
The resultant of the forces $10\sqrt{2}$, $10\sqrt{2}$, 20 newton acts in direction $\dots\dots\dots$

- (a) the eastern north. (b) the north.
(c) the western north. (d) the western south.



(11) In the opposite figure :

ABCDEF is a regular hexagon , forces of magnitudes 15 , $5\sqrt{3}$, $5\sqrt{3}$ and 15 act along \overrightarrow{AB} , \overrightarrow{CA} , \overrightarrow{EA} and \overrightarrow{AF} , then the magnitude of the resultant R = newton.



- (a) 5 (b) 10
(c) 25 (d) zero.

(12) If three forces meeting at a point and acting up on a particle are in equilibrium , then the magnitude of each force is proportional to the of the included angle between the two other forces.

- (a) cosine (b) sine (c) tangent (d) cotangent

(13) The ratio between the lateral surface area of the triangular regular faces pyramid to its total surface area =

- (a) 1 : 3 (b) 1 : 4 (c) 3 : 4 (d) 1 : 2

(14) The radius length of the base of a right circular cone = 5 cm. and its total surface area = $90\pi \text{ cm}^2$, then its volume = cm^3 .

- (a) 105π (b) 95π (c) 100π (d) 120π

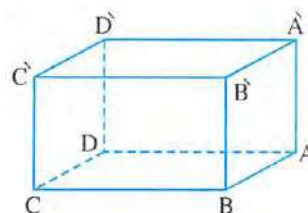
(15) The general form of the equation of the circle which its centre is $(-2, 5)$ and passes through $(3, 2)$ is

- (a) $x^2 + y^2 - 4x + 10y - 5 = 0$ (b) $x^2 + y^2 + 4x - 10y - 5 = 0$
(c) $x^2 + y^2 + 2x - 5y - 5 = 0$ (d) $x^2 + y^2 + 4x - 10y - 25 = 0$

(16) In the opposite figure :

The plane $\overleftrightarrow{AA'B'}$ \cap the plane $\overleftrightarrow{ACC'}$ =

- (a) $\overleftrightarrow{AA'}$ (b) $\overleftrightarrow{BB'}$
(c) $\overleftrightarrow{CC'}$ (d) \overleftrightarrow{AC}



(17) The equation $(x^2 + y^2 + 8x - 2y - 8) = 0$ represents a circle its diameter length = length unit.

- (a) 2 (b) 4 (c) 6 (d) 8

(18) The least number of planes can determine a solid is planes.

- (a) three (b) four (c) two (d) five

(19) The centre of the circle : $x^2 + y^2 - 6x + 8y = 0$ is the point

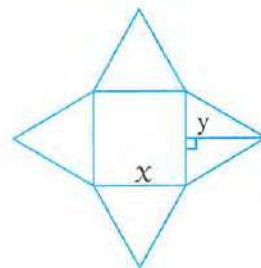
- (a) $(3, -4)$ (b) $(-4, 3)$ (c) $(-3, 4)$ (d) $(-3, -4)$

(20) Any three points non-collinear identify

- (a) one plane. (b) two planes. (c) 3 planes. (d) 4 planes.

(21) the opposite figure represents a regular quadrilateral pyramid its height (h), then the relation between x , y and h is

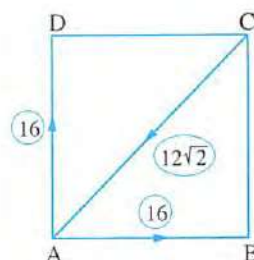
- (a) $x^2 + y^2 = h^2$ (b) $x^2 + h^2 = y^2$
 (c) $\left(\frac{x}{2}\right)^2 + h^2 = y^2$ (d) $\left(\frac{x}{2}\right)^2 + y^2 = h^2$



Second Essay questions

Answer the following questions :

- 1 A body of weight 18 newton is placed on a smooth plane inclines to the horizontal by an angle of measure 30° and kept in equilibrium by a horizontal force of magnitude F newton. Find the magnitude of this force and the reaction of the plane on the body.
- 2 Write the equation of the circle which touches the x -axis at the point $(-2, 0)$ and intercepts from the positive part of y -axis a chord of length $4\sqrt{3}$ length unit.
- 3 The opposite figure represents the forces 16, 16, $12\sqrt{2}$ newton which act in the square ABCD in the directions \overrightarrow{AB} , \overrightarrow{AD} , \overrightarrow{CA} respectively. Find the magnitude and direction of their resultant.



5

Giza Governorate



Omrana Directorate
Math Inspection

First Multiple choice questions

Choose the correct answer from the given ones :

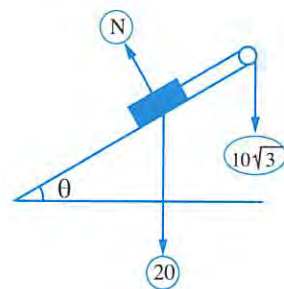
- (1) If the resultant of the two forces F and $3F$ kg.wt. is perpendicular to one of them, then the magnitude of their resultant equals kg.wt.
 (a) $\sqrt{2} F$ (b) $4 F$ (c) $2\sqrt{2} F$ (d) $2 F$

- (2) Two different straight lines are skew if they are
 (a) different parallel. (b) not intersected.
 (c) not parallel. (d) not contained in the same plane.
- (3) If the resultant of two forces meeting at one point is $R \in [5, 19]$, where $F_1 > F_2$, then $F_1 =$
 (a) 7 (b) 12 (c) 5 (d) 19
- (4) The number of planes which passes through three-collinear points equals
 (a) 1 (b) 3 (c) 6 (d) infinite numbers.
- (5) Three coplanar forces 6 , F , k the first act in direction of east, the second in direction of 30° west of north, the third in direction of south and their resultant $5\sqrt{3}$ newton and act in north direction, then $F + \sqrt{3}k =$ newton.
 (a) 11 (b) $12\sqrt{3}$ (c) 15 (d) $9\sqrt{3}$
- (6) If the equation of a circle is $(2a + 1)x^2 + (a + 2)y^2 + (b - 1)xy - 6ax + 12by - 12 = 0$, then its radius length equals length unit.
 (a) 3 (b) 4 (c) 5 (d) 6
- (7) Force of magnitude 10 kg.wt. in direction 30° with line of greatest slope of a plane makes θ with the horizontal, then its components in the perpendicular direction of the line of greatest slope equals gm.wt.
 (a) $5\sqrt{3}$ (b) 5000 (c) 5 (d) $5000\sqrt{3}$
- (8) A regular quadrilateral pyramid its lateral edge = 10 cm. , and its base area = 100 cm^2 , then its lateral surface area equals square unit.
 (a) $100\sqrt{3}$ (b) $100\sqrt{2}$ (c) $50\sqrt{2}$ (d) $50\sqrt{3}$
- (9) If the slant height of a triangular regular faces pyramid equals $10\sqrt{3} \text{ cm.}$, then its volume equals cm^3 .
 (a) $\frac{2000\sqrt{3}}{3}$ (b) $\frac{2000\sqrt{2}}{3}$ (c) $\frac{100\sqrt{3}}{3}$ (d) $\frac{100\sqrt{2}}{3}$

(10) In the opposite figure :

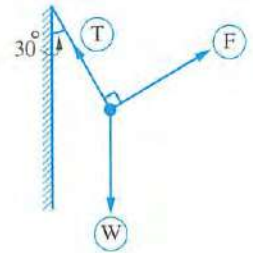
Inclined smooth plane makes with the horizontal an angle of measure θ and a smooth pulley on the top of the plane if the system is in equilibrium, then $\theta =$

- (a) 60° (b) 30°
 (c) 45° (d) 15°



(11) In the opposite figure :

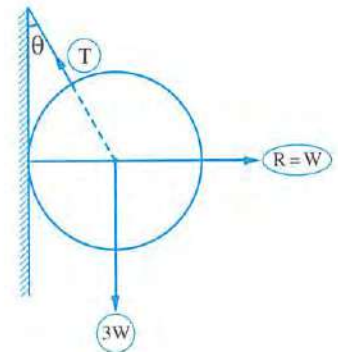
A body of weight W gm.wt. suspended in one end of string and the other end is fixed in a point in vertical wall, the body pulled by a perpendicular force (F) to the string till the string makes 30° with the vertical in equilibrium, then $T : F = \dots\dots\dots$



- (a) $\sqrt{3} : 1$ (b) $1 : 1$ (c) $1 : \sqrt{3}$ (d) $\sqrt{3} : 2$

(12) In the opposite figure :

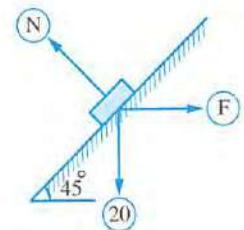
A smooth uniform sphere of weight $3W$ kg.wt. suspended in equilibrium by one end of string its other end fixed in a vertical smooth wall above the tangency point, the string makes angle θ with the vertical, then $\tan \theta = \dots\dots\dots$



- (a) 3 (b) 2
(c) $\frac{1}{3}$ (d) $\frac{1}{2}$

(13) In the opposite figure :

A body of weight 20 N. is placed on smooth inclined plane makes 45° with the horizontal the body kept in equilibrium by horizontal force F , then $\frac{F}{N} = \dots\dots\dots$



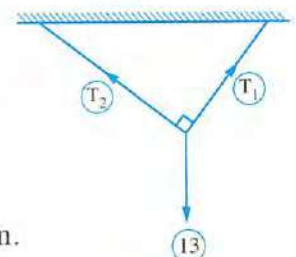
- (a) 5 (b) $\sqrt{2}$ (c) 1 (d) $\frac{1}{\sqrt{2}}$

(14) If y -axis intersect the circle whose equation $(x - 3)^2 + (y + 2)^2 = 18$ at two points A and B, then the length of $\overline{AB} = \dots\dots\dots$ unit length.

- (a) 6 (b) 4 (c) 8 (d) 7

(15) In the opposite figure :

A body of weight 13 N. is attached by two strings their other ends on the same horizontal line and distant between them is 1.3 m. and one of the two strings is of length 0.5 m. while the other is of length 1.2 m., then $T_1 - T_2 = \dots\dots\dots$ newton.



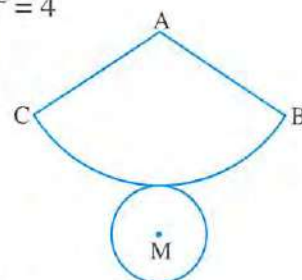
- (a) 5 (b) 7 (c) 12 (d) 19

- (16) A right circular cone its base touch the two positive axes in X y-plane and its drawer is twice its radius base length , the volume of a cone is $72\sqrt{3}\pi \text{ cm}^3$.

, then the equation of its base is

- (a) $(X-5)^2 + (y-5)^2 = 25$ (b) $(X-3)^2 + (y-3)^2 = 9$
(c) $(X-6)^2 + (y-6)^2 = 36$ (d) $(X-2)^2 + (y-2)^2 = 4$

- (17) The opposite figure represents the net of a solid
, $\widehat{BC} = 20\pi \text{ cm}$. , $AB = 15 \text{ cm}$. , then the total area of
the solid = cm^2 .



- (a) 250π (b) 150π
(c) 100π (d) 200π

- (18) ABCD $\hat{A} \hat{B} \hat{C} \hat{D}$ is a cube , then the plane $A \hat{A} \hat{B} \hat{B} \cap$ the plane $C \hat{C} \hat{B} \hat{B} =$

- (a) \overrightarrow{BB} (b) \overline{BB} (c) \overleftrightarrow{BB} (d) $\{\hat{B}\}$

- (19) The ratio between the volume of right circular cone inside cube and the volume of this cube equals such that the base of a cone touch all interior sides of the base of the cube and its vertex on the opposite base of the cube.

- (a) $\frac{12}{\pi}$ (b) $\frac{6}{\pi}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{12}$

- (20) A quadrilateral pyramid whose height 10 cm. , and its base as parallelogram its base length 5 cm. and its height 12 cm. , then its volume = cm^3 .

- (a) 200 (b) 400 (c) 600 (d) 900

- (21) A right circular cone its total surface area $27\pi \text{ cm}^2$ its drawer twice its radius length , then its volume cm^3 .

- (a) 27π (b) $27\sqrt{3}\pi$ (c) $9\sqrt{3}\pi$ (d) $3\sqrt{3}\pi$

Second

Essay questions

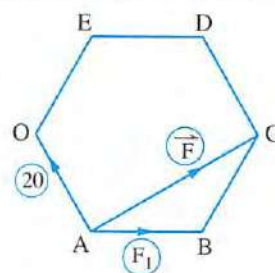
Answer the following questions :

- 1 How many circles you can draw in different quadrantes in X y-plane which touches the two co-ordinates axes and its radius length is 5 unit length.

Write down the equation of two of them which lies in first and third quadrant.

What is the distant between their centers ?

- 2 ABCDEO is a regular hexagon , a force of magnitude (F) in the direction of \overrightarrow{AC} is resolved into to components one of them in the direction \overrightarrow{AB} and the other in the direction \overrightarrow{AO} of magnitude 20 newton. , find F and the component in direction of \overrightarrow{AB}

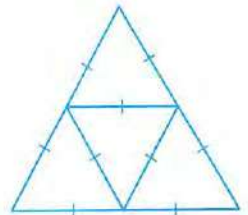


- 3** \overline{AB} is a uniform rod of weight 20 newton is hanged from (A) in a vertical wall and the rod is in equilibrium in horizontal position by attached its end B by string its other end is fixed in a wall above (A) at point (C) given that $AB = 0.6$ m. , $AC = 0.8$ m.
Find the magnitude of each of the tension in the string and the reaction of the hinge.

6**Giza Governorate**
Dokki Educational Administration
Narmer Language School
First Multiple choice questions

Choose the correct answer from the given ones :

- (1) Two perpendicular forces of magnitudes $2F - 5$, $F + 2$ newton act at a particle , the magnitude of their resultant is $3\sqrt{5}$ newton. , then $F = \dots\dots\dots$ newton.
 (a) 2 (b) 3 (c) 4 (d) 5
- (2) Which solid , its net is the opposite figure ?
 (a) Quadrilateral pyramid.
 (b) Regular quadrilateral pyramid.
 (c) Triangular pyramid with regular faces.
 (d) Otherwise.
- (3) Volume of right circular cone is 100 cm^3 , then its volume when its height is doubled becomes $\dots\dots\dots \text{ cm}^3$
 (a) 100 (b) 200 (c) 400 (d) 800
- (4) A body of weight 18 kg.wt. is placed on a smooth plane inclines to the horizontal at angle of measure 30° , the body kept in equilibrium by a force F inclines to line of greatest slope upward by an angle of measure 30° , then the magnitude of this force = $\dots\dots\dots$ kg.wt.
 (a) 12 (b) 9 (c) $3\sqrt{3}$ (d) $6\sqrt{3}$
- (5) Force of magnitude $4\sqrt{2}$ acts in east direction it was resolved into two perpendicular component , then the magnitude of the component in direction of eastern north equals $\dots\dots\dots$ newton.
 (a) 4 (b) $4\sqrt{2}$ (c) 8 (d) $8\sqrt{2}$
- (6) A regular quadrilateral pyramid , the perimeter of its base = 40 cm. and its height 12 cm. , then its lateral surface area = $\dots\dots\dots \text{ cm}^2$
 (a) 200 (b) 240 (c) 260 (d) 320

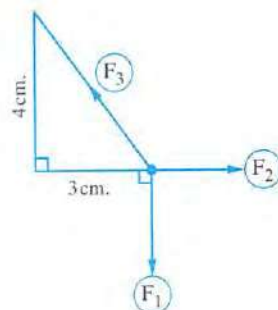


- (7) The equation of the circle which the straight line : $x + y = 2$ touches it and its centre is (3 , 5) is
- (a) $(x - 3)^2 + (y - 5)^2 = 3\sqrt{2}$ (b) $(x + 3)^2 + (y + 5)^2 = 18$
 (c) $(x - 3)^2 + (y - 5)^2 = 12$ (d) $(x - 3)^2 + (y - 5)^2 = 18$
- (8) Three coplanar forces of magnitudes 60 , 88 , 60 newton act at a point in the directions , the first due to north and the second due to 30° south of west and the thrid due to 30° south of east , then the magnitude of their resultant = newton.
- (a) 30 (b) 23 (c) 32 (d) 28
- (9) Three forces meeting at one point are in equilibrium , if the magnitudes of two forces of them equals 7 , 3 newton , then the value of the third force may be equals newton.
- (a) 11 (b) 2 (c) 5 (d) 3
- (10) A body of weight 200 gm.wt. is suspended by two strings of a length 60 cm. , 80 cm. , from the two points on the same horizontal line with 100 cm. apart , then the magnitude of the tension in each string gm.wt.
- (a) 160 , 120 (b) 180 , 120 (c) 150 , 160 (d) 100 , 130
- (11) The least number of a coplanar and non-equal forces in magnitude can be in equilibrium is
- (a) 1 (b) 2 (c) 3 (d) 4
- (12) The length of the diameter of the circle $4x^2 + 4y^2 + 16x - 8y - 16 = 0$ equals
- (a) 3 (b) 6 (c) 12 (d) 24
- (13) If the force F is in equilibrium with the two forces 5 , 3 newton whose included an angle of measure 60° , then F = newton.
- (a) 7 (b) 15 (c) 19 (d) 34
- (14) If the forces $\vec{F}_1 = (a , 7)$, $\vec{F}_2 = (-5 , -b)$, $\vec{F}_3 = (1 , 1)$ are equilibrium , then (a , b) =
- (a) (4 , 2) (b) (1 , 2) (c) (4 , 8) (d) (-4 , -8)
- (15) The two straight lines are skew if they are
- (a) not parallel. (b) not intersecting.
 (c) not coincident. (d) not contained in the same plane.
- (16) If the volume of hemisphere of radius length (r) equals the volume of a cone the length of radius of its base = r and its height = h , then
- (a) $h = \frac{2}{3} r$ (b) $h = 2 r$ (c) $h = 2 r^2$ (d) $h = 4 r$

- (17) If the length of the base side of a regular quadrilateral pyramid is doubled , then its volume =
 (a) will doubled. (b) will be three times its first volume.
 (c) will be four times its first volume. (d) will not change.
- (18) The circumference of the circle whose equation : $x^2 + y^2 = 8$ is
 (a) 8π (b) 64π (c) $2\sqrt{2}\pi$ (d) $4\sqrt{2}\pi$
- (19) Two planes coincide if they have in common.
 (a) one point (b) two points
 (c) three collinear points (d) three non-collinear points
- (20) The length of the tangent segment which drawn of the circle : $x^2 + y^2 = r^2$ from the point $(0, 2r)$ is
 (a) r (b) $2r$ (c) $\sqrt{3}r$ (d) $\frac{\sqrt{3}}{2}r$

(21) In the opposite figure :

A body is in equilibrium under the action of three forces meeting at a point of magnitudes F_1 , F_2 and F_3 newton and the sides of the right-angled triangle are parallel to the lines of action of the forces in the same cyclic order , then $F_1 : F_2 : F_3 =$



- (a) 3 : 4 : 5 (b) 3 : 5 : 4 (c) 4 : 5 : 3 (d) 4 : 3 : 5

Second Essay questions

Answer the following questions :

- 1 Three forces of magnitudes 10 , 20 , 30 newton act at a particle , the first in direction of east and the second in direction of 30° west of north and third in direction of 60° south of west , find the magnitude and direction of the resultant of these forces.
- 2 Prove that the two circles : $x^2 + y^2 - 6x - 4y + 12 = 0$, $x^2 + y^2 + 2x - 4y - 4 = 0$ touch each other and find the coordinates of the point of tangency , then find the circle equation whose centre is the point of tangency and passes through the center of the second circle.
- 3 AB is a uniform rod of weight 20 kg.wt, the end A attached to a hinge fixed on a vertical wall a horizontal force F acts at B , the rod is in equilibrium when it inclined by angle 30° with vertical , find the magnitude of each of the force and reaction of the hinge.

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Alexandria Governorate


 Al-Agamy Educational Zone
 Al-Agamy Language School

First

Multiple choice questions

Choose the correct answer from the given ones :

- (1) The circumference of the circle of equation : $x^2 + y^2 = 8$ is
- (a) 8π (b) 64π (c) $2\sqrt{2}\pi$ (d) $4\sqrt{2}\pi$
- (2) The ratio between the maximum and minimum value of the resultant of two forces is $7:3$, then the ratio between the two forces is
- (a) $7:4$ (b) $7:3$ (c) $5:3$ (d) $5:2$
- (3) The two planes are coincide if they have in common.
- (a) one point. (b) two points.
(c) 3 collinear points. (d) 3 non-collinear points.
- (4) A ball of pendulum of weight 600 dyne is in equilibrium when the string makes an angle of measure 30° with the vertical under the effect of a force perpendicular to the string , then the magnitude of the force = Dyne.
- (a) 1200 (b) 300 (c) $300\sqrt{2}$ (d) $300\sqrt{3}$
- (5) Two perpendicular forces of magnitude 6 N. , 8 N. , then sine the angle included between the resultant and the first force =
- (a) $\frac{3}{4}$ (b) $\frac{3}{5}$ (c) $\frac{4}{5}$ (d) $\frac{4}{3}$
- (6) The centre of the circle : $x^2 + y^2 - 6x + 8y = 0$ is the point
- (a) $(3, -4)$ (b) $(-4, 3)$ (c) $(-3, 4)$ (d) $(-3, -4)$
- (7) ABCDOH is a regular hexagon , force 20 N. acts on \overline{AD} , then the components of the force in directions \overrightarrow{AC} , \overrightarrow{AH} respectively are N.
- (a) $10\sqrt{3}, 10$ (b) $5\sqrt{3}, 10$ (c) $10, 10\sqrt{3}$ (d) $20\sqrt{3}, 20$
- (8) If the length of the base of a regular quadrilateral pyramid is doubled , then its volume
- (a) does not change. (b) doubled.
(c) becomes 3 times its first volume. (d) becomes 4 times its first volume.
- (9) Two forces of magnitude 16 , F gm.wt. the measure of the angle between them $\in]0, \pi[$ and the resultant bisects the angle between them , then F = gm.wt.
- (a) 4 (b) 8 (c) 12 (d) 16

- (10) A body of weight w is placed on an inclined plane inclined to the horizontal by angle θ if the components of the weight in the directions of the plane and the perpendicular to it are 7, 24 N, then $w = \dots\dots\dots$
- (a) 7 (b) 24 (c) 25 (d) 31
- (11) Two forces of magnitude 5, 3 newton act at a point, the measure of the angle between them is 60° , then the magnitude of the resultant = $\dots\dots\dots$ N.
- (a) 3 (b) 5 (c) 7 (d) 8
- (12) The total area of the regular faces triangular pyramid of edge length $\ell = \dots\dots\dots$
- (a) $2\sqrt{3}\ell^2$ (b) $\sqrt{3}\ell^2$ (c) $3\sqrt{2}\ell^2$ (d) $4\ell^2$
- (13) A regular quadrilateral pyramid of height 9 cm. and volume 300 cm^3 , the length of its base side = $\dots\dots\dots$ cm.
- (a) 5 (b) 10 (c) 15 (d) 20
- (14) The force which is in equilibrium with the two perpendicular forces of magnitude 8 N, 15 N, = $\dots\dots\dots$ N.
- (a) 7 (b) 23 (c) 17 (d) $7\sqrt{2}$
- (15) Two forces of magnitudes $5F$, $2F$ newton act at a point, the magnitude of their resultant is $7F$, then the measure of the angle between them = $\dots\dots\dots$
- (a) 0° (b) 90° (c) 180° (d) 60°
- (16) Three equal and concurrent forces are in equilibrium then the measure of the angle between each two forces = $\dots\dots\dots^\circ$
- (a) 60 (b) 120 (c) 90 (d) 150
- (17) The lateral area of a right cone of base radius length 6 cm. and of height 8 cm. equals $\dots\dots\dots\text{ cm}^2$
- (a) 60π (b) 28π (c) 10π (d) 48π
- (18) Two equal forces their resultant = 3 N. and the measure of the angle between them is 60° , then the magnitude of each force = $\dots\dots\dots$ N.
- (a) 3 (b) $\frac{3}{2}$ (c) $\sqrt{3}$ (d) $3\sqrt{3}$
- (19) The equation $x^2 + y^2 = 36$ is a circle of radius length = $\dots\dots\dots$ units.
- (a) 3 (b) 9 (c) 6 (d) 18
- (20) Two forces of magnitude 4, F newton act at a point, the measure of the angle between them is 120° , if the resultant is perpendicular to the first force, then the resultant = $\dots\dots\dots$ N.
- (a) $4\sqrt{2}$ (b) $4\sqrt{3}$ (c) $4\sqrt{5}$ (d) 4
- (21) Any three non-collinear points determine $\dots\dots\dots$
- (a) one plane. (b) two planes. (c) 3 planes. (d) 4 planes.

Second Essay questions

Answer the following questions :

- 1 A sphere of radius length 10 cm. and of weight 30 gm.wt. is suspended from a point on its surface by a string of length 10 cm. , the other end of the string is fixed on a smooth vertical wall. Find in case of equilibrium the tension of the string and the reaction of the wall.
- 2 Resolve the force 100 N. acts due north into two components , the first in direction 30° east of north , the second in the direction 30° north of west.
- 3 Find the general equation of the circle of radius length 3 units and its centre lies in the first quadrant and touches the two lines $x = 1$, $y = 2$

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El-Kalyoubia Governorate

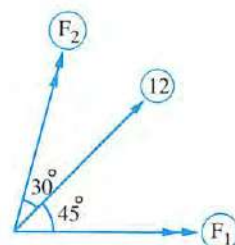


Maths Inspection

First Multiple choice questions

Choose the correct answer from the given ones :

- (1) Two forces its magnitudes are 9 and 8 newton , then the maximum value of its resultant N.
 (a) 1 (b) 17 (c) 15 (d) 16
- (2) Two forces its magnitude are 7 and F kg.wt. , the resultant bisect the angle between them , then $F =$ kg.wt.
 (a) 7 (b) 8 (c) 9 (d) 5
- (3) Two forces its magnitude are $8\sqrt{3}$ and 8 newton act at a point the angle between them is of measure 150° , then the resultant of two forces = newton.
 (a) 64 (b) 32 (c) 16 (d) 8
- (4) If the force 12 N. , resolved into two component F_1 and F_2 , then $F_1 =$ N
 (a) $12 \cos 75^\circ$ (b) $12 \cos 45^\circ$
 (c) $6 \csc 45^\circ$ (d) $6 \csc 75^\circ$
- (5) A body (w) is placed on plane inclined by angle (θ) to the horizontal , then the magnitude of the component of its weight in direction perpendicular to the plane
 (a) $w \sin \theta$ (b) $w \cos \theta$ (c) $w \tan \theta$ (d) $w \csc \theta$



- (6) The center of the circle $x^2 + y^2 - 6x + 8y = 0$ is
- (a) (3, -4) (b) (-4, 3) (c) (-3, 4) (d) (-3, -4)
- (7) The volume of a right cone its base circumference 44 cm. and its height 15 cm.
= cm^3 ($\pi = \frac{22}{7}$)
- (a) 235 (b) 110 (c) 245 (d) 770
- (8) A pendulum its weight 600 dyne is in equilibrium, if the string make angle 30° to the vertical under the effect of force perpendicular to the string, then the magnitude of the force = dyne.
- (a) $300\sqrt{3}$ (b) 1200 (c) 300 (d) $300\sqrt{2}$
- (9) A regular quadrilateral pyramid its base side length 18 cm. and its slant height 15 cm., then its volume = cm^3 .
- (a) 1156 (b) 1254 (c) 1308 (d) 1296
- (10) The equation $\left| \frac{x}{y} - \frac{y}{x} \right| = 36$ represents a circle its radius = length unit.
- (a) 3 (b) 6 (c) 9 (d) 18
- (11) A right circular cone its base radius length 5 cm. and its total area $90\pi \text{ cm}^2$, then its volume = cm^3 .
- (a) 105π (b) 95π (c) 100π (d) 120π
- (12) Three coplanar forces meeting at a point and they are in equilibrium, if two of these forces of magnitude 7 N. and 3 N., then the magnitude of the third force may equal N.
- (a) 11 (b) 2 (c) 5 (d) 3
- (13) The point which lies on the circle : $x^2 + (y - 3)^2 = 16$ is
- (a) (0, 3) (b) (3, -2) (c) (2, 0) (d) (4, 3)
- (14) The length of tangent to the circle $x^2 + y^2 = r^2$ from point (0, 2r) is length unit.
- (a) r (b) 2r (c) $\sqrt{3}r$ (d) $\frac{\sqrt{3}}{2}r$
- (15) Which of the following equations represent a circle ?
- (a) $x^2 - y^2 + x - y = 9$ (b) $3x^2 + y^2 - x + y = 8$
(c) $x^2 + y^2 - x = 25$ (d) $x^2 + y^2 - xy = 9$
- (16) A regular pyramid its base is hexagon its side length 8 cm. and its height 10 cm., then its volume equal = cm^3 .
- (a) $320\sqrt{3}$ (b) $960\sqrt{3}$ (c) $160\sqrt{3}$ (d) 554.35

(17) A force its magnitude $4\sqrt{2}$ act in east direction , it resolved into two perpendicular component , then its component in eastern north newton.

- (a) 4 (b) $4\sqrt{2}$ (c) 8 (d) $8\sqrt{2}$

(18) If $\vec{F}_1 = 2\hat{i} + 3\hat{j}$, $\vec{F}_2 = \hat{i} + \hat{j}$, then the magnitude of its resultant = newton.
(Two forces measured by newton).

- (a) $\sqrt{2}$ (b) $\sqrt{5}$ (c) $\sqrt{13}$ (d) 5

(19) Two perpendicular forces $(2F - 5)$, $(F + 2)$ N and the magnitude of its resultant $3\sqrt{5}$, then $F =$ N.

- (a) 7 (b) 4 (c) 6 (d) 3

(20) If θ is the angle between two forces 2 N. , 6 N. and $\theta \in]0, \pi]$, then the magnitude of their resultant in N. \in

- (a) $]4, 8[$ (b) $[4, 8[$ (c) $]4, 8]$ (d) $[4, 8]$

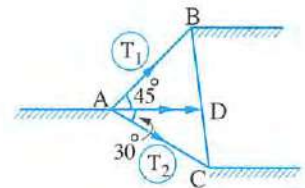
(21) The magnitude of two forces 3 N. , 4 N. and its resultant 7 N. , then the angle between two forces =°

- (a) 0 (b) 60 (c) 180 (d) 90

Second Essay questions

Answer the following questions :

- 1 A battleship is wanted to pull by two locomotive connected by two wires attached by a hook fixed at point A on the battleship and the angle between them 75° , if the inclination angle of one of two wires on \overrightarrow{AD} equals 45° and the resultant of the two forces to pull it equals 5000 N. in direction of \overrightarrow{AD} , find the tension force of each of two wires.



- 2 An engineer designed a building its base in form of a square inscribed in a circle its equation $x^2 + y^2 - 12x + 6y + 20 = 0$, find the area of the square.

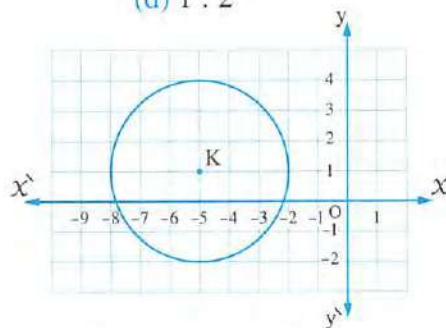
- 3 \overline{AB} is a uniform rod with length 60 cm. and weight 40 newton is connected to a hinge on the vertical wall at A , if the rod kept in equilibrium horizontally by a light string connected to the rod at B and with point C on the wall just above A and at a distance 60 cm. from A , find the tension on the string and the reaction on the hinge at A



First Multiple choice questions

Choose the correct answer from the given ones :

- (1) Two perpendicular forces of magnitudes 5 newton , 12 newton act at a particle , then the magnitude of their resultant is newton.
 (a) 7 (b) 13 (c) 14 (d) 17
- (2) Right circular cone , area of its base = $25 \pi \text{ cm}^2$, length of its drawer = 13 cm. , then its lateral area = cm^2 .
 (a) 50π (b) 65π (c) 90π (d) 100π
- (3) The maximum resultant of the two forces of magnitudes 8 , 13 newton is newton.
 (a) 15 (b) 17 (c) 20 (d) 21
- (4) The number of different planes which passes three non-collinear points =
 (a) 1 (b) 3 (c) 6 (d) infinite number.
- (5) The center of the circle whose equation : $X^2 + y^2 - 6X + 8y = 0$ is
 (a) (3 , -4) (b) (-4 , 3) (c) (-3 , 4) (d) (-3 , -4)
- (6) If \vec{F}_1 , \vec{F}_2 , \vec{F}_3 are three coplanar forces meeting at a point and in equilibrium , then the resultant force of \vec{F}_1 , \vec{F}_2 =
 (a) \vec{F}_1 (b) $\vec{F}_1 + \vec{F}_2$ (c) $-\vec{F}_3$ (d) zero
- (7) The ratio between the lateral surface area of the triangular pyramid of regular faces to its total surface area =
 (a) 1 : 3 (b) 1 : 4 (c) 3 : 4 (d) 1 : 2
- (8) The equation of the circle in the opposite figure is
 (a) $(X + 5)^2 + (y - 1)^2 = 3$
 (b) $(X + 5)^2 + (y - 1)^2 = 9$
 (c) $(X - 5)^2 + (y + 1)^2 = 3$
 (d) $(X - 5)^2 + (y + 1)^2 = 9$
- (9) A force of magnitude $4\sqrt{2}$ newton acts in direction of east , it is resolved into two perpendicular components , so its component in the direction of northern east = newton.
 (a) 1 (b) 2 (c) 3 (d) 4



- (10) The resultant of two forces acting at a point is minimum when the included angle between them is equal to°

(a) zero (b) 30 (c) 90 (d) 180

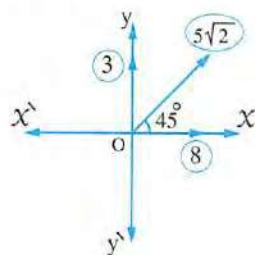
- (11) A triangular regular faces pyramid , its edge = 6 cm. , then its volume = cm³

(a) $27\sqrt{3}$ (b) $36\sqrt{3}$ (c) $54\sqrt{2}$ (d) $18\sqrt{2}$

- (12) In the opposite figure :

A coplanar forces of magnitudes by newton act at the point O , then the resultant of these forces = newton.

(a) $\sqrt{230}$ (b) $\sqrt{232}$
(c) $\sqrt{231}$ (d) $\sqrt{233}$



- (13) If the height of triangular pyramid is $4\sqrt{5}$ cm. and the side lengths of its triangular base are 7 cm. , 8 cm. , 9 cm. , then its volume = cm³

(a) 50 (b) 60 (c) 70 (d) 80

- (14) The two circles $(X - 3)^2 + (y - 2)^2 = 4$, $X^2 + y^2 + 2X + 2y + 1 = 0$ are

(a) touching externally. (b) touching internally.
(c) intersecting. (d) distant.

- (15) If three coplanar forces meeting at a point and acting up on particle are in equilibrium , then the magnitude of each force is proportional to the of the angle between the two other forces.

(a) cosine (b) sine (c) tangent (d) cotan

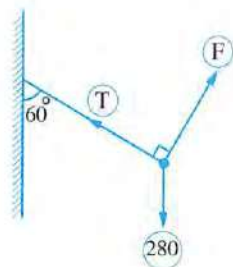
- (16) All of the following cases form a plane except

(a) a straight line and a point don't belong to it.
(b) two different parallel straight lines.
(c) two skew straight lines.
(d) two intersecting straight lines.

- (17) In the opposite figure :

A lamp of weight 280 gm.wt. is attached the end of a string. It is in equilibrium under the effect of a force perpendicular to the string when it is inclined to the vertical by an angle of measure 60° , then $\frac{T}{F}$ =

(a) $\frac{1}{\sqrt{3}}$ (b) $\sqrt{3}$
(c) $-\frac{1}{\sqrt{3}}$ (d) $-\sqrt{3}$



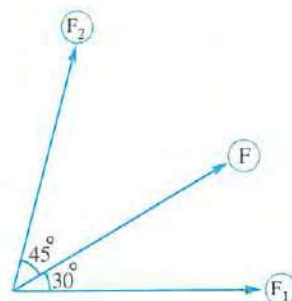
- (18) An inclined plane of length 130 cm. and its height 50 cm. , a rigid body of weight 390 gm.wt. is placed on it , then the components of the weight in the direction of the line of greatest slope of the plane = newton.

(a) 50 (b) 100 (c) 150 (d) 200

- (19) The force \vec{F} is the resultant of \vec{F}_1, \vec{F}_2

, then $\frac{F_1 + F_2}{F} = \dots\dots\dots$

(a) $\sin 30^\circ + \sin 45^\circ$ (b) $\frac{\sin 75^\circ + \sin 30^\circ}{\sin 75^\circ}$
 (c) $\frac{\sin 45^\circ + \sin 30^\circ}{\sin 75^\circ}$ (d) $\frac{\sin 75^\circ}{\sin 30^\circ} + \frac{\sin 75^\circ}{\sin 45^\circ}$



- (20) Two equal forces of magnitude 6 newton concurrent at a point and the magnitude of their resultant is 6 newton , then the measure of the angle between them =°

(a) 30 (b) 60 (c) 120 (d) 150

- (21) Two forces of magnitude 3 newton , F newton act at a particle and the measure of the angle between them 120° , if their resultant is perpendicular to the first force , then F = newton.

(a) 1.5 (b) 3 (c) $3\sqrt{3}$ (d) 6

Second

Essay questions

Answer the following questions :

- A metallic sphere of weight 15 kg.wt. is put such that it touches two smooth planes one of them is vertical and the other inclines to the vertical by an angle of measure 30° , find the reaction of each of the two planes.
- ABCDHE is a regular hexagon , forces of magnitudes 2 , $4\sqrt{3}$, 8 , $2\sqrt{3}$ and 4 kg.wt. act at point A in directions \vec{AB} , \vec{AC} , \vec{AD} , \vec{AH} , \vec{AE} respectively , find the magnitude and the direction of their resultant.
- A regular quadrilateral pyramid , the side length of its base is 18 cm. , its volume 1296 cm^3 find its slant height and its lateral surface area.

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El-Menia Governorate



Menia Educational Directorate
Maths Inspection

First

Multiple choice questions

Choose the correct answer from the given ones :

- (1) The volume of the right cone is $27 \pi \text{ cm}^3$ and the circumference of its base is $6 \pi \text{ cm}$.
 , then its height is cm.
 (a) 27 (b) 18 (c) 9 (d) 6
- (2) Two equal forces act at a point and the measure of the angle between them is 90°
 and their resultant is 8 newton , then the magnitude of each is newton.
 (a) $2\sqrt{2}$ (b) 4 (c) $4\sqrt{2}$ (d) 8
- (3) The circumference of the circle which its equation is : $(x - 3)^2 + (y + 2)^2 = 25$
 equal length unit.
 (a) 2π (b) 3π (c) 10π (d) 25π
- (4) Number of planes that can pass through three non-collinear points equals
 (a) 1 (b) 2 (c) 3 (d) an infinite number.
- (5) Three equal forces in magnitude , concurrent at a point and in equilibrium , then the
 measure of the angle between any two forces of them =
 (a) 60° (b) 90° (c) 120° (d) 150°
- (6) The length of the drawer of a right circular cone is 17 cm. , and its height 15 cm.
 , then its total surface area = cm^2
 (a) 200π (b) 136π (c) 320π (d) 400π
- (7) A body of weight 18 kg.wt. is placed on a smooth plane inclines to the horizontal at angle
 of measure 30 , the body kept in equilibrium by a force \vec{F} inclines to line of greatest slope
 upward by an angle of measure 30° , then the magnitude of this force = kg.wt.
 (a) 12 (b) 9 (c) $3\sqrt{3}$ (d) $6\sqrt{3}$
- (8) The volume of the regular quadrilateral pyramid , where the perimeter of its base = 36 cm.
 and its height 10 cm. is cm^3
 (a) 810 (b) 180 (c) 360 (d) 270

- (9) Two forces of magnitudes 4, F newton act at a particle, the measure of include angle is 120° , if line of action of the resultant is perpendicular to the first force, then magnitude of the resultant = newton.

(a) $4\sqrt{2}$ (b) $4\sqrt{3}$ (c) 4 (d) $4\sqrt{5}$

- (10) Any three points are non-collinear identify

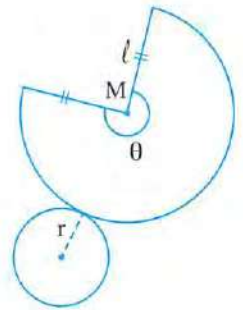
(a) 1 plane. (b) 2 plane. (c) 3 plane. (d) 4 plane.

- (11) Which of the following system of forces cannot be equilibrium ?

(a) 10 newton, 10 newton, 5 newton (b) 4 newton, 6 newton, 8 newton
(c) 11 newton, 7 newton, 8 newton (d) 8 newton, 4 newton, 14 newton

- (12) The opposite figure represents net of a cone where the central angle of its circular sector $= \theta$, $180^\circ < \theta < 360^\circ$, then

(a) $l < 2r$ (b) $l = r$
(c) $l = 2r$ (d) $l > 2r$



- (13) $\vec{F}_1, \vec{F}_2, \vec{F}_3$ three forces meeting a point and equilibrium, then magnitude of the resultant of \vec{F}_1 and $\vec{F}_2 = \dots\dots\dots$

(a) F_1 (b) $F_1 + F_2$ (c) zero (d) F_3

- (14) Two forces of magnitudes 6 and 8 newton and the magnitude of their resultant is 10 newton, then the measure of the angle between them =

(a) 0° (b) 60° (c) 90° (d) 180°

- (15) The centre of the circle whose equation is $x^2 + y^2 - 6x + 8y = 0$ is the point

(a) $(-4, 3)$ (b) $(3, -4)$ (c) $(-3, 4)$ (d) $(-4, -3)$

- (16) The general form of the equation of the circle its centre is $(5, -4)$ and touches x -axis is

(a) $x^2 + y^2 - 10x + 8y + 25 = 0$ (b) $x^2 + y^2 - 5x + 4y = 0$
(c) $x^2 + y^2 - 10x + 8y = 25$ (d) $x^2 + y^2 + 10x - 8y + 25 = 0$

- (17) If the resultant of two forces acting at a point reaches the maximum value, then the measures of the angle between their lines of action equals

(a) 180° (b) 120° (c) zero (d) 60°

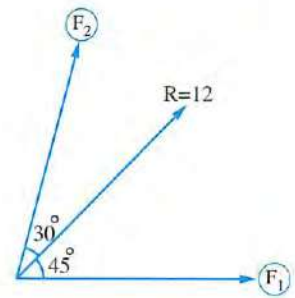
- (18) If $\vec{F}_1 = 2\hat{i} + 3\hat{j}$, $\vec{F}_2 = \hat{i} + \hat{j}$, then $R = \dots\dots\dots$ newton.

(a) $\sqrt{2}$ (b) $\sqrt{5}$ (c) $\sqrt{13}$ (d) 5

(19) In the opposite figure :

$F_1 = \dots\dots\dots$

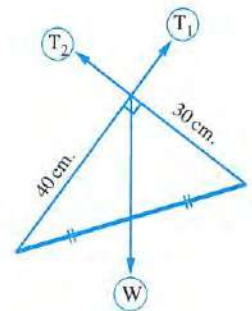
- (a) $12 \csc 75^\circ$
- (b) $12 \csc 45^\circ$
- (c) $6 \csc 45^\circ$
- (d) $6 \csc 75^\circ$



(20) In the opposite figure :

$T_1 : T_2 : W = \dots\dots\dots$

- (a) $5 : 3 : 4$
- (b) $8 : 5 : 4$
- (c) $4 : 3 : 5$
- (d) $5 : 4 : 3$



(21) Which two forces from the following pairs , could not have resultant of magnitude 3 newton ?

- | | |
|-------------------------|-------------------------|
| (a) 2 newton , 4 newton | (b) 3 newton , 3 newton |
| (c) 3 newton , 6 newton | (d) 3 newton , 8 newton |

Second Essay questions

Answer the following questions :

- 1 A particle is in equilibrium under the effect of three forces of magnitudes F_1 , F_2 and 5 newton they are represented by the line segments \overline{AB} , \overline{BC} and \overline{CA} of $\triangle ABC$ respectively , where $AB = 3 \text{ cm.}$, $BC = 4 \text{ cm.}$ and $CA = 5 \text{ cm.}$, find the value of each of : F_1 and F_2
- 2 A regular quadrilateral pyramid the length of its base is 20 cm. and its height is $10\sqrt{3}$
Find : (1) Its lateral surface area.
(2) Its volume.
- 3 Two forces are of magnitudes 9 and 6 kg.wt. act at a particle , the measure of the included angle is α , find α if the magnitude of the resultant is $3\sqrt{7} \text{ kg.wt.}$, then find the measure of the angle between the resultant and the greatest force.

Model 1

Interactive test 1



First Multiple choice questions

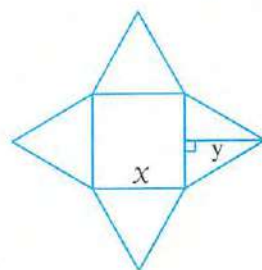
Choose the correct answer from the given ones :

- 1 The total surface area of a right circular cone which its slant height equal the diameter length of its base is
 (a) $4\pi r^2$ (b) $3\pi r^2$ (c) $3\pi r^3$ (d) $4\pi r^3$

- 2 If A , B and C are three points identify a plane , then
 (a) $AB = BC = CA$ (b) $AB + BC = AC$
 (c) $AB + BC > AC$ (d) $AB + BC < AC$

- 3 Two forces are equal act at a point and the measure of the angle between them is $\frac{\pi}{3}$ and their resultant is 3 newton , then the magnitude of each is newton.
 (a) $\sqrt{3}$ (b) 3 (c) $\frac{3}{2}$ (d) $3\sqrt{3}$

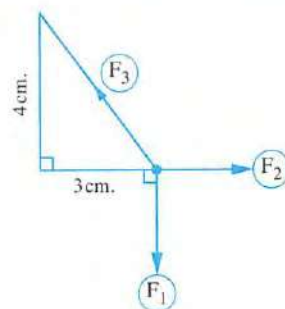
- 4 The opposite figure represents a net of regular quadrilateral pyramid its height (h) , then the relation between x , y and h is
 (a) $x^2 + y^2 = h^2$ (b) $x^2 + h^2 = y^2$
 (c) $\left(\frac{x}{2}\right)^2 + h^2 = y^2$ (d) $\left(\frac{x}{2}\right)^2 + y^2 = h^2$



5 In the opposite figure :

A body is in equilibrium under the action of three forces meeting at a point of magnitudes F_1 , F_2 and F_3 newton , and the sides of the right-angled triangle are parallel to the lines of action of the forces in the same cyclic order , then $F_1 : F_2 : F_3 = \dots\dots\dots$

- (a) 3 : 4 : 5 (b) 3 : 5 : 4
 (c) 4 : 5 : 3 (d) 4 : 3 : 5



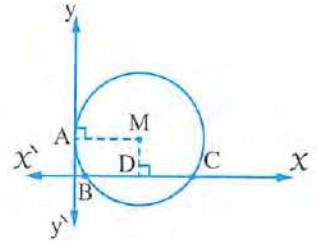
- 6** Volume of a regular quadrilateral pyramid is 400 cm^3 and its height is 12 cm , then its lateral surface area = cm^2

(a) 240 (b) 260 (c) 300 (d) 360

- 7** In the opposite figure :

If B (2 , 0) , C (8 , 0) , then the equation of the circle is

- (a) $(X - 5)^2 + (y - 4)^2 = 25$
 (b) $(X + 5)^2 + (y - 4)^2 = 36$
 (c) $(X - 5)^2 + (y - 4)^2 = 36$
 (d) $(X + 5)^2 + (y - 4)^2 = 25$



- 8** Two forces of magnitude 6 , F kg.wt. act at a point and measure of the angle between them 135° , if the line of action of resultant inclined by an angle of measure 45° , with the line of action of the force F , the magnitude of the resultant = kg.wt.

(a) 6 (b) $6\sqrt{2}$ (c) $6\sqrt{3}$ (d) 10

- 9** A body of weight (W) newton is placed on a smooth plane inclined with the horizontal at an angle of measure 30° and kept in equilibrium by the effect of force of magnitude 36 newton acts in the direction of the line of greatest slope of the plane upwards , then the magnitude of the weight = newton.

(a) 36 (b) $72\sqrt{3}$ (c) 72 (d) $36\sqrt{3}$

- 10** If \vec{R} is the resultant of the two forces \vec{F}_1 , \vec{F}_2 and \vec{R} is the resultant of the two forces \vec{F}_1 , $-\vec{F}_2$, then

- (a) $\vec{R} + \vec{R} = 2\vec{F}_1$ (b) $\vec{R} = \vec{R} + 2\vec{F}_2$
 (c) $R^2 + \vec{R}^2 = 2(F_1^2 + F_2^2)$ (d) all the previous.

- 11** The equation of the circle which is the image of the circle :

$X^2 + y^2 - 12X + 6y + 20 = 0$ by translation $(X + 2 , y - 2)$

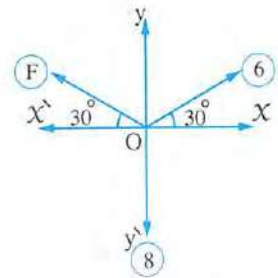
- (a) $X^2 + y^2 - 10X + 4y + 20 = 0$ (b) $X^2 - 16X + 10y + 20 = 0$
 (c) $(X - 6)^2 + (y + 3)^2 = 20$ (d) $(X - 8)^2 + (y + 5)^2 = 25$

- 12** A force of magnitude $5\sqrt{3}$ newton act in direction 30° east of north , is resolved into two perpendicular components , then the magnitude of its component in direction the east = newton.

(a) 5 (b) 7.5 (c) $\frac{5\sqrt{3}}{2}$ (d) 15

13 In the opposite figure :

If the resultant of the forces (in newton) acts along y-axis
 , then $F = \dots\dots\dots$ newton.

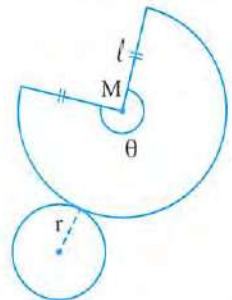


- (a) 8 (b) 6
 (c) 14 (d) 2

14 A cube made of wax , its edge length = 20 cm. it is melted and converted to a right circular cone of height 21 cm. then the radius length of the base of the cone = $\dots\dots\dots$ cm. given that 12% from wax had been lost during melting and reforming. ($\pi \approx \frac{22}{7}$)

- (a) $\frac{20\sqrt{110}}{11}$ (b) $10\sqrt{2}$ (c) 160 (d) $8\sqrt{5}$

15 The opposite figure represents net of a cone where the central angle of its circular sector = θ , $180^\circ < \theta < 360^\circ$
 , then $\dots\dots\dots$



- (a) $l < 2r$ (b) $l = r$
 (c) $l = 2r$ (d) $l > 2r$

16 If $(x \ y \ 25) \begin{pmatrix} x \\ y \\ -4 \end{pmatrix} = 0$ represents a circle , then the length of its diameter is $\dots\dots\dots$ length unit.

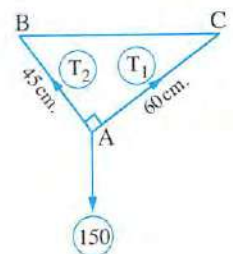
- (a) 10 (b) 20 (c) 100 (d) 200

17 The magnitudes of two forces, meeting at a point, are $5F$, $3F$, then the magnitude of their resultant can not be equal to $\dots\dots\dots$

- (a) $2F$ (b) $3\sqrt{2}F$ (c) $8F$ (d) $5\sqrt{3}F$

18 In the opposite figure :

A body of weight 150 gm. wt.
 is in equilibrium by suspended by two perpendicular strings their lengths are 60 cm. , 45 cm.
 and the other ends are fixed at C and B
 on the same horizontal line, then $T_2 - T_1 = \dots\dots\dots$ gm.wt.



- (a) 120 (b) 90 (c) 60 (d) 30

19 Two lines are skew if

- (a) they are not parallel. (b) they are not intersecting.
(c) they are not coincident. (d) they are not on the same plane.

20 The point which lies on the circle $(x - 2)^2 + y^2 = 13$ is

- (a) (2, 3) (b) (3, -2) (c) (2, 0) (d) (4, 3)

21 The ratio between the length of the edge of the regular faces pyramid : its slant height =

- (a) $2\sqrt{2} : \sqrt{3}$ (b) $2\sqrt{3} : 3$ (c) $\sqrt{6} : 2$ (d) $\sqrt{6} : 3$

Second Essay questions

Answer the following questions :

- 1** ABCDHE is a regular hexagon. Forces of magnitudes 2, $4\sqrt{3}$, 8, $2\sqrt{3}$ and 4 kg.wt. act at point A in directions \overrightarrow{AB} , \overrightarrow{AC} , \overrightarrow{AD} , \overrightarrow{AH} , \overrightarrow{AE} respectively.
Find the magnitude and the direction of their resultant.
- 2** The area of base of a right circular cone is $36\pi \text{ cm}^2$ and the length of its slant height is 10 cm., find its :
(1) Lateral surface area. (2) Total surface area. (3) Volume.
- 3** \overline{AB} is a uniform rod of weight 20 kg.wt. the end A attached to a hinge fixed on a vertical wall a horizontal force \vec{F} acts at B, the rod is in equilibrium when it inclined by angle of measure 30° with vertical, find the magnitude of each of the force and reaction of the hinge.

Model

2

Interactive test **2**



First Multiple choice questions

Choose the correct answer from the given ones :

- 1** Any three points are non-collinear identify
(a) 1 plane. (b) 2 planes. (c) 3 planes. (d) 4 planes.
- 2** When the two forces 6 and 8 newton are perpendicular, then the sine of inclination angle of the resultant with the first force equals
(a) $\frac{3}{5}$ (b) $\frac{4}{5}$ (c) $\frac{3}{4}$ (d) $\frac{4}{3}$

- 3** The centre of the circle : $x^2 + y^2 - 6x + 8y = 0$ is the point
 (a) (3, -4) (b) (-4, 3) (c) (-3, 4) (d) (-3, -4)
-
- 4** Three forces are equal in magnitude and meeting at a point are in equilibrium, then the measure of the angle between any two of them is
 (a) 60° (b) 120° (c) 90° (d) 150°
-
- 5** The volume of the right cone, if the circumference of its base is 44 cm. and its height is 15 cm. equals cm^3 ($\pi \approx \frac{22}{7}$)
 (a) 77 (b) 105 (c) 110 (d) 770
-
- 6** The forces of magnitudes $2F$, $3F$ and $4F$ newton act on a particle in the directions parallel to the sides of an equilateral triangle in the same cyclic order. then the magnitude of the resultant of these forces = newton.
 (a) $5F$ (b) $2\sqrt{3}F$ (c) $\sqrt{3}F$ (d) F
-
- 7** ABCDEF is a regular hexagon, the force of magnitude 20 newton acts along \overline{AD} , is resolved into two components in directions \overline{AC} , \overline{AF} , then the component in direction \overline{AF} equals newton.
 (a) 10 (b) $10\sqrt{3}$ (c) 20 (d) $20\sqrt{3}$
-
- 8** The equation of the circle which its centre is (2, -3) and touches the straight line which its equation is : $3x - 4y + 2 = 0$ is
 (a) $(x-2)^2 + (y+3)^2 = 2$ (b) $(x+2)^2 + (y-3)^2 = 4$
 (c) $x^2 + y^2 - 4x + 6y = 12$ (d) $(x-2)^2 + (y+3)^2 = 16$
-
- 9** If the length of the base side of a regular quadrilateral pyramid is doubled, then its volume
 (a) will doubled. (b) will be three times.
 (c) will be four times. (d) will not change.
-
- 10** If $\vec{F}_1 = 5\hat{i} - 3\hat{j}$, $\vec{F}_2 = -7\hat{i} + 2\hat{j}$, $\vec{F}_3 = 2\hat{i} + \hat{j}$, then $\vec{R} =$
 (a) $7\hat{i} - 2\hat{j}$ (b) $14\hat{i} - 4\hat{j}$ (c) $-14\hat{i} + 4\hat{j}$ (d) $\vec{0}$

- 11** The ball of a pendulum of weight 600 dyne, is displaced until the string makes an angle of measure 30° with the vertical under the action of a force perpendicular to the string. Then the magnitude of the force = dyne.

(a) $300\sqrt{3}$ (b) 1200 (c) 300 (d) $300\sqrt{2}$

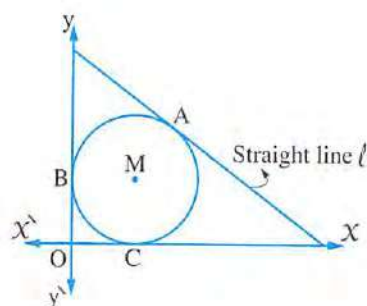
- 12** Two forces F , F act at a particle and the magnitude of their resultant is F , then the measure of the included angle of the two forces =

(a) 60° (b) 45° (c) 120° (d) 135°

- 13** In the opposite figure :

If the equation of the straight line ℓ is $\frac{x}{8} + \frac{y}{6} = 1$, then the equation of the circle is

(a) $(x-2)^2 + (y-2)^2 = 4$
 (b) $(x-2)^2 + (y-2)^2 = 16$
 (c) $(x+2)^2 + (y+2)^2 = 4$
 (d) $(x+2)^2 + (y+2)^2 = 16$



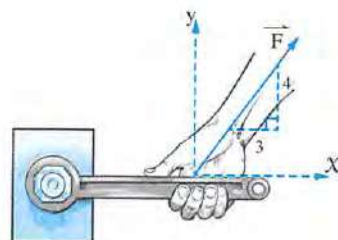
- 14** The ratio between volume of a regular triangular pyramid and volume of greatest cone can put it inside the pyramid equals

(a) $\frac{3\sqrt{3}}{\pi}$ (b) $\frac{3\sqrt{3}}{2\pi}$ (c) $\frac{\sqrt{3}}{\pi}$ (d) $\frac{3\sqrt{3}}{4\pi}$

- 15** In the opposite figure :

If vertical component of the force (\vec{F}) of a person uses a spanner is 60 newton, then the horizontal component of \vec{F} equals newton.

(a) 30 (b) 45
 (c) 60 (d) 75

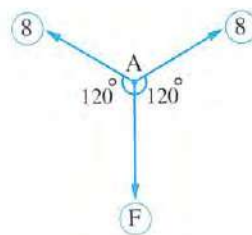


- 16** The lateral area of a right cone, its base radius length is r , and its slant height ℓ equals

(a) $2\pi r \ell$ (b) $2\pi r^2 \ell$ (c) $\pi r \ell$ (d) $\pi r^2 \ell$

17 In the opposite figure :

Particle A is kept in equilibrium under action of the three forces, as shown in the figure, where \vec{F} is in equilibrium with two forces each of magnitude 8 N. and it makes with each an angle of measure 120° , then $F = \dots\dots\dots$ N.



- (a) zero (b) 8 (c) 16 (d) $8 \sin 120^\circ$

18 Which of the following statements is not true ?

- (a) Any two points in the space have only one plane passing through them.
 (b) Any three non-collinear points in the space determine a plane.
 (c) The vertices of a triangle determine a plane.
 (d) Every two intersecting straight lines are contained in one plane.

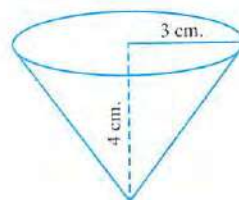
19 The least number of planes that determine a solid is

- (a) 1 (b) 2 (c) 3 (d) 4

20 In the opposite figure :

The length of the drawer = cm.

- (a) 2 (b) 3
 (c) 4 (d) 5


21 If the number of the faces of a pyramid = m and the number of its vertices = n , then the number of its edges =

- (a) $m + n$ (b) $m + n - 1$ (c) $m + n - 2$ (d) $m + n + 2$

Second Essay questions
Answer the following questions :

1 Two forces of equal magnitude meeting at a point and the magnitude of their resultant equals 12 kg.wt. if the direction of one of them is reversed then the magnitude of the resultant becomes 6 kg.wt. Find the magnitude of each force.

2 A regular quadrilateral pyramid , the side length of its base = 40 cm. , and its slant height is 25 cm. , **find :**

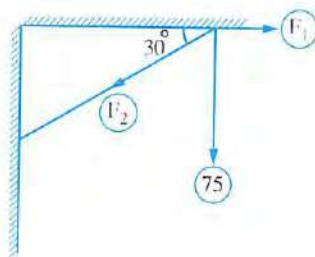
- (1) Height of the pyramid. (2) The lateral surface area.
 (3) The total surface area. (4) Its volume.

- 3** \overline{AB} is a uniform rod with length 60 cm. and weight 40 newton is connected to a hinge on the vertical wall at A. If the rod kept in equilibrium horizontally by a light string connected to the rod at B and with point C on the wall just above A and at a distance 60 cm. from A. Find the tension on the string and the reaction on the hinge at A.

Model**3**Interactive test **3****First Multiple choice questions**

Choose the correct answer from the given ones :

- 1** Two forces of magnitudes 5 , 3 newton and the measure of the angle enclosed between them is 60° , then the magnitude of their resultant R equals
- (a) 2 (b) 5 (c) 7 (d) 8
-
- 2** The height of a right circular cone is 12 cm. and the length of its drawer is 15 cm. , then its volume = cm^3
- (a) 324π (b) 715π (c) 32π (d) 180π
-
- 3** The minimum value of resultant of two forces of magnitudes 5 , 9 newton and meeting at a point equals newton.
- (a) zero (b) 9 (c) 4 (d) 5
-
- 4** The least number of planes can determine a solid is planes.
- (a) three (b) four (c) two (d) five
-
- 5** The volume of a regular quadrilateral pyramid if the side length of its base = 18 cm. and the slant height is 15 cm. is equal to cm^3
- (a) 1156 (b) 1254 (c) 1308 (d) 1296
-
- 6 In the opposite figure :**
- A vertical force of magnitude 75 newton is resolved into two components , one of them is horizontal (F_1) and the other F_2 , then $F_2 =$ newton.
- (a) 75 (b) $75\sqrt{3}$
(c) 150 (d) $150\sqrt{3}$



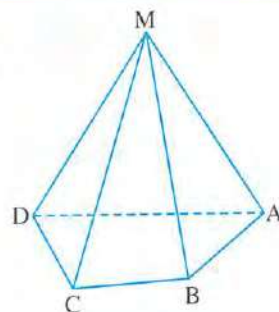
- 7** Two forces of magnitudes 6 , 12 newton act at a particle , enclosed between them an angle of measure 120° , then the measure of the angle between the resultant and the first forces =

(a) 120° (b) 60° (c) 90° (d) 30°

- 8** In the opposite figure :

The plane $ABD \cap$ the plane $MCD =$

(a) \overrightarrow{AM} (b) \overrightarrow{CD}
(c) $\{D\}$ (d) \overrightarrow{MC}



- 9** If the geometric centre of a regular hexagon is the origin and its area $= 3\sqrt{3} \text{ cm}^2$, then the equation of its circumcircle is

(a) $x^2 + y^2 = 2$ (b) $x^2 + y^2 = 4$
(c) $x^2 + y^2 = 6$ (d) $x^2 + y^2 = 8$

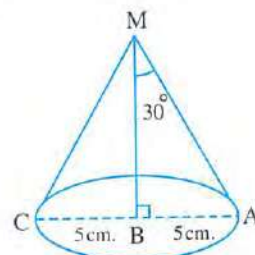
- 10** In the opposite figure :

A right circular cone in which $m(\angle AMB) = 30^\circ$

, the radius length of the base = 5 cm.

, then its total area = cm^2

(a) 50π (b) 75π
(c) 100π (d) 125π



- 11** Two forces of magnitudes 6 , 2.5 newton , the magnitude of their resultant = 6.5 newton , then the included angle between the two forces is

(a) acute. (b) obtuse. (c) right. (d) straight.

- 12** A body of weight 100 newton is placed on a smooth plane inclines to the horizontal by an angle 30° , the body kept in equilibrium by a horizontal force. F N. and the reaction of the plane on the body. is R N. then $F + R =$ N.

(a) $100\sqrt{3}$ (b) $\frac{100\sqrt{3}}{3}$ (c) $200\sqrt{3}$ (d) $\frac{200\sqrt{3}}{3}$

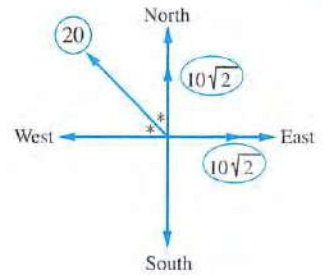
- 13** In a triangular pyramid of regular faces , if the sum of lengths of its edges = 36 cm. , then the height of the pyramid = cm.

(a) $\sqrt{6}$ (b) $2\sqrt{6}$ (c) 6 (d) 4

14 In the opposite figure :

The resultant of the forces $10\sqrt{2}$, $10\sqrt{2}$, 20 newton acts in direction

- (a) the eastern north. (b) the north.
(c) the western north. (d) the western south.



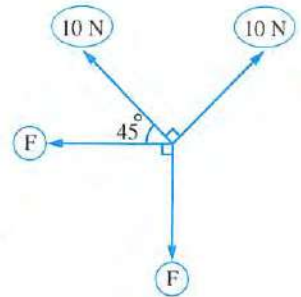
15 If the volume of hemisphere of radius length “r” equals the volume of a cone the length of radius of its base = r and its height = h , then

- (a) $h = \frac{2}{3} r$ (b) $h = 2 r$ (c) $h = 2 r^2$ (d) $h = 4 r$

16 In the opposite figure :

The condition of equilibrium of the given forces is

- (a) $F = 10$ newton. (b) $F = 10\sqrt{2}$ newton.
(c) $F = 5\sqrt{2}$ newton. (d) the system will not be equilibrium.



17 The circumference of the circle whose equation : $x^2 + y^2 = 8$ is

- (a) 8π (b) 64π (c) $2\sqrt{2}\pi$ (d) $4\sqrt{2}\pi$

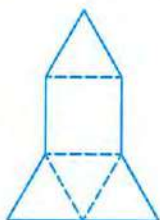
18 Two planes coincide if they have in common.

- (a) one point (b) two points
(c) three collinear points (d) three non-collinear points

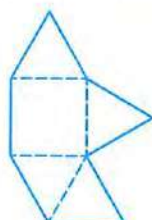
19 Force of magnitude $4\sqrt{2}$ N. acts in east direction. It is resolved into two perpendicular directions , then its component in the direction of north of the east equals N.

- (a) zero (b) $4\sqrt{2}$ (c) 4 (d) 6

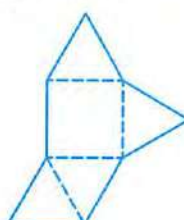
20 Which of the following nets cannot form a pyramid ?



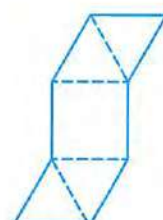
(a)



(b)



(c)



(d)

- 21** The equation of the circle with area 49π square units and centre lies on the X -axis and touches the y -axis is

(a) $(X - 2)^2 + (y - 7)^2 = 49$

(b) $(X + 5)^2 + y^2 = 25$

(c) $(X - 7)^2 + y^2 = 49$

(d) $(X + 7)^2 + (y - 7)^2 = 49$

Second Essay questions

Answer the following questions :

- 1** A weight of magnitude 200 gm.wt. is suspended by two strings of lengths 60 cm. and 80 cm. , from two points on one horizontal line where the distance between them is 100 cm. Find the magnitude of tension in each string.

- 2** The forces 8 , $4\sqrt{3}$, $6\sqrt{3}$ and 14 newton act at a point , the measure of the angle between the first force and the second force is 30° , between the second and the third is 120° and between the third and the fourth is 90° taken in the same cyclic order. Find the magnitude and direction of the resultant of these forces.

- 3** Prove that the two circles :

$$X^2 + y^2 - 2X + 6y + 1 = 0 \quad , \quad 4X^2 + 4y^2 - 8X + 24y + 15 = 0$$

are concentric circles , and find length of radius of each of them.

Model

4

Interactive test 4



First Multiple choice questions

Choose the correct answer from the given ones :

- 1** In the opposite figure :

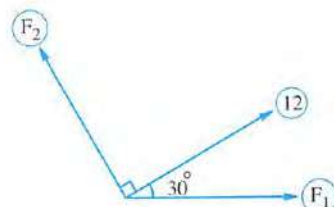
The force of magnitude 12 newton is resolved into two components \vec{F}_1 , \vec{F}_2 make angles of measures 30° , 90° , then $F_2 = \dots\dots\dots$ newton.

(a) 10

(b) $10\sqrt{3}$

(c) $6\sqrt{3}$

(d) $4\sqrt{3}$



- 2** The height of a regular quadrilateral pyramid is 9 cm. and its volume = 300 cm^3 , then the side length of its base equals cm.

(a) 5

(b) 10

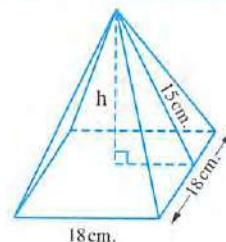
(c) 15

(d) 20

- 3** All of the following cases form a plane except
- (a) a straight line and a point do not belong to it. (b) two different parallel straight lines.
(c) two intersected straight lines. (d) two skew straight lines.
-
- 4** ABC is right-angled triangle at B where $AB = 3$ cm. , $BC = 4$ cm. , then the volume of the solid which generated by rotation of the triangle complete turn about \overrightarrow{BC} is cm^3 .
- (a) 16π (b) 18π (c) 15π (d) 12π
-
- 5** The equation $(x^2 + y^2 - 8x - 6y + 8) = 0$ represents a circle its diameter length = length unit.
- (a) 2 (b) 4 (c) 6 (d) 8
-
- 6** Two forces of magnitudes 4 , F newton act at a particle , the measure of included angle is 120° , if line of action of the resultant is perpendicular to the first force , then magnitude of the resultant = newton.
- (a) $4\sqrt{2}$ (b) $4\sqrt{3}$ (c) 4 (d) $4\sqrt{5}$
-
- 7** A body of weight (W) newton is suspended by two light strings inclined to the vertical by angles θ° and 30° the body becomes in equilibrium when the tension of the first string equal 12 newton. and the other is $12\sqrt{3}$ newton , then the weight of the body $W =$ N.
- (a) 60 (b) 25 (c) 36 (d) 24
-
- 8** If \vec{F}_1, \vec{F}_2 are two forces , then the measure of the angle enclosed between \vec{F}_1 and the resultant of the two forces $(\vec{F}_1 + \vec{F}_2), (\vec{F}_1 - \vec{F}_2)$ equals
- (a) zero. (b) $\tan^{-1}\left(\frac{F_1}{F_2}\right)$ (c) $\tan^{-1}\left(\frac{F_2}{F_1}\right)$ (d) $\tan^{-1}\left(\frac{F_1 - F_2}{F_1 + F_2}\right)$
-

9 In the opposite figure :

The volume of the regular quadrilateral pyramid in which the side length of its base = 18 cm. and the slant height = 15 cm. is cm^3

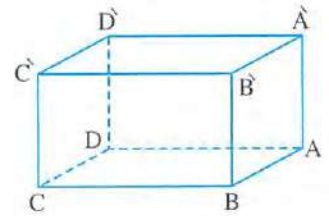


- (a) 1296 (b) 1620
(c) 540 (d) 1944

10 In the opposite figure :

The plane $\overleftrightarrow{AA'B'B} \cap \text{the plane } \overleftrightarrow{ACC'C} = \dots\dots\dots$

- (a) $\overleftrightarrow{AA'}$ (b) $\overleftrightarrow{BB'}$
(c) $\overleftrightarrow{CC'}$ (d) \overleftrightarrow{AC}

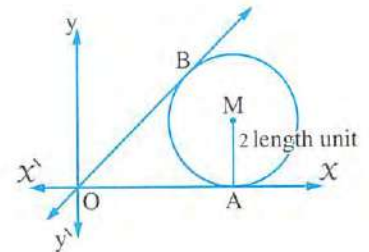


11 In the opposite figure :

If $OB = 5$ length unit ,

then the equation of the circle M is

- (a) $(x-2)^2 + (y-5)^2 = 25$
(b) $(x-2)^2 + (y-5)^2 = 4$
(c) $(x-5)^2 + (y-2)^2 = 25$
(d) $(x-5)^2 + (y-2)^2 = 4$

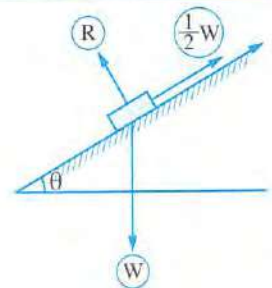


12 In the opposite figure :

If the body is in equilibrium under acting

of the shown forces , then $m (\angle \theta) = \dots\dots\dots$

- (a) 30° (b) 60°
(c) 45° (d) 15°



13 The radius length of the base of a right circular cone = 5 cm. and its total surface area = $90 \pi \text{ cm}^2$, then its volume = cm^3 .

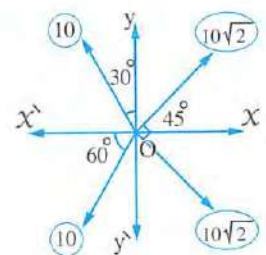
- (a) 105π (b) 95π (c) 100π (d) 120π

14 In the opposite figure :

The resultant of the system

of forces "R" = newton.

- (a) 20 (b) $10\sqrt{2}$
(c) 10 (d) zero.



15 The equation $\left| \frac{x}{y} - \frac{y}{x} \right| = 36$ represents the equation of a circle with radius length equals length unit.

- (a) 3 (b) 6 (c) 9 (d) 18

- 16** Three equal forces, intersecting at one point, are in equilibrium, then the measure of the angle between any two forces =

(a) 60° (b) 90° (c) 120° (d) 150°

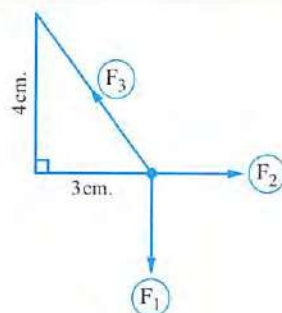
- 17** The ratio between the edge length of a triangular pyramid of regular faces and its height =

(a) $\sqrt{2} : \sqrt{3}$ (b) $\sqrt{3} : 2$
(c) $\sqrt{6} : 2$ (d) $\sqrt{3} : 3$

- 18** Two forces of magnitude 8, F newton, the measure of the angle between them $\in]0, \pi[$. If their resultant bisects the angle between them, then $F = \dots\dots\dots$ newton.

(a) $2\sqrt{2}$ (b) 4 (c) 8 (d) 16

- 19** The opposite figure represents a body remains in equilibrium under action of three forces F_1, F_2, F_3 N. act at a point and the sides of the right angled triangle are parallel to the lines of action of these forces and in one cyclic order then $F_1 : F_2 : F_3 = \dots\dots\dots$



(a) 3 : 4 : 5 (b) 3 : 5 : 4 (c) 4 : 5 : 3 (d) 4 : 3 : 5

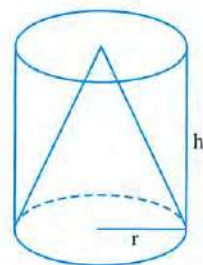
- 20** The equation of the circle passing through two points $(1, 3)$, $(2, -4)$ and its centre lies on the X -axis is

(a) $(x-5)^2 + y^2 = 25$ (b) $x^2 + (y-5)^2 = 25$
(c) $(x+5)^2 + (y-5)^2 = 25$ (d) $(x-5.1)^2 + (y+0.5)^2 = 25$

- 21** In the opposite figure :

$\frac{\text{The volume of the cone}}{\text{The volume of the cylinder}} = \dots\dots\dots$

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$
(c) $\frac{1}{4}$ (d) $\frac{3}{1}$



Second**Essay questions****Answer the following questions :**

- 1** ABCD is a rectangle in which $AB = 6$ cm. , $BC = 8$ cm. , a point $E \in \overline{AD}$ where $AE = 6$ cm. , the forces of magnitudes F , 5 , K , $6\sqrt{10}$ newton act along \overrightarrow{CB} , \overrightarrow{CA} , \overrightarrow{CD} and \overrightarrow{EC} respectively. If the system of forces are in equilibrium , then find value of each of F and K
- 2** A right circle cone , its base on the coordinate plane with equation $x^2 + y^2 = 36$ if the height of the cone = 8 length unit , **find :**
(1) Volume of the cone. **(2)** Total surface area.
- 3** \overline{AB} is a uniform rod of length $2L$ cm. and weight 8 kg.wt. acting at its midpoint. its end A is hinged at a point in a vertical wall where its end B is attached to a light string and the other end of the string is fixed to a point C on the wall situated vertically above A
 If $AB = AC = BC$ and the rod is in equilibrium. Find the tension in the string and the reaction of the hinge at A

Model**5**Interactive test **5****First****Multiple choice questions****Choose the correct answer from the given ones :**

- 1** The two straight lines are skew if they are
 (a) not parallel. (b) not intersecting.
 (c) not coincident. (d) not contained in the same plane.
- 2** The lateral surface area of a right circular cone , radius length of its base = 6 cm. and its height = 8 cm. equals cm^2
 (a) 60π (b) 28π (c) 10π (d) 48π
- 3** Two forces of magnitudes $5F$, $2F$ and their resultant is $7F$ newton , then the measure of the angle between them =
 (a) 180° (b) 60° (c) 20° (d) zero.

- 4 If the three coplanar forces $\vec{F}_1 = 5\hat{i} + 3\hat{j}$, $\vec{F}_2 = a\hat{i} + 6\hat{j}$, $\vec{F}_3 = -14\hat{i} + b\hat{j}$ act at a point and their resultant $\vec{R} = \left(10\sqrt{2}, \frac{3}{4}\pi\right)$, then $a + b = \dots\dots\dots$

(a) -1 (b) 1 (c) zero. (d) 14

- 5 The total surface area of a triangular regular faces pyramid which its edge length = ℓ cm. is equal to $\dots\dots\dots \text{cm}^2$

(a) $2\sqrt{3}\ell^2$ (b) $\sqrt{3}\ell^2$ (c) $\frac{\sqrt{3}}{3}\ell^2$ (d) $3\sqrt{2}\ell^2$

- 6 The area of any of the lateral faces of a regular quadrilateral pyramid equals to the area of its base, if side length of its base = 6 cm., then its volume = $\dots\dots\dots \text{cm}^3$

(a) 36 (b) $6\sqrt{3}$ (c) $36\sqrt{15}$ (d) $216\sqrt{15}$

- 7 Two forces of magnitude F , $F\sqrt{3}$ newton, meeting at a point and magnitude of their resultant = R_1 when the measure of included angle = 90° and the resultant became R_2 when the measure of the included angle = 150° , then $\dots\dots\dots$

(a) $R_1 = R_2$ (b) $R_1 = 2R_2$ (c) $R_1 = \frac{3}{5}R_2$ (d) $R_1 = \frac{1}{2}R_2$

- 8 The general form of the circle which its diameter \overline{AB} , where $A(2, 3)$, $B(-4, 9)$ is $\dots\dots\dots$

(a) $x^2 + y^2 - 4x - 6y + 18 = 0$ (b) $(x+4)^2 + (y-9)^2 = 72$
(c) $x^2 + y^2 - 2x + 12y + 19 = 0$ (d) $x^2 + y^2 + 2x - 12y + 19 = 0$

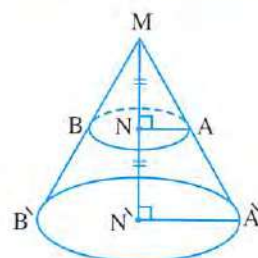
- 9 The maximum value of the resultant of two forces is 25 newton and minimum value of their resultant is 13 newton of two forces, then their magnitudes are $\dots\dots\dots$

(a) 25, 13 (b) 19, 6 (c) 13, 12 (d) 7, 20

- 10 In the opposite figure :

$MN = NN'$, then the ratio between the lateral surface area of the cone MAB to the lateral surface area of the cone $M\hat{A}\hat{B}$ equals $\dots\dots\dots$

(a) 1 : 2 (b) 1 : 4
(c) 1 : 6 (d) 1 : 8



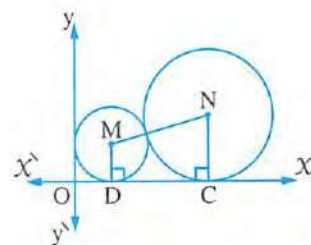
11 In the opposite figure :

M, N are two circles touching externally their

equations are $(x - 2)^2 + (y - 2)^2 = 4$ and

$(x - a)^2 + (y - b)^2 = 64$, then $a + b = \dots\dots\dots$

- (a) 8 (b) 10
(c) 18 (d) 28

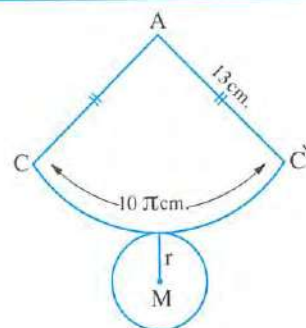


12 The opposite net describes

a solid, its volume

$= \dots\dots\dots \text{cm}^3$

- (a) 25π (b) 50π
(c) 75π (d) 100π



13 A force of magnitude 40 newton acts vertically upwards was resolved into two components one of them is horizontal and its magnitude 20 newton, then the magnitude of the other = $\dots\dots\dots$ newton.

- (a) 20 (b) $20\sqrt{2}$ (c) 10 (d) $20\sqrt{5}$

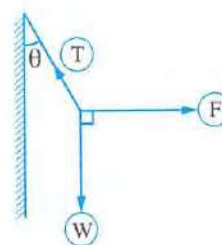
14 In the opposite figure :

A weight of magnitude (W) newton is suspended in one end of a string and the other end of the string fixed in a point on a vertical wall

, the weight is pulled by a horizontal force of magnitude F newton till the string makes an angle θ with vertical.

Which of the following statements is not correct in equilibrium state ?

- (a) $F = W \tan \theta$ (b) $\vec{W} + \vec{F} + \vec{T} = \vec{O}$
(c) $T^2 = F^2 + W^2$ (d) $T = F + W$



15 Three coplanar forces intersecting at one point and in equilibrium. If 3 and 7 N. are magnitudes of two forces of them, then the magnitude of the third force could be equals $\dots\dots\dots$ N.

- (a) 11 (b) 2 (c) 5 (d) 3

16 If a plane intersects a regular quadrilateral pyramid, parallel to its base, then the cross section shape is

- (a) a triangle. (b) a square. (c) a rectangle. (d) a circle.

17 The point which lies on the circle $x^2 + (y - 3)^2 = 16$ is

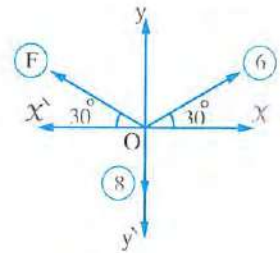
- (a) (0 , 3) (b) (3 , -2) (c) (2 , 0) (d) (4 , 3)

18 If the resultant (measured in newton) of the forces

shown in the figure acts along the y-axis

, then $F = \dots\dots\dots$ N.

- (a) 2 (b) 6
(c) 8 (d) 14



19 In the opposite figure :

\overline{AB} is a uniform rod fixed to a hinge at A to a vertical wall.

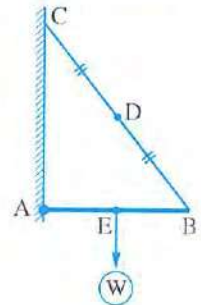
It's kept horizontally by a string fixed to point B and

the other end of the string is fixed to point C

on the wall above A.

Which of the following is the triangle of force ?

- (a) $\triangle DBE$ (b) $\triangle DEA$ (c) $\triangle ADE$ (d) $\triangle ACD$



20 The ratio between the edge length of the triangular regular faces pyramid :

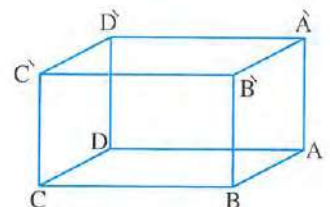
its height =

- (a) $\sqrt{2} : \sqrt{3}$ (b) $\sqrt{3} : 2$ (c) $\sqrt{6} : 2$ (d) $\sqrt{3} : 3$

21 In the opposite figure :

The plane $A\hat{A}B \cap$ the plane $A\hat{C}C = \dots\dots\dots$

- (a) \overleftrightarrow{AA} (b) \overleftrightarrow{BB}
(c) \overleftrightarrow{CC} (d) \overleftrightarrow{AC}



Second Essay questions

Answer the following questions :

- 1 The total surface area of a right circular cone is $96\pi \text{ cm}^2$, the length of its slant height is 10 cm. Find the radius length of its base and its volume.
- 2 A homogeneous smooth sphere its radius length is 10 cm. , its weight = 30 gm.wt. is in equilibrium by a string of length 10 cm. attached to a point of its surface and the other end of the string is fixed at the point in vertical smooth wall , find the tension of the string and the reaction of the wall on the sphere.
- 3 ABCD is a square of side length = 10 cm. , E is the midpoint of \overline{AB} , forces of magnitudes 2 , $7\sqrt{5}$, $4\sqrt{2}$ and 4 newton in directions \overrightarrow{CB} , \overrightarrow{CE} , \overrightarrow{CA} and \overrightarrow{CD} respectively , find magnitude and direction of resultant of this forces.

Model

6

Interactive test **6**



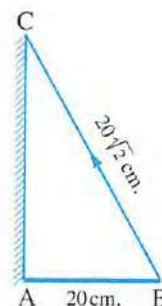
First Multiple choice questions

Choose the correct answer from the given ones :

- 1 The lateral surface area of right cone , which the radius length of its base is r , and length of its slant height l equals
 (a) $2\pi lr$ (b) $2\pi l r^2$ (c) πlr (d) $\pi l r^2$
- 2 Which two forces from the following pairs , could not have resultant with magnitude = 4 newton ?
 (a) 2 newton , 4 newton (b) 3 newton , 3 newton
 (c) 2 newton , 6 newton (d) 3 newton , 8 newton
- 3 The point which lies on the circle $(x-2)^2 + y^2 = 13$ is
 (a) (2 , 3) (b) (3 , -2) (c) (2 , 5) (d) (4 , 3)
- 4 Number of planes which carry faces of pentagon pyramid is
 (a) 5 (b) 6 (c) 10 (d) infinite.

5 In the opposite figure :

\overline{AB} is a uniform rod with length 20 cm. and its weight = 30 newton attached by a smooth hinge fixed on a vertical wall in the end A and at the end B suspended by a light string with length $20\sqrt{2}$ cm. its other end fixed at point C on the wall above point A, if the rod is in equilibrium in the horizontal position , then the reaction of the hinge



- (a) act in direction \overrightarrow{AB}
- (b) its line of action distant 10 cm. from the wall.
- (c) bisects \overline{BC}
- (d) is of magnitude = 15 newton.

6 The general form of the equation of circle its centre is (5 , - 4) and touches X-axis is

- (a) $x^2 + y^2 - 10x + 8y + 25 = 0$
- (b) $x^2 + y^2 - 5x + 4y = 0$
- (c) $x^2 + y^2 - 10x + 8y = 25$
- (d) $x^2 + y^2 + 10x - 8y + 25 = 0$

7 If \vec{R} is the resultant of $\vec{F}_1, \vec{F}_2, \vec{R} \perp \vec{F}_1$ and $R = \frac{1}{2} F_2$, then the measure of the angle between the two forces \vec{F}_1, \vec{F}_2 is

- (a) 40°
- (b) 120°
- (c) 135°
- (d) 150°

8 Three coplanar forces of magnitudes 60 , F and K newton meeting at a point and in equilibrium. If the angle between the 1st and the 2nd forces measures 120° and between the 2nd and the 3rd measures 90° , then the value of K = newton.

- (a) $30\sqrt{3}$
- (b) $30\sqrt{2}$
- (c) 30
- (d) 60

9 A right cone of volume $27\pi \text{ cm}^3$, circumference of its base $6\pi \text{ cm}$, then its height. = cm.

- (a) 27
- (b) 3
- (c) $3\sqrt{3}$
- (d) 9

10 The ratio between the lateral surface area of the triangular pyramid of regular faces to its total surface area =

- (a) 1 : 3
- (b) 1 : 4
- (c) 3 : 4
- (d) 1 : 2

- 11** ABCDEO is regular hexagon, the forces of magnitudes $2, 4\sqrt{3}, 4\sqrt{3}, 2$ kg.wt. act at point A in directions $\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AE}, \overrightarrow{AO}$ respectively.

, then the resultant of this forces acts in direction of

- (a) \overrightarrow{AC} (b) \overrightarrow{AE} (c) \overrightarrow{AD} (d) \overrightarrow{AO}

- 12** The length of the tangent segment which drawn from the point $(0, 2r)$ to the circle $x^2 + y^2 = r^2$ is length unit.

- (a) r (b) $2r$ (c) $\sqrt{3}r$ (d) $\frac{\sqrt{3}}{2}r$

- 13** ABC is an isosceles triangle where $AB = AC = 10$ cm., $BC = 12$ cm., rotates a complete revolution about \overline{BC} , then the volume of the solid which generated by rotation = cm^3 .

- (a) 128π (b) 256π (c) 384π (d) 512π

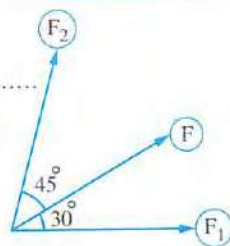
- 14** ABCD $\hat{A}\hat{B}\hat{C}\hat{D}$ is a cube of edge length = 20 cm. a right circular cone is put inside the cube such that the vertex of the cone is the centre of cube base $\hat{A}\hat{B}\hat{C}\hat{D}$, and base of the cone touches the sides of the base ABCD, then the ratio between volume of each the cone and cube is

- (a) $\frac{\pi}{12}$ (b) $\frac{\pi}{3}$ (c) $\frac{1}{3}$ (d) $\frac{12}{\pi}$

- 15** In the opposite figure :

The force \vec{F} is the resultant of the two forces \vec{F}_1, \vec{F}_2 , then $\frac{F_1 + F_2}{F} = \dots\dots\dots$

- (a) $\sin 30^\circ + \sin 45^\circ$ (b) $\frac{\sin 75^\circ + \sin 30^\circ}{\sin 75^\circ}$
(c) $\frac{\sin 45^\circ + \sin 30^\circ}{\sin 75^\circ}$ (d) $\frac{\sin 75^\circ}{\sin 30^\circ} + \frac{\sin 75^\circ}{\sin 45^\circ}$

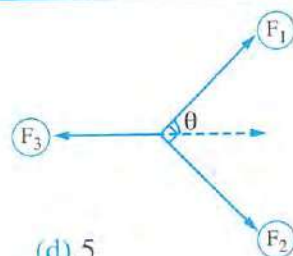


- 16** Two forces meeting at a point of magnitudes F_1, F_2 where $0 \leq F_1 \leq 13, 8 \leq F_2 \leq 17$, the measure of the included angle is 180° and magnitude of their resultant R , then

- (a) $3 \leq R \leq 4$ (b) $0 \leq R \leq 4$ (c) $0 \leq R \leq 17$ (d) $5 \leq R \leq 17$

- 17** The opposite figure represents three forces \vec{F}_1, \vec{F}_2 and \vec{F}_3 of magnitudes 4, 3 and 2 newton respectively, if $\sin \theta = \frac{3}{5}$, then magnitude of their resultant equals newton.

- (a) 1 (b) 2 (c) 3 (d) 5

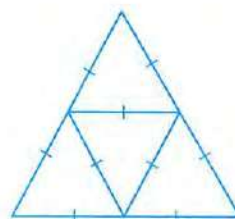


18 The vertical straight lines in the space are

- (a) parallel. (b) skew. (c) lie in one plane. (d) intersecting.

19 Which solid represents the opposite net ?

- (a) quadrilateral pyramid.
(b) regular quadrilateral pyramid.
(c) triangular pyramid with regular faces.
(d) otherwise.



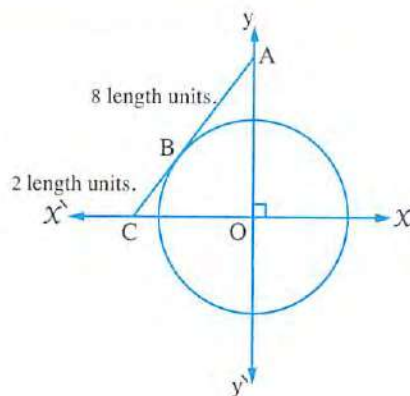
20 The magnitude of two forces 3 and 5 N. and thier resultant is 2 N. , then the measure of the angle between the resultant and the second force =

- (a) 180° (b) 90° (c) 0° (d) 30°

21 In the opposite figure :

The equation of the circle is

- (a) $x^2 + y^2 = 4$
(b) $x^2 + y^2 = 16$
(c) $x^2 + y^2 = 64$
(d) $x^2 + y^2 = 100$



Second Essay questions

Answer the follwoing questions :

- 1** A uniform rod of length 100 cm. , and its weight 150 gm.wt. is suspended from its ends by two strings , the other end of each string fixed on the same point , if the lengths of the two strings are 80 cm. , 60 cm. , then find the magnitude of the tension of each of them.
- 2** A regular quadrilateral pyramid , the side length of its base 18 cm. If its volume is 1296 cm^3 , then find the slant height and lateral surface area.
- 3** Two forces of magnitudes 16 and F kg.wt. act on a particle and the measure of the angle between them is 120° . If their resultant is inclined to the force 16 kg.wt. by an angle whose measure is 30° , find the magnitude of F and the resultant.

Model

7

Interactive test **7**



First Multiple choice questions

Choose the correct answer from the given ones :

- 1** If the resultant of two forces acting at a point reaches the maximum value , then the measure of the angle between their line of actions equals
 (a) 180° (b) 120° (c) 0° (d) 60°

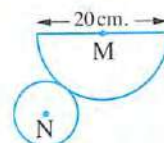
- 2** A regular quadrilateral pyramid , the side length of its base 10 cm. , and its slant height 13 cm. , then its lateral area = cm^2 .
 (a) 260 (b) 360 (c) 130 (d) 520

- 3** A body of weight W newton is placed on smooth inclined plane , where the angle of the inclination of the plane with the horizontal is 30° , the body kept in equilibrium by a force of magnitude 36 newton and acts in the direction of the line of greatest slope upward , then the magnitude of the weight = N.
 (a) $36\sqrt{3}$ (b) $36\sqrt{2}$ (c) 72 (d) $72\sqrt{3}$

- 4** The general form of the equation of the circle which its centre is $(-2, 5)$ and passes through $(3, 2)$ is
 (a) $x^2 + y^2 - 4x + 10y - 5 = 0$ (b) $x^2 + y^2 + 4x - 10y - 5 = 0$
 (c) $x^2 + y^2 + 2x - 5y - 5 = 0$ (d) $x^2 + y^2 + 4x - 10y - 25 = 0$

- 5** If the straight line $L \parallel$ the plane X , the point $A \in X$, then $L \cap X =$
 (a) \emptyset (b) L (c) $\{A\}$ (d) X

- 6** If we folded the opposite net to become a cone
 , then the radius length of its base = cm.
 (a) 10 (b) 8
 (c) 5 (d) 2.5



- 7** The volume of right cone , where the length of its drawer 15 cm. and the total surface area = $216\pi \text{ cm}^2$ equals cm^3 .
 (a) 205π (b) 320π (c) 380π (d) 324π

- 8 If \vec{R} is the resultant of the two forces \vec{F}_1, \vec{F}_2 where $F_2 > F_1$, then which of the following conditions is enough to make $\vec{R} \perp \vec{F}_1$?

(a) $R^2 = F_1^2 + F_2^2$ (b) $R^2 = F_2^2 - F_1^2$ (c) $\vec{F}_1 \perp \vec{F}_2$ (d) All of previous.

- 9 If $x^2 + y^2 + 2(\cos \theta)x - 2(\sin \theta)y - 8 = 0$ represents the equation of a circle, then $r = \dots\dots\dots$ length unit.

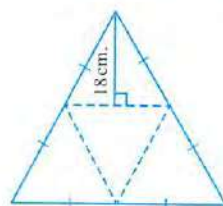
(a) $\sqrt{2}$ (b) $2\sqrt{2}$ (c) 3 (d) 8

- 10 Four coplanar forces of magnitudes $F_1, 6\sqrt{2}, 6\sqrt{2}, F_2$ gm.wt. acting at a point in direction of east, the eastern north, western north and south respectively. If the resultant of this forces equal 7 gm.wt. and acts in direction of east, then $(F_1, F_2) = \dots\dots\dots$

(a) (7, 0) (b) (7, 12) (c) $(7, 12\sqrt{2})$ (d) $(6\sqrt{2}, 6\sqrt{2})$

- 11 When we fold the opposite net, then the total surface area of the produced solid is $\dots\dots\dots \text{cm}^2$

(a) $108\sqrt{3}$ (b) $324\sqrt{3}$
(c) 758 (d) $432\sqrt{3}$



- 12 ABCDE is a regular pentagon, a force of magnitude 20 newton acts along \vec{AC} , then was resolved in two directions \vec{AB} and \vec{AE} , then the magnitude of the component in direction \vec{AB} equals $\dots\dots\dots$ newton.

(a) 10 (b) 20 (c) $20\sqrt{3}$ (d) 12.4

- 13 The radius length of the base of a right circular cone is 15 cm., and its height = 20 cm., then its lateral area = $\dots\dots\dots \text{cm}^2$

(a) 600π (b) 375π (c) 1875π (d) 5625π

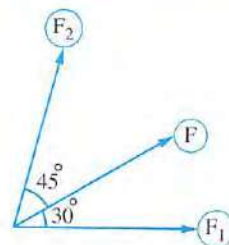
- 14 If the force of magnitude F is in equilibrium with the two forces 5, 3 N. and their included angle is 60° , then $F = \dots\dots\dots$ N.

(a) $\sqrt{19}$ (b) $\sqrt{34}$ (c) 7 (d) 15

- 15 In the opposite figure :

The force \vec{F} is the resultant of the two forces \vec{F}_1, \vec{F}_2 , then $F_2 = \dots\dots\dots$

(a) $\frac{F \sin 45^\circ}{\sin 75^\circ}$ (b) $F \sin 30^\circ$
(c) $\frac{F \sin 30^\circ}{\sin 75^\circ}$ (d) $F \cos 45^\circ$



- 16** The centre of the circle : $2x^2 + 2y^2 - 6x + 8y = 0$ is the point
 (a) $(3, -4)$ (b) $(-4, 3)$ (c) $\left(\frac{3}{2}, -2\right)$ (d) $(-3, -4)$
-
- 17** Two perpendicular forces of magnitude 6, 8 N., then the sine of angle between the resultant and first force =
 (a) $\frac{3}{5}$ (b) $\frac{4}{5}$ (c) $\frac{3}{4}$ (d) $\frac{4}{3}$
-
- 18** The least number of planes could form a solid is
 (a) 1 (b) 2 (c) 3 (d) 4
-
- 19** If $\vec{F}_1 = 2\hat{i} - 2\hat{j}$, $\vec{F}_2 = 4\hat{i} - 8\hat{j}$ and their resultant $\vec{R} = 2a\hat{i} - 3b\hat{j}$, then $a + b =$
 (a) 3 (b) $3\frac{1}{3}$ (c) $6\frac{1}{3}$ (d) 12
-
- 20** The equation of the circle with area 81π square units and its centre lies in the second quadrant and touches the y-axis could be
 (a) $(x+3)^2 + (y-5)^2 = 25$ (b) $(x+4)^2 + (y-3)^2 = 25$
 (c) $(x-6)^2 + (y+4)^2 = 36$ (d) $(x+9)^2 + (y-14)^2 = 81$
-
- 21** If a right circular cone intersected by a plane parallel to its base , then the resulted sector is
 (a) an isosceles triangle. (b) an equilateral triangle.
 (c) a circle. (d) a trapezoid.

Second Essay questions

Answer the following questions :

- 1** The lateral surface area of a regular quadrilateral pyramid 240 cm^2 , and its slant height is 12 cm., find :
 (1) Height of the pyramid. (2) Volume of the pyramid.
-
- 2** A metal sphere of weight 400 kg.wt. acts in its centre, placed between two smooth planes, one of them is vertical and the other inclined 60° with vertical, then find the reaction of each plane.
-
- 3** ABCD is a square of side length 12 cm. $H \in \overline{BC}$ where $BH = 5$ cm. forces of magnitudes 2, 13, $4\sqrt{2}$, 9 gm.wt. act in directions of \overrightarrow{AB} , \overrightarrow{AH} , \overrightarrow{CA} and \overrightarrow{AD} respectively. Find the resultant of these forces.

Model

8

Interactive test 8



First Multiple choice questions

Choose the correct answer from the given ones :

- 1 The volume of the regular quadrilateral pyramid , where the perimeter of its base = 36 cm. and its height 10 cm. is cm^3 .
 (a) 810 (b) 180 (c) 360 (d) 270
- 2 The circumference of the circle which its equation is $x^2 + y^2 = 8$ is
 (a) 8π (b) 64π (c) $2\sqrt{2}\pi$ (d) $4\sqrt{2}\pi$
- 3 Two forces are equal in magnitude and each of them equal F newton if the magnitude of the resultant is F newton , then the measure of the included angle =
 (a) 0° (b) 30° (c) 60° (d) 120°
- 4 Force of magnitude $10\sqrt{2}$ gm.wt. acts in direction of the eastern south , it was resolved into two perpendicular components , then the component in the south direction = gm.wt.
 (a) $10\sqrt{3}$ (b) $10\sqrt{2}$ (c) 10 (d) 5
- 5 The general form of the equation of the circle where its centre is (2 , - 1) and radius length is 3 cm. is
 (a) $x^2 + y^2 - 4x + 2y - 4 = 0$ (b) $x^2 + y^2 - 2x + y - 4 = 0$
 (c) $x^2 + y^2 + 4x - 2y - 4 = 0$ (d) $x^2 + y^2 - 4x + 2y - 16 = 0$
- 6 A body of weight 24 newton is suspended at one end of a string of length 130 cm. , the other end is fixed at a point of a vertical wall. A horizontal force acts on the body to become in equilibrium. , then the magnitude of horizontal force = newton when the body is at a distance = 50 cm. from the wall.
 (a) 26 (b) 22 (c) 13 (d) 10

7 In the opposite figure :

The central angle of the circular sector which if it is folded becomes this right circular cone is

- (a) acute. (b) obtuse. (c) straight. (d) reflex.



8 Number of the planes which passes through two given points is

- (a) zero. (b) 1 (c) 2 (d) infinite.

9 A right circular cone , length of its drawer 17 cm. , and its height 15 cm. , then the radius length of its base = cm.

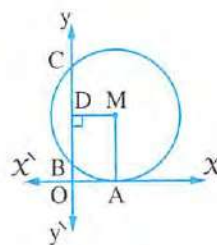
- (a) 8 (b) 13 (c) 7 (d) 12

10 In the opposite figure :

Circle M touches X -axis at A , $OB = 2$ length units

, $BC = 6$ length units , then equation of the circle M is

- (a) $(X + 4)^2 + (y + 5)^2 = 16$
 (b) $(X - 4)^2 + (y - 5)^2 = 25$
 (c) $(X - 4)^2 + (y - 5)^2 = 16$
 (d) $(X + 4)^2 + (y + 5)^2 = 25$



11 A body of weight 6 kg.wt. is placed on a smooth plane inclines to the horizontal by an angle of measure 30° and kept in equilibrium by a horizontal force. , then the magnitude of the reaction of the plane on the body = kg.wt.

- (a) $2\sqrt{3}$ (b) $4\sqrt{3}$ (c) $12\sqrt{3}$ (d) $8\sqrt{3}$

12 The value of K which makes the two circle $C_1 : (X + 2)^2 + (y + 11)^2 = K$, $C_2 : (X - 3)^2 + (y - 1)^2 = 16$ are touching each other is =

- (a) 9 or 17 (b) 81 or 289 (c) 3 or 17 (d) 17 or 81

13 If the resultant of two perpendicular forces , inclined to the greatest one by angle of measure θ , then which of the following values is suitable value of θ ?

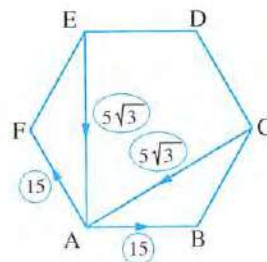
- (a) 90° (b) 70° (c) 45° (d) 10°

14 In the opposite figure :

ABCDEF is a regular hexagon , forces of magnitudes

15 , $5\sqrt{3}$, $5\sqrt{3}$ and 15 newton act along \overrightarrow{AB} , \overrightarrow{CA} , \overrightarrow{EA} and \overrightarrow{AF} , then the magnitude of the resultant R = newton.

- (a) 5 (b) 10
 (c) 25 (d) zero.

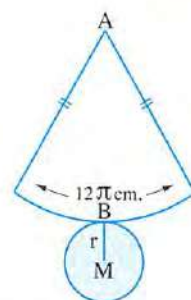


- 15** ABCDEF is a regular hexagon. A force of magnitude 20 newton acts in direction of \overrightarrow{AD} , then the components of the force in direction of \overrightarrow{AC} , \overrightarrow{AF} respectively are newton.

(a) $10\sqrt{3}$, 10 (b) $5\sqrt{3}$, 10 (c) 10, $10\sqrt{3}$ (d) $20\sqrt{3}$, 20

- 16** The opposite net describes a solid its volume = $96\pi \text{ cm}^3$, then its total area = cm^2 .

(a) 16π (b) 32π
(c) 48π (d) 96π



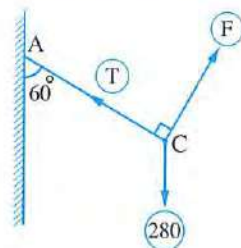
- 17** Which of the following statements is true ?

(a) The lateral faces of the right pyramid are congruent.
(b) The regular pyramid is a right pyramid.
(c) The heights of the lateral faces of the right pyramid are equal.
(d) The least number of planes that can determine a solid = 3 planes.

- 18** In the opposite figure :

A lamp of weight 280 gm.wt. is attached to the end of a string. It is in equilibrium under the effect of a force perpendicular to the string when it is inclined to the vertical by an angle of measure 60° , then $\frac{F}{T} = \dots\dots\dots$

(a) 2 (b) $\frac{1}{2}$ (c) $\frac{1}{\sqrt{3}}$ (d) $\sqrt{3}$

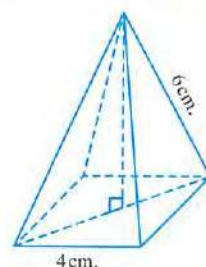


- 19** The resultant of two forces F , $2F$ is perpendicular to one of them, then $R = \dots\dots\dots$

(a) $\sqrt{5}F$ (b) $\sqrt{3}F$ (c) $3F$ (d) F

- 20** The opposite figure represents a regular quadrilateral pyramid of height = cm.

(a) $7\sqrt{2}$ (b) $2\sqrt{7}$
(c) $4\sqrt{2}$ (d) $2\sqrt{5}$



- 21** The ratio between the edge length of the triangular regular faces pyramid :
its height =

(a) $\sqrt{2} : \sqrt{3}$

(b) $\sqrt{3} : 2$

(c) $\sqrt{6} : 2$

(d) $\sqrt{3} : 3$

Second Essay questions

Answer the following questions :

- 1** The volume of regular hexagon pyramid is $8\sqrt{3} \text{ cm}^3$ and its height is 4 cm. , find the perimeter of its base.
- 2** The forces of magnitudes F , 80 , K , 50 , $80\sqrt{3}$ newton act at a point in the directions of east , 30° east of north , north , west and south respectively.
Find the values of F and K if the resultant is 40 newton in magnitude in the direction of 60° north of east.
- 3** A sphere in which M is its centre and its radius length is 12 cm. and its weight is (W) newton rests at B against a smooth vertical wall , from a point C on its surface , it is tied by a string , its other end is fixed at A of the wall lies directly above B
If the tension in the string is 50 newton. Find the length of the string and the weight of the sphere when the reaction of the wall to the sphere equals 25 newton.

Model

9

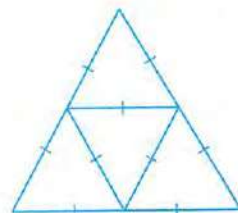
Interactive test **9**



First Multiple choice questions

Choose the correct answer from the given ones :

- 1** Two perpendicular forces of magnitudes $2F - 5$, $F + 2$ newton act at a particle , the magnitude of their resultant is $3\sqrt{5}$ newton , then $F = \dots\dots\dots$ newton
- (a) 2 (b) 3 (c) 4 (d) 5
- 2** Which solid , its net is the opposite figure ?
- (a) Quadrilateral pyramid.
(b) Regular quadrilateral pyramid.
(c) Triangular pyramid with regular faces.
(d) Otherwise.



- 3** Volume of right circular cone is 100 cm^3 , then its volume when its height is doubled becomes cm^3 .
 (a) 100 (b) 200 (c) 400 (d) 800
-
- 4** A body of weight 18 kg.wt. is placed on a smooth plane inclines to the horizontal at angle of measure 30° , the body kept in equilibrium by a force \vec{F} inclines to line of greatest slope upward by an angle of measure 30° , then the magnitude of this force = kg.wt.
 (a) 12 (b) 9 (c) $3\sqrt{3}$ (d) $6\sqrt{3}$
-
- 5** Force of magnitude $4\sqrt{2}$ newton acts in east direction it was resolved into two perpendicular component, then the magintude of the component in direction of eastern north equals newton.
 (a) 4 (b) $4\sqrt{2}$ (c) 8 (d) $8\sqrt{2}$
-
- 6** A regular quadrilateral pyramid. The perimeter of its base = 40 cm. and its height 12 cm. , then its lateral surface area = cm^2 .
 (a) 200 (b) 240 (c) 260 (d) 320
-
- 7** The equation of the circle which the straight line : $X + y = 2$ touches it, and its centre is $(3, 5)$ is
 (a) $(X - 3)^2 + (y - 5)^2 = 3\sqrt{2}$ (b) $(X + 3)^2 + (y + 5)^2 = 18$
 (c) $(X - 3)^2 + (y - 5)^2 = 12$ (d) $(X - 3)^2 + (y - 5)^2 = 18$
-
- 8** A weight of 16 newton is suspended at the end of a light string and the other end is fixed at a point of a vertical wall. A force of magnitude $F \text{ newton}$ acts on the weight in a perpendicular direction of the string till it becomes in equilibrium when the string is inclined to the wall with an angle of measure 30° , then the magnitude of the tension in the string. = newton.
 (a) 8 (b) $8\sqrt{2}$ (c) $8\sqrt{3}$ (d) 12
-
- 9** The equation of the circle which touches the X -axis at the point $(-2, 0)$ and intersect from the positive part of y -axis a chord of length $4\sqrt{3}$ length unit is
 (a) $(X + 2)^2 = 48$ (b) $(X + 2)^2 + (y - 4)^2 = 48$
 (c) $(X - 2)^2 + (y + 4)^2 = 24$ (d) $(X + 2)^2 + (y - 4)^2 = 16$

- 10** Two forces are equal in magnitude and the magnitude of their resultant is 24 newton and the measure of the angle between the resultant and one of the two forces is 30° , then the magnitude of each of the two forces = newton.

(a) 8 (b) $8\sqrt{3}$ (c) $8\sqrt{2}$ (d) 12

- 11** The ratio between length of the edge of triangular pyramid of regular faces to its height =

(a) $\sqrt{2} : \sqrt{3}$ (b) $\sqrt{3} : 2$ (c) $\sqrt{6} : 2$ (d) $\sqrt{3} : 3$

- 12** Right circular cone, area of its base = $25\pi \text{ cm}^2$, length of its drawer = 13 cm., then its lateral area = cm^2 .

(a) 50π (b) 65π (c) 90π (d) 100π

- 13** Two forces of magnitudes F , $2F$ newton act at a particle, and the line of action of its resultant is perpendicular to one of the two forces, then the measure of the included angle between the two forces =

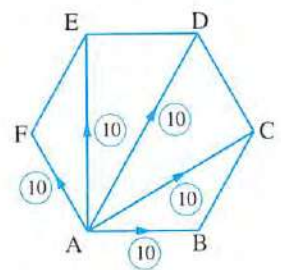
(a) 60° (b) 90° (c) 120° (d) 135°

- 14** The point which lies on the circle : $(x - 2)^2 + y^2 = 13$ is

(a) (2, 3) (b) (3, -2) (c) (2, 5) (d) (4, 3)

- 15** Five forces equal in magnitude each equals 10 newton act on one of vertices of a regular hexagon in directions of the other vertices as shown in the opposite figure, then the resultant of this forces is newton.

(a) 50 (b) 20
(c) $30\sqrt{3}$ (d) $(20 + 10\sqrt{3})$

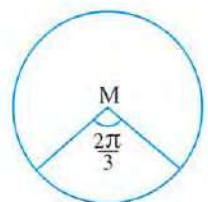


- 16** In the opposite figure :

A circle is divided into two circular sectors such that they form two right cone nets

, then $\frac{\text{the lateral area of the smallest cone}}{\text{the lateral area of the greatest cone}} = \dots\dots\dots$

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{8}$ (d) $\frac{1}{16}$



- 17** Two forces of magnitudes 4 and 6 newton. The measure of the angle between them is 90° , then the tangent of the angle between the resultant and the first force equals

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) $2\sqrt{13}$ (d) $\frac{\sqrt{6}}{2}$

- 18** Two forces equal in magnitude, the measure of the angle between them is 90° and the magnitude of their resultant is 8 N. , then the magnitude of each one = N.

(a) $2\sqrt{2}$ (b) 4 (c) $4\sqrt{2}$ (d) 8

- 19** The centre of the circle : $x^2 + y^2 - 6x + 8y = 0$ is the point

(a) (3 , -4) (b) (4 , -3) (c) (-3 , 4) (d) (-4 , 3)

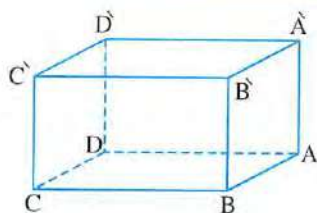
- 20** Two non parallel planes intersect at

(a) a point. (b) a straight line. (c) a plane. (d) a ray.

- 21** In the opposite figure :

The plane ABCD // the plane

(a) \vec{ABC} (b) \vec{ABD}
(c) \vec{ABB} (d) \vec{ABC}



Second Essay questions

Answer the following questions :

- 1** A uniform rod \overline{AB} of length 6 metres and weight 8 kg.wt. is attached to a hinge fixed in a vertical wall at its end A. The rod is kept horizontally by attaching it at a point C on the rod (where $AC = 4$ metres) by a string which its other end is fixed at the point D on the wall above A exactly and at a distance 4 metres from it. Calculate the magnitude of the tension in the string and the reaction of the hinge.
- 2** A circular sector , the radius length of its circle is 18 cm. and the measure of its central angle = 60° , it is folded and their radii are connected to form greatest lateral area of a right circular cone. Find the volume of this cone.
- 3** Three forces of magnitudes 10 , 20 , 30 newton act at a particle , the first in direction of east and the second in direction of 30° west of north and third in direction of 60° south of west. Find the magnitude and direction of the resultant of these forces.

Model

10

Interactive test 10



First Multiple choice questions

Choose the correct answer from the given ones :

- 1 The point which lies on the circle : $x^2 + (y - 5)^2 = 20$ is
 (a) (2 , 3) (b) (3 , - 2) (c) (2 , 5) (d) (4 , 3)

- 2 Two forces of magnitudes 3 , 4 newton their resultant is 7 newton , then the measure of the angle between them is
 (a) 0° (b) 60° (c) 180° (d) 90°

- 3 Two forces of magnitudes 8 , F newton act at a particle , if the measure of the included angle is 120° , and their resultant $F\sqrt{3}$ newton , then $F =$ newton.
 (a) 4 (b) $4\sqrt{2}$ (c) $4\sqrt{3}$ (d) 8

- 4 The length of the drawer of a right circular cone is 17 cm. and its height = 15 cm. , then its total surface area = cm^2
 (a) 200π (b) 136π (c) 320π (d) 400π

- 5 If O is the origin of perpendicular Cartesian coordinate plane and $\vec{F} = (8 \text{ kg.wt.} , 135^\circ)$ is a force acts at the point O , then the component of \vec{F} in direction of y-axis equals
 (a) $-4\sqrt{2}$ (b) $4\sqrt{2}$ (c) $4\sqrt{3}$ (d) 4

- 6 ABCDEF is a regular hexagon , forces of magnitudes $6\sqrt{3}$, 5 , $6\sqrt{3}$ newton act in directions \overrightarrow{AC} , \overrightarrow{AD} , \overrightarrow{AE} respectively. then the magnitude and direction of the resultant of these forces is
 (a) 18 N. in \overrightarrow{AD} direction. (b) 23 N. in \overrightarrow{AD} direction.
 (c) 20 N. in \overrightarrow{AE} direction. (d) 23 N. in \overrightarrow{AC} direction.

- 7 A body of weight 32 newton is suspended at the end of a string with length 10 cm. and the other end of the string is fixed at a point on a vertical wall and the body is pulled by horizontal force to make the body in equilibrium when it distant 6 cm. from the wall. , then the magnitude of this force = newton.
 (a) 24 (b) 40 (c) 36 (d) 28

- 8** A body of weight 18 newton is placed on a smooth plane inclines to the horizontal by angle of measure 30° and kept in equilibrium by a horizontal force of magnitude F newton.
 , then the magnitude of the reaction of the plane on the body = newton.

(a) $6\sqrt{3}$ (b) $8\sqrt{3}$ (c) $12\sqrt{3}$ (d) $10\sqrt{3}$

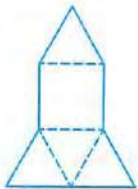
- 9** The equation of the circle which its centre $(-4, 3)$ and passes through the origin point is

(a) $(x+4)^2 + (y-3)^2 = 5$ (b) $(x-4)^2 + (y+3)^2 = 25$
 (c) $(x+4)^2 + (y-3)^2 = 625$ (d) $(x+4)^2 + (y-3)^2 = 25$

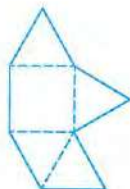
- 10** A cylindrical shaped vessel contains water , a metallic body in the form of a right cone , its height is 12 cm. and the length of its base radius is 2 cm. is completely immersed in it raising the surface of the water in the vessel with 1 cm.
 , then the length of base diameter of the vessel = cm.

(a) 2 (b) 4 (c) 6 (d) 8

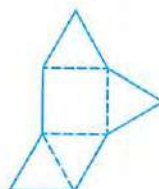
- 11** Which of the following nets does not make a regular quadrilateral pyramid when it is folded ?



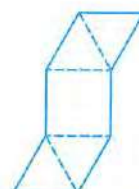
(a)



(b)



(c)

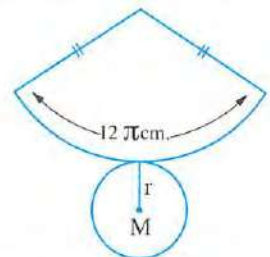


(d)

- 12** The opposite net describes a solid , its volume = $96\pi \text{ cm}^3$

, its total surface area = cm^2

(a) 96π (b) 48π
 (c) 32π (d) 16π



- 13** In the opposite figure :

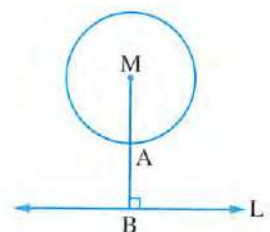
If the equation of the circle is : $x^2 + y^2 - 6x + 4y - 12 = 0$

, $\overline{MB} \perp$ the straight line L where $L : 3x - 4y + 23 = 0$

, \overline{MB} intersects the circle at A ,

then length of \overline{AB} = length units.

(a) 3 (b) 5 (c) 8 (d) 12

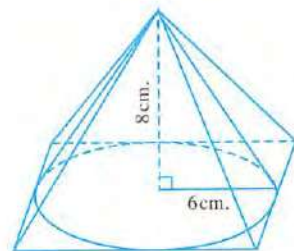


- 14** Two forces of magnitudes F , $F\sqrt{3}$ newton act at a particle, the magnitude of their resultant = F newton, and θ_1 is the measure of the angle between 1st force and the resultant and θ_2 is the angle between the 2nd force and the resultant, then
- (a) $\theta_1 = \theta_2$ (b) $\theta_1 = \frac{1}{2} \theta_2$ (c) $\theta_1 = 3 \theta_2$ (d) $\theta_1 = 4 \theta_2$

- 15** Which of the following statements is not true ?
- (a) Any two different parallel straight line identify a plane ?
- (b) Any two intersecting different straight lines have a common point.
- (c) The two skew lines aren't contained in one plane.
- (d) Any three non collinear points, there is at least one plane passes through them.

16 In the opposite figure :

A right regular pyramid and right circular cone are common in the vertex and base such that base of the cone touches the sides of the base of the pyramid internally, then the ratio between the lateral area of the right circular cone and the lateral area of the pyramid =

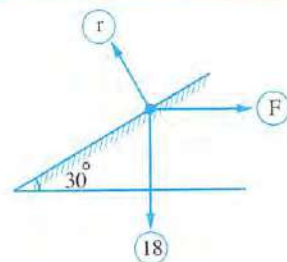


- (a) $\frac{4}{\pi}$ (b) $\frac{5}{6}$ (c) $\frac{7}{8}$ (d) $\frac{\pi}{4}$

- 17** Force of magnitude $5\sqrt{3}$ N. acts in direction 30° east of the north. It is resolved into two perpendicular components, then the component in direction of the east equals N.
- (a) $\frac{5\sqrt{3}}{2}$ (b) $\frac{15}{2}$ (c) $\frac{15\sqrt{3}}{2}$ (d) $15\sqrt{3}$

18 In the opposite figure :

A body of weight 18 N. placed on a smooth inclined plane, inclined to the horizontal at an angle of measure 30° and kept in equilibrium by a horizontal force F N., then $F + r =$ N.



- (a) $6\sqrt{3}$ (b) $12\sqrt{3}$
- (c) $18\sqrt{3}$ (d) $24\sqrt{3}$

- 19** The diameter length of the circle : $4x^2 + 4y^2 + 16x - 8y - 16 = 0$ equals length units.
- (a) 3 (b) 6 (c) 12 (d) 24

20 The ratio between the edge length of regular triangular pyramid and its height =

- (a) $\sqrt{2} : \sqrt{3}$ (b) $\sqrt{3} : 2$ (c) $\sqrt{6} : 2$ (d) $\sqrt{3} : 3$

21 The right circular cone is formed from rotation of a right-angled triangle a complete rotation about

- (a) its hypotenuse.
 (b) one of its right sides.
 (c) any straight line in the plane of the triangle.
 (d) any straight line passes through one of its vertices and parallel to the opposite side of this vertex.

Second Essay questions

Answer the following questions :

1 The length of the base side of a regular quadrilateral pyramid is 20 cm. and its height is $10\sqrt{3}$ cm. , then find :

- (1) The lateral surface area. (2) The volume of the pyramid.

2 Five coplanar forces meeting at a point their magnitudes are 12 , 9 , $5\sqrt{2}$, $7\sqrt{2}$ and 7 kg.wt. act due east , north , western north , western south and south respectively , prove that the system is in equilibrium.

3 \overline{AB} is a uniform rod of length 80 cm. and weight 24 kg.wt. The end A is attached to a hinge fixed on a vertical wall , and the end B is tied by a light string of length $80\sqrt{3}$ cm. fixed at a point C on the wall which lies directly above A and at a distance 80 cm. If the rod is in equilibrium, find the magnitude of the tension in the string and the reaction of the hinge.

Answers of schools Examinations

1 Cairo

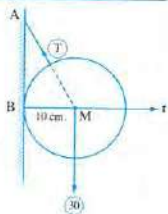
First Multiple choice questions

- | | | |
|----------|----------|----------|
| (1) (d) | (2) (a) | (3) (c) |
| (4) (a) | (5) (b) | (6) (c) |
| (7) (b) | (8) (c) | (9) (b) |
| (10) (c) | (11) (b) | (12) (d) |
| (13) (d) | (14) (a) | (15) (b) |
| (16) (c) | (17) (a) | (18) (d) |
| (19) (d) | (20) (d) | (21) (d) |

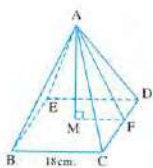
Second Essay questions

- 1
Let \overrightarrow{AB} in direction of \overrightarrow{OX}
 $\therefore X = 8 \cos 0^\circ + 6\sqrt{3} \cos 30^\circ$
 $+ 5 \cos 60^\circ + 4\sqrt{3} \cos 90^\circ$
 $= 19.5$
 $\therefore Y = 8 \sin 0^\circ + 6\sqrt{3} \sin 30^\circ$
 $+ 5 \sin 60^\circ + 4\sqrt{3} \sin 90^\circ$
 $= 9.5\sqrt{3}$
 $\therefore \vec{R} = 19.5\vec{i} + 9.5\sqrt{3}\vec{j}$
 $\therefore \|\vec{R}\| = \sqrt{651}$ newtons
 $\therefore \tan \theta = \frac{19\sqrt{3}}{39} \therefore \theta = 40^\circ \hat{9}$
 \therefore Magnitude of the resultant is $\sqrt{651}$ newtons and makes an angle of measure $40^\circ \hat{9}$ with \overrightarrow{AB}

- 2
From the figure :
 $\therefore \Delta ABM$ is the triangle of forces
 $\therefore AB = \sqrt{(20)^2 - (10)^2} = 10\sqrt{3}$
 $\therefore \frac{30}{AB} = \frac{r}{BM} = \frac{T}{AM}$
 $\therefore \frac{30}{10\sqrt{3}} = \frac{r}{10} = \frac{T}{20}$
 $\therefore r = 10\sqrt{3}$ gm.wt. $\therefore T = 20\sqrt{3}$ gm.wt.



- 3
 $\therefore v = \frac{1}{3} l^2 h$
 $\therefore 1296 = \frac{1}{3} (18)^2 (AM)$
 $\therefore AM = 12$ cm.
 \therefore Slant height = AF
 $= \sqrt{(AM)^2 + (MF)^2}$
 $= \sqrt{(12)^2 + 9^2} = 15$ cm.
 \therefore lateral surface area = $\frac{1}{2} (18 \times 4) \times 15 = 540$ cm²



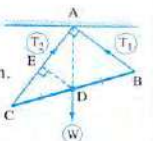
2 Cairo

First Multiple choice questions

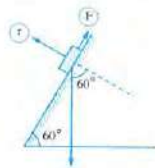
- | | | |
|----------|----------|----------|
| (1) (b) | (2) (b) | (3) (a) |
| (4) (a) | (5) (b) | (6) (c) |
| (7) (b) | (8) (c) | (9) (a) |
| (10) (c) | (11) (b) | (12) (b) |
| (13) (a) | (14) (b) | (15) (c) |
| (16) (c) | (17) (d) | (18) (a) |
| (19) (d) | (20) (d) | (21) (c) |

Second Essay questions

- 1
Side length of the base of the pyramid = $16 \div 4 = 4$
 volume of the pyramid = $\frac{1}{3} \times 4^2 \times 9 = 48$ cm³
 \therefore Volume of raises water = 48 cm³
 $\therefore \frac{22}{7} \times r^2 \times \frac{21}{88} = 48 \therefore r = 8$ cm.
 2
In ΔABC :
 $AB = \sqrt{(100)^2 - (50\sqrt{3})^2} = 50$ cm.
 $\therefore D$ is the midpoint of \overline{BC}
 $\therefore AD = \frac{1}{2} BC = 50$ cm.
 $\therefore ED = \frac{1}{2} AB = 25$ cm. $\therefore AE = \frac{1}{2} AC = 25\sqrt{3}$ cm.
 $\therefore \Delta ADE$ is the triangle of forces
 $\therefore \frac{W}{AD} = \frac{T_1}{ED} = \frac{T_2}{EA} \therefore \frac{W}{50} = \frac{T_1}{25} = \frac{T_2}{25\sqrt{3}}$
 $\therefore T_1 = \frac{1}{2} W, T_2 = \frac{\sqrt{3}}{2} W$



- 3
Using Lami's rule
 $\therefore \frac{12}{\sin 90^\circ} = \frac{F}{\sin 120^\circ} = \frac{r}{\sin 150^\circ}$
 $\therefore F = 6\sqrt{3}$ newton
 $\therefore r = 6$ newton



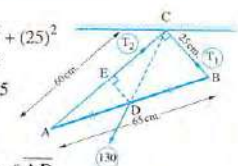
3 Cairo

First Multiple choice questions

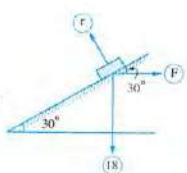
- | | | |
|----------|----------|----------|
| (1) (d) | (2) (b) | (3) (a) |
| (4) (a) | (5) (a) | (6) (c) |
| (7) (c) | (8) (c) | (9) (b) |
| (10) (d) | (11) (d) | (12) (c) |
| (13) (b) | (14) (a) | (15) (c) |
| (16) (a) | (17) (d) | (18) (d) |
| (19) (a) | (20) (c) | (21) (a) |

Second Essay questions

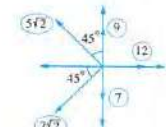
- 1
 $(CA)^2 + (CB)^2 = (60)^2 + (25)^2$
 $= 4225$
 $\therefore (AB)^2 = (65)^2 = 4225$
 $\therefore \Delta ABC$ is right-angled at C
 $\therefore D$ is the midpoint of \overline{AB}
 $\therefore CD = \frac{1}{2} AB = 32.5$ cm.
 $\therefore ED = \frac{1}{2} BC = 12.5$ cm. $\therefore EC = \frac{1}{2} AC = 30$ cm.
 $\therefore \Delta ECD$ is the triangle of forces
 $\therefore \frac{130}{32.5} = \frac{T_1}{12.5} = \frac{T_2}{30}$
 $\therefore T_1 = 50$ gm.wt. $\therefore T_2 = 120$ gm.wt.



- 2
 $AB = \sqrt{(4+1)^2 + (2+3)^2} = 5\sqrt{2}$
 $\therefore r = \frac{5\sqrt{2}}{2}$ length units
 \therefore centre of the circle = $(\frac{3}{2}, -\frac{1}{2})$
 \therefore Equation of the circle is :
 $(x - \frac{3}{2})^2 + (y + \frac{1}{2})^2 = \frac{25}{2}$
 $\therefore x^2 + y^2 - 3x + y - 10 = 0$



- 3
 $X = 12 \cos 0^\circ + 9 \cos 90^\circ + 5\sqrt{2} \cos 135^\circ$
 $+ 7\sqrt{2} \cos 225^\circ + 7 \cos 270^\circ = 0$
 $\therefore Y = 12 \sin 0^\circ + 9 \sin 90^\circ$
 $+ 5\sqrt{2} \sin 135^\circ$
 $+ 7\sqrt{2} \sin 225^\circ$
 $+ 7 \sin 270^\circ = 0$
 \therefore The forces are in equilibrium.



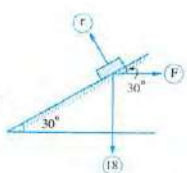
4 Giza

First Multiple choice questions

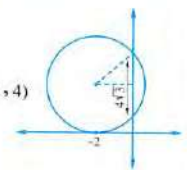
- | | | |
|----------|----------|----------|
| (1) (c) | (2) (b) | (3) (b) |
| (4) (d) | (5) (c) | (6) (c) |
| (7) (a) | (8) (d) | (9) (d) |
| (10) (b) | (11) (d) | (12) (b) |
| (13) (c) | (14) (c) | (15) (b) |
| (16) (a) | (17) (d) | (18) (b) |
| (19) (a) | (20) (a) | (21) (c) |

Second Essay questions

- 1
Using Lami's rule
 $\frac{18}{\sin 120^\circ} = \frac{r}{\sin 90^\circ} = \frac{F}{\sin 150^\circ}$
 $\therefore r = 12\sqrt{3}$ newton
 $\therefore F = 6\sqrt{3}$ newton



- 2
 $r = \sqrt{(2)^2 + (2\sqrt{3})^2} = 4$
 \therefore Centre of the circle = $(-2, 4)$
 \therefore Equation of the circle is
 $(X + 2)^2 + (y - 4)^2 = 16$



- 3
Let \overrightarrow{AB} in the direction of \overrightarrow{OX}

$$\therefore X = 16 \cos 0^\circ + 16 \cos 90^\circ$$

$$+ 12\sqrt{2} \cos 225^\circ = 4$$

$$\therefore Y = 16 \sin 0^\circ + 16 \sin 90^\circ$$

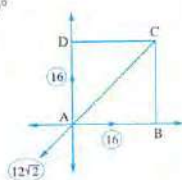
$$+ 12\sqrt{2} \sin 225^\circ = 4$$

$$\therefore \vec{R} = 4\hat{i} + 4\hat{j}$$

$$\therefore \tan \theta = 1$$

$$\therefore \theta = 45^\circ$$

\therefore Magnitude of resultant $= \sqrt{4^2 + 4^2} = 4\sqrt{2}$ newton
and its direction makes an angle of measure 45°
with \vec{OX}



5 Giza

First Multiple choice questions

- | | | |
|----------|----------|----------|
| (1) (c) | (2) (d) | (3) (b) |
| (4) (d) | (5) (c) | (6) (a) |
| (7) (b) | (8) (a) | (9) (b) |
| (10) (a) | (11) (a) | (12) (c) |
| (13) (d) | (14) (a) | (15) (b) |
| (16) (c) | (17) (a) | (18) (c) |
| (19) (d) | (20) (a) | (21) (c) |

Second Essay questions

1

(1) Four different circles

(2) • In the 1st quadrant : $(X-5)^2 + (y-5)^2 = 25$

• In the 3rd quadrant : $(X+5)^2 + (y+5)^2 = 25$

• The distance between their centres

$$= \sqrt{(5+5)^2 + (5+5)^2} = 10\sqrt{2} \text{ length unit}$$

2

$$\therefore 20 = \frac{F \sin 30^\circ}{\sin 120^\circ} \quad \therefore F = 20\sqrt{3}$$

$$\therefore F_1 = \frac{F \sin 90^\circ}{\sin 120^\circ} = 40$$

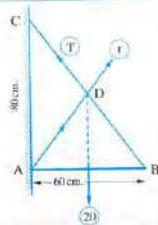
3

$$BC = \sqrt{(80)^2 + (60)^2} = 100 \text{ cm.}$$

$\therefore \Delta ACD$ is triangle of forces

$$\therefore \frac{20}{80} = \frac{r}{50} = \frac{T}{50}$$

$$\therefore r = T = 12.5 \text{ newton}$$



6 Giza

First Multiple choice questions

- | | | |
|----------|----------|----------|
| (1) (c) | (2) (c) | (3) (b) |
| (4) (d) | (5) (a) | (6) (c) |
| (7) (d) | (8) (d) | (9) (c) |
| (10) (a) | (11) (c) | (12) (b) |
| (13) (a) | (14) (c) | (15) (d) |
| (16) (b) | (17) (c) | (18) (d) |
| (19) (d) | (20) (c) | (21) (d) |

Second Essay questions

1

$$X = 10 \cos 0^\circ + 20 \cos 120^\circ$$

$$+ 30 \cos 240^\circ = -15$$

$$\therefore Y = 10 \sin 0^\circ + 20 \sin 120^\circ$$

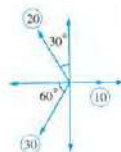
$$+ 30 \sin 240^\circ = -5\sqrt{3}$$

$$\therefore \vec{R} = -15\hat{i} - 5\sqrt{3}\hat{j}$$

$$\therefore \|\vec{R}\| = \sqrt{225 + 75} = 10\sqrt{3}$$

$$\therefore \tan \theta = \frac{-5\sqrt{3}}{-15} = \frac{1}{\sqrt{3}}$$

\therefore Magnitude of resultant is $10\sqrt{3}$ newton and in
direction of 30° south of west



2

The point of tangency :

$$X^2 + y^2 - 6X - 4y + 12 = X^2 + y^2 + 2X - 4y - 4$$

$$\therefore -6X - 4y + 12 = 2X - 4y - 4$$

$$\therefore -6X - 2X = -12 - 4$$

$$\therefore -8X = -16 \quad \therefore X = 2$$

By substitution in the first circle equation by $X = 2$

$$\therefore (2)^2 + y^2 - 6 \times 2 - 4y + 12 = 0$$

$$\therefore y^2 - 4y + 4 = 0 \quad \therefore y = 2$$

\therefore The two circles intersected at one point (2, 2)

\therefore The two circles touch each other

\therefore the equation of the circle whose center (2, 2) and
passes through the center of the second circle (-1, 2)

$$r = \sqrt{(2+1)^2 + (0)^2} = 3$$

$$\therefore \text{The equation : } (X-2)^2 + (y-2)^2 = 9$$

3

Suppose that :

$$AB = 4\ell$$

$$\therefore CB = 2\ell$$

$$\therefore CE = \ell$$

$$\therefore AC = 2\sqrt{3}\ell$$

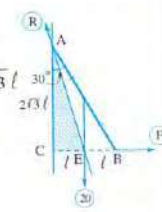
$$\text{In } \Delta ACE : AE = \sqrt{13}\ell$$

$\therefore \Delta ACE$ is the Δ of forces

$$\frac{20}{2\sqrt{3}\ell} = \frac{F}{\ell} = \frac{R}{\sqrt{13}\ell}$$

$$\therefore F = \frac{10\sqrt{3}}{3} \text{ kg.wt.}$$

$$R = \frac{10\sqrt{39}}{3} \text{ kg.wt.}$$



7 Alexandria

First Multiple choice questions

- | | | |
|----------|----------|----------|
| (1) (d) | (2) (d) | (3) (d) |
| (4) (b) | (5) (c) | (6) (a) |
| (7) (a) | (8) (d) | (9) (d) |
| (10) (c) | (11) (c) | (12) (b) |
| (13) (b) | (14) (c) | (15) (a) |
| (16) (b) | (17) (a) | (18) (c) |
| (19) (c) | (20) (b) | (21) (a) |

Second Essay questions

1

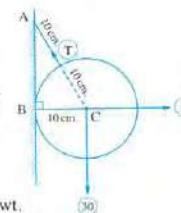
From the figure ΔABC
represents a triangle of
forces

$$\therefore AB = \sqrt{20^2 - 10^2} = 10\sqrt{3}$$

$$\therefore \frac{30}{10\sqrt{3}} = \frac{R}{10} = \frac{T}{20}$$

$$\therefore T = \frac{30 \times 20}{10\sqrt{3}} = 20\sqrt{3} \text{ gm.wt.}$$

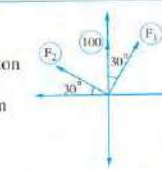
$$\therefore R = \frac{30 \times 10}{10\sqrt{3}} = 10\sqrt{3} \text{ gm.wt.}$$



2

$$F_1 = \frac{100 \sin 60^\circ}{\sin 90^\circ} = 50\sqrt{3} \text{ newton}$$

$$\therefore F_2 = \frac{100 \sin 30^\circ}{\sin 90^\circ} = 50 \text{ newton}$$



3

Centre of the circle

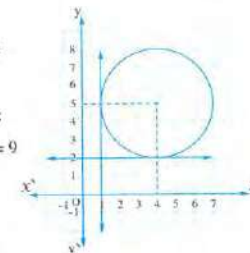
is (4, 5), $r = 3$

\therefore The equation is :

$$(X-4)^2 + (y-5)^2 = 9$$

i.e. :

$$X^2 + y^2 - 8X - 10y + 32 = 0$$



8 El-Kalyoubia

First Multiple choice questions

- | | | |
|----------|----------|----------|
| (1) (b) | (2) (a) | (3) (d) |
| (4) (d) | (5) (b) | (6) (a) |
| (7) (d) | (8) (c) | (9) (d) |
| (10) (b) | (11) (c) | (12) (c) |
| (13) (d) | (14) (c) | (15) (c) |
| (16) (a) | (17) (a) | (18) (d) |
| (19) (b) | (20) (b) | (21) (a) |

Second Essay questions

1

$$T_1 = \frac{5000 \times \sin 30^\circ}{\sin 75^\circ} \approx 2588.2 \text{ newton}$$

$$\therefore T_2 = \frac{5000 \times \sin 45^\circ}{\sin 75^\circ} \approx 3660.3 \text{ newton}$$

2

Centre of the circle = (6, -3)

$$\therefore r = \sqrt{36 + 9 - 20} = 5 \text{ unit length.}$$

\therefore Diagonal of the square = $2r = 10$ unit length.

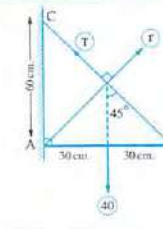
$$\therefore \text{Area of the square} = \frac{1}{2} \times 10 \times 10 = 50 \text{ cm}^2$$

3

Using Lami's rule

$$\frac{40}{\sin 90^\circ} = \frac{r}{\sin 135^\circ} = \frac{T}{\sin 135^\circ}$$

$$\therefore T = r = 20\sqrt{2} \text{ newton}$$



9 El-Dakhla

First Multiple choice questions

- (1) (b) (2) (b) (3) (d)
 (4) (a) (5) (a) (6) (c)
 (7) (c) (8) (b) (9) (d)
 (10) (d) (11) (d) (12) (d)
 (13) (d) (14) (d) (15) (b)
 (16) (c) (17) (a) (18) (c)
 (19) (c) (20) (c) (21) (d)

Second Essay questions

1 Since the two planes are smooth

$\therefore r_1$ and r_2 are perpendicular to the two planes and pass through the center of the sphere
 Applying lamé's rule

$$\therefore \frac{r_1}{\sin 120^\circ} = \frac{r_2}{\sin 90^\circ} = \frac{15}{\sin 150^\circ}$$

$\therefore r_1$ (The reaction of the vertical plane) = $15\sqrt{3}$ kg.wt.

r_2 (The reaction of the inclined plane) = 30 kg.wt.

2

Let \vec{AB} in the direction of \vec{OX}

$$\therefore X = 2 \cos 0^\circ + 4\sqrt{3} \cos 30^\circ$$

$$+ 8 \cos 60^\circ + 2\sqrt{3} \cos 90^\circ$$

$$+ 4 \cos 120^\circ$$

$$= 2 \times 1 + 4\sqrt{3} \times \frac{\sqrt{3}}{2} + 8 \times \frac{1}{2}$$

$$+ 2\sqrt{3} \times 0 + 4 \times \frac{-1}{2} = 10$$

$$\therefore Y = 2 \sin 0^\circ + 4\sqrt{3} \sin 30^\circ + 8 \sin 60^\circ$$

$$+ 2\sqrt{3} \sin 90^\circ + 4 \sin 120^\circ$$

$$= 2 \times 0 + 4\sqrt{3} \times \frac{1}{2} + 8 \times \frac{\sqrt{3}}{2} + 2\sqrt{3} \times 1 + 4 \times \frac{\sqrt{3}}{2}$$

$$= 10\sqrt{3}$$

$$\therefore \vec{R} = 10\vec{i} + 10\sqrt{3}\vec{j}$$

$$\therefore R = \sqrt{(10)^2 + (10\sqrt{3})^2} = 20 \text{ kg.wt.}$$

$$\therefore \tan \theta = \frac{10\sqrt{3}}{10} = \sqrt{3} \quad \therefore \theta = 60^\circ$$

$$\therefore \text{The magnitude of } \vec{R} = 20 \text{ kg.wt. and makes}$$

$$\text{an angle of measure } 60^\circ \text{ with } \vec{AB}$$

3

Volume of the pyramid = $\frac{1}{3}$ base area \times height

$$1296 = \frac{1}{3} \times (18)^2 \times \text{height}$$

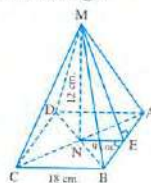
height = 12 cm.

$$\text{The slant height} = \sqrt{9^2 + (12)^2}$$

$$= 15 \text{ cm.}$$

The lateral area

$$= \frac{1}{2} \times (4 \times 18) \times 15 = 540 \text{ cm}^2.$$



10 El-Menia

First Multiple choice questions

- (1) (c) (2) (c) (3) (c)
 (4) (a) (5) (c) (6) (a)
 (7) (d) (8) (d) (9) (b)
 (10) (a) (11) (d) (12) (a)
 (13) (d) (14) (c) (15) (b)
 (16) (a) (17) (c) (18) (d)
 (19) (d) (20) (c) (21) (d)

Second Essay questions

1 $\triangle ABC$ is the triangle of forces

$$\therefore \frac{F_1}{3} = \frac{F_2}{4} = \frac{5}{5}$$

$$\therefore F_1 = 3 \text{ newton}$$

$$\therefore F_2 = 4 \text{ newton}$$

2

$$\text{The slant height} = \sqrt{(10)^2 + (10\sqrt{3})^2} = 20 \text{ cm.}$$

$$(1) \text{ The lateral area} = \frac{1}{2} \times (4 \times 20)$$

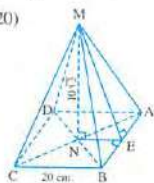
$$\times 20 = 800 \text{ cm}^2.$$

$$(2) \text{ Volume of the pyramid}$$

$$= \frac{1}{3} \text{ base area} \times \text{height}$$

$$= \frac{1}{3} \times (20)^2 \times 10\sqrt{3}$$

$$= \frac{4000}{3} \sqrt{3} \text{ cm}^3.$$



3

$$(3\sqrt{7})^2 = (9)^2 + (6)^2 + 2 \times 9 \times 6 \cos \alpha$$

$$\therefore \cos \alpha = -\frac{1}{2} \quad \therefore \alpha = 120^\circ$$

$$\tan \theta = \frac{6 \sin 120^\circ}{9 + 6 \cos 120^\circ} = \frac{\sqrt{3}}{2} \quad \therefore \theta = 40^\circ 53' 36''$$

Answers of final models

Model 1

First Multiple choice questions

- 1 (b) 2 (c) 3 (a) 4 (c)
 5 (d) 6 (b) 7 (a) 8 (a)
 9 (c) 10 (d) 11 (d) 12 (c)
 13 (b) 14 (d) 15 (a) 16 (b)
 17 (d) 18 (d) 19 (d) 20 (d)
 21 (b)

Second Essay questions

1

Let \vec{AB} in the direction of \vec{OX}

$$\therefore X = 2 \cos 0^\circ + 4\sqrt{3} \cos 30^\circ$$

$$+ 8 \cos 60^\circ$$

$$+ 2\sqrt{3} \cos 90^\circ$$

$$+ 4 \cos 120^\circ$$

$$= 2 \times 1 + 4\sqrt{3} \times \frac{\sqrt{3}}{2}$$

$$+ 8 \times \frac{1}{2} + 2\sqrt{3} \times 0 + 4 \times \frac{-1}{2} = 10$$

$$\therefore Y = 2 \sin 0^\circ + 4\sqrt{3} \sin 30^\circ + 8 \sin 60^\circ$$

$$+ 2\sqrt{3} \sin 90^\circ + 4 \sin 120^\circ$$

$$= 2 \times 0 + 4\sqrt{3} \times \frac{1}{2} + 8 \times \frac{\sqrt{3}}{2} + 2\sqrt{3} \times 1 + 4 \times \frac{\sqrt{3}}{2}$$

$$= 10\sqrt{3}$$

$$\therefore \vec{R} = 10\vec{i} + 10\sqrt{3}\vec{j}$$

$$\therefore R = \sqrt{(10)^2 + (10\sqrt{3})^2} = 20 \text{ kg.wt.}$$

$$\therefore \tan \theta = \frac{10\sqrt{3}}{10} = \sqrt{3} \quad \therefore \theta = 60^\circ$$

$$\therefore \text{The magnitude of } \vec{R} = 20 \text{ kg.wt. and makes}$$

$$\text{an angle of measure } 60^\circ \text{ with } \vec{OX}$$

2

$$(1) \text{ The area of the base} = \pi r^2$$

$$\therefore 36 \pi = \pi r^2 \quad \therefore r = 6 \text{ cm.}$$

$$\therefore \text{the lateral area} = \pi r L = \pi \times 6 \times 10 = 60 \pi \text{ cm}^2.$$

$$(2) \text{ The total area} = \pi r (L + r) = \pi \times 6 (10 + 6)$$

$$= 96 \pi \text{ cm}^2.$$

$$(3) h = \sqrt{(10)^2 - (6)^2} = 8 \text{ cm.}$$

$$\therefore \text{Volume} = \frac{1}{3} \pi r^2 h = \frac{1}{3} \pi \times 6^2 \times 8 = 96 \pi \text{ cm}^3.$$

3

Suppose that:

$$AB = 4l$$

$$\therefore CB = 2l$$

$$\therefore CE = l$$

$$\therefore AC = 2\sqrt{3}l$$

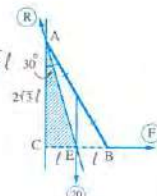
$$\text{In } \triangle ACE : AE = \sqrt{13}l$$

$$\therefore \triangle ACE \text{ is the } \Delta \text{ of forces}$$

$$\therefore \frac{20}{2\sqrt{3}l} = \frac{F}{l} = \frac{R}{\sqrt{13}l}$$

$$\therefore F = \frac{10\sqrt{3}}{3} \text{ kg.wt.}$$

$$\therefore R = \frac{10\sqrt{39}}{3} \text{ kg.wt.}$$



Model 2

First Multiple choice questions

- 1 (a) 2 (b) 3 (a) 4 (b)
 5 (d) 6 (c) 7 (a) 8 (d)
 9 (c) 10 (d) 11 (c) 12 (c)
 13 (a) 14 (a) 15 (b) 16 (c)
 17 (b) 18 (a) 19 (d) 20 (d)
 21 (c)

Second Essay questions

1

$$R_1 = 2F \cos \frac{\alpha}{2} = 12$$

$$\therefore F \cos \frac{\alpha}{2} = 6 \quad (1)$$

$$\therefore R_2 = 2F \cos \left(\frac{180^\circ - \alpha}{2} \right) = 6$$

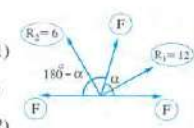
$$\therefore F \sin \frac{\alpha}{2} = 3 \quad (2)$$

$$\therefore \text{squaring the two equations and adding them}$$

$$\therefore F^2 \cos^2 \frac{\alpha}{2} + F^2 \sin^2 \frac{\alpha}{2} = 6^2 + 3^2$$

$$\therefore F^2 \left(\cos^2 \frac{\alpha}{2} + \sin^2 \frac{\alpha}{2} \right) = 45$$

$$\therefore F = \sqrt{45} = 3\sqrt{5} \text{ kg.wt.}$$





2

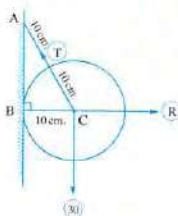
From the figure $\triangle ABC$ represents a triangle

$$\text{forces } AB = \sqrt{20^2 - 10^2} = 10\sqrt{3}$$

$$\therefore \frac{30}{10\sqrt{3}} = \frac{R}{10} = \frac{T}{20}$$

$$\therefore T = \frac{30 \times 20}{10\sqrt{3}} = 20\sqrt{3} \text{ gm.wt.}$$

$$\therefore R = \frac{30 \times 10}{10\sqrt{3}} = 10\sqrt{3} \text{ gm.wt.}$$



3

Consider \vec{CB} , \vec{CD}
axes \vec{OX} , \vec{OY}

$$\therefore X = 2 \cos 0^\circ + 7\sqrt{5} \cos \theta^\circ + 4\sqrt{2} \cos 45^\circ + 4 \cos 90^\circ$$

$$= 2 + 7\sqrt{5} \times \frac{10}{5\sqrt{5}} + 4\sqrt{2} \times \frac{1}{\sqrt{2}} + 0$$

$$\therefore X = 20 \text{ newton}$$

$$\therefore Y = 2 \sin 0^\circ + 7\sqrt{5} \sin \theta + 4\sqrt{2} \sin 45^\circ + 4 \sin 90^\circ$$

$$= 0 + 7\sqrt{5} \times \frac{5}{5\sqrt{5}} + 4\sqrt{2} \times \frac{1}{\sqrt{2}} + 4$$

$$\therefore Y = 15 \text{ newton}$$

$$\therefore R = \sqrt{X^2 + Y^2} = \sqrt{20^2 + 15^2} = 25 \text{ newton}$$

$$\therefore \tan \theta = \frac{Y}{X} = \frac{15}{20} = \frac{3}{4}$$

$$\therefore \theta = 36^\circ 52' 12'' \text{ with first force.}$$

Model 6

First Multiple choice questions

- | | | | |
|--------|--------|--------|--------|
| 1 (c) | 2 (d) | 3 (d) | 4 (b) |
| 5 (c) | 6 (a) | 7 (d) | 8 (a) |
| 9 (d) | 10 (c) | 11 (c) | 12 (c) |
| 13 (b) | 14 (a) | 15 (c) | 16 (c) |
| 17 (c) | 18 (a) | 19 (c) | 20 (c) |
| 21 (b) | | | |

Second Essay questions

1

$$\therefore (AB)^2 = (BC)^2 + (AC)^2$$

$$\therefore m(\angle ACB) = 90^\circ$$

$$\therefore CD = \frac{1}{2} AB = 50 \text{ cm.}$$

$$\therefore CD = DB,$$

$$\therefore m(\angle B) = \theta_1$$

$$\therefore CD = AD,$$

$$\therefore m(\angle A) = \theta_2$$

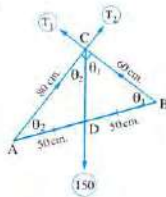
$$\frac{T_1}{\sin(90^\circ + \theta_1)} = \frac{T_2}{\sin(90^\circ + \theta_2)}$$

$$= \frac{150}{\sin 90^\circ}$$

$$\therefore \frac{T_1}{\cos \theta_1} = \frac{T_2}{\cos \theta_2} = \frac{150}{1}$$

$$\therefore \frac{T_1}{\frac{6}{10}} = \frac{T_2}{\frac{8}{10}} = 150$$

$$\therefore T_1 = 90 \text{ gm.wt.}, T_2 = 120 \text{ gm.wt.}$$



2

$$\therefore \text{The volume of the pyramid} = 1296$$

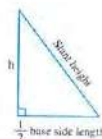
$$\therefore \frac{1}{3} \times (18)^2 \times h = 1296$$

$$\therefore h = 12 \text{ cm.}$$

$$\therefore \text{Slant height} = \sqrt{h^2 + \left(\frac{1}{2} \text{ side}\right)^2}$$

$$= \sqrt{12^2 + \left(\frac{18}{2}\right)^2} = 15 \text{ cm.}$$

$$\therefore \text{Lateral surface area} = \frac{1}{2} \times \text{perimeter of the base} \times \text{slant height} = \frac{1}{2} \times (4 \times 18) \times 15 = 540 \text{ cm}^2$$



3

$$\therefore \tan 30^\circ = \frac{F \sin 120^\circ}{16 + F \cos 120^\circ} \therefore \frac{1}{\sqrt{3}} = \frac{\frac{\sqrt{3}}{2} F}{16 - \frac{1}{2} F}$$

$$\therefore 16 - \frac{1}{2} F = \frac{3}{2} F \therefore 2F = 16$$

$$\therefore F = 8 \text{ kg.wt.}$$

$$\therefore R = \sqrt{(16)^2 + (8)^2} + 2 \times 16 \times 8 \cos 120^\circ = 8\sqrt{3} \text{ kg.wt.}$$

Model 7

First Multiple choice questions

- | | | | |
|--------|--------|--------|--------|
| 1 (c) | 2 (a) | 3 (c) | 4 (b) |
| 5 (a) | 6 (c) | 7 (d) | 8 (b) |
| 9 (c) | 10 (b) | 11 (d) | 12 (b) |
| 13 (b) | 14 (c) | 15 (c) | 16 (c) |
| 17 (b) | 18 (d) | 19 (c) | 20 (d) |
| 21 (c) | | | |

Second Essay questions

1

$$\therefore \text{Lateral surface area} = 240$$

$$\therefore \frac{1}{2} (\text{perimeter of the base}) \times \text{slant height} = 240$$

If ℓ is side length of the base.

$$\therefore \frac{1}{2} \times 4 \ell \times 12 = 240$$

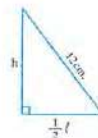
$$\therefore \ell = 10 \text{ cm.}$$

$$(1) \therefore \text{Height of pyramid}$$

$$= \sqrt{12^2 - 5^2} = \sqrt{119} \text{ cm.}$$

$$(2) \text{Volume of pyramid} = \frac{1}{3} \times \text{area of base} \times h$$

$$= \frac{1}{3} \times (10)^2 \times \sqrt{119} = \frac{100}{3} \sqrt{119} \approx 363.6 \text{ cm}^3$$

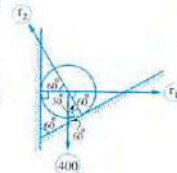


2

\therefore The two planes are smooth

$\therefore r_1$ and r_2 are

perpendicular to the two planes and passes through the centre of the sphere.



Applying lami's rule

$$\therefore \frac{r_1}{\sin 150^\circ} = \frac{r_2}{\sin 90^\circ} = \frac{400}{\sin 120^\circ}$$

$$\therefore r_1 = \frac{400 \times \sin 150^\circ}{\sin 120^\circ} = \frac{400\sqrt{3}}{3} \text{ kg.wt.}$$

$$\therefore r_2 = \frac{400 \times \sin 90^\circ}{\sin 120^\circ} = \frac{800\sqrt{3}}{3} \text{ kg.wt.}$$

3

$\therefore \triangle AHB$ is right-angled at B

$$\therefore AH = \sqrt{(5)^2 + (12)^2} = 13 \text{ cm.}$$

$$\therefore \sin(\angle BAH) = \frac{5}{13}$$

$$\therefore \cos(\angle BAH) = \frac{12}{13}$$

$\therefore \vec{AC}$ is a diagonal of the square ABCD

$$\therefore \alpha = 45^\circ$$

$$\therefore X = 2 \cos 0^\circ + 13 \cos(\angle BAH) + 9 \cos 90^\circ$$

$$+ 4\sqrt{2} \cos 225^\circ$$

$$= 2 \times 1 + 13 \times \frac{12}{13} + 9 \times 0 + 4\sqrt{2} \times \frac{-1}{\sqrt{2}} = 10$$

$$\therefore Y = 2 \sin 0^\circ + 13 \sin(\angle BAH) + 9 \sin 90^\circ$$

$$+ 4\sqrt{2} \sin 225^\circ$$

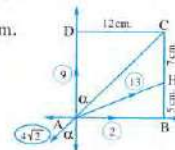
$$= 2 \times 0 + 13 \times \frac{5}{13} + 9 \times 1 + 4\sqrt{2} \times \frac{-1}{\sqrt{2}} = 10$$

$$\therefore \vec{R} = 10\vec{i} + 10\vec{j}$$

$$\therefore R = \sqrt{(10)^2 + (10)^2} = 10\sqrt{2} \text{ gm.wt.}$$

$$\therefore \tan \theta = \frac{10}{10} = 1 \therefore X > 0, Y > 0 \therefore \theta = 45^\circ$$

$$\therefore \vec{R} \text{ acts due to } \vec{AC}$$



Model 8

First Multiple choice questions

- | | | | |
|--------|--------|--------|--------|
| 1 (d) | 2 (d) | 3 (d) | 4 (c) |
| 5 (a) | 6 (d) | 7 (d) | 8 (d) |
| 9 (a) | 10 (b) | 11 (b) | 12 (b) |
| 13 (d) | 14 (d) | 15 (a) | 16 (d) |
| 17 (b) | 18 (d) | 19 (b) | 20 (b) |
| 21 (c) | | | |

Second Essay questions

1

$$\therefore \text{Volume of the pyramid} = \frac{1}{3} \times \text{base area} \times \text{height}$$

$$\therefore 8\sqrt{3} = \frac{1}{3} \times \text{base area} \times 4$$

$$\therefore \text{Base area} = 6\sqrt{3} \text{ cm}^2$$

$$\therefore \frac{6}{4} \times X^2 \times \cot \frac{\pi}{6} = 6\sqrt{3}$$

$$\therefore X^2 = 4 \quad \therefore X = 2$$

\therefore Side length of the hexagon = 2 cm.

\therefore Base perimeter = $6 \times 2 = 12$ cm.

2

$$\therefore X = F \cos 0^\circ + 80 \cos 60^\circ$$

$$+ K \cos 90^\circ + 50 \cos 180^\circ$$

$$+ 80\sqrt{3} \cos 270^\circ$$

$$= F \times 1 + 80 \times \frac{1}{2} + K \times 0$$

$$+ 50 \times -1 + 80\sqrt{3} \times 0$$

$$= F - 10$$

$$\therefore Y = F \sin 0^\circ + 80 \sin 60^\circ + K \sin 90^\circ + 50 \sin 180^\circ$$

$$+ 80\sqrt{3} \sin 270^\circ$$

$$= F \times 0 + 80 \times \frac{\sqrt{3}}{2} + K \times 1 + 50 \times 0$$

$$+ 80\sqrt{3} \times -1 = K - 40\sqrt{3}$$

$$\therefore \vec{R} = (F - 10)\hat{i} + (K - 40\sqrt{3})\hat{j}$$

$$\therefore R = 40 \text{ newton due to } 60^\circ \text{ North of East}$$

$$\therefore \vec{R} = 40 \cos 60^\circ \hat{i} + 40 \sin 60^\circ \hat{j}$$

$$= 20\hat{i} + 20\sqrt{3}\hat{j}$$

From (1) and (2): $\therefore F - 10 = 20$

$$\therefore F = 30 \text{ newton}$$

$$\therefore K - 40\sqrt{3} = 20\sqrt{3}$$

$$\therefore K = 60\sqrt{3} \text{ newton}$$

3

ΔMAB is the triangle of forces

$$\therefore \frac{50}{MA} = \frac{25}{BM} = \frac{W}{AB}$$

$$\therefore \frac{MA}{MB} = \frac{50}{25} = 2$$

$$\therefore MA = 2 \times 12 = 24$$

The length of the string

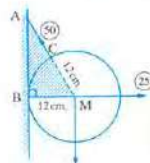
$$CA = 24 - 12 = 12 \text{ cm.}$$

From ΔMAB which is right-angled at B

$$\therefore AB = 12\sqrt{3}$$

$$\therefore \frac{25}{12} = \frac{W}{12\sqrt{3}}$$

$$\therefore W = 25\sqrt{3} \text{ newton}$$



Model 9

First Multiple choice questions

- | | | | |
|---------------|---------------|---------------|---------------|
| 1 (c) | 2 (c) | 3 (b) | 4 (d) |
| 5 (a) | 6 (c) | 7 (d) | 8 (c) |
| 9 (d) | 10 (b) | 11 (c) | 12 (b) |
| 13 (c) | 14 (d) | 15 (d) | 16 (a) |
| 17 (b) | 18 (c) | 19 (a) | 20 (b) |
| 21 (b) | | | |

Second Essay questions

1

$\therefore AC = AD = 4$ metres

$\therefore m(\angle ACD) = 45^\circ$

$\therefore CD = 4\sqrt{2}$ metres

From ΔMCN :

$MN = 1$ metre $\therefore m(\angle NCM) = 45^\circ$

$\therefore m(\angle NMC) = 90^\circ$

$\therefore NC = \sqrt{2}$ metres

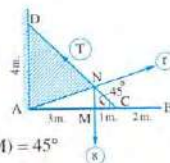
$\therefore DN = 4\sqrt{2} - \sqrt{2} = 3\sqrt{2}$ metres

From ΔAMN : $AN = \sqrt{10}$ metres (Pythagoras)

\therefore Since ΔAND is the triangle of forces

$$\therefore \frac{r}{\sqrt{10}} = \frac{T}{3\sqrt{2}} = \frac{8}{4}$$

$$\therefore r = 2\sqrt{10} \text{ kg.wt.}, T = 6\sqrt{2} \text{ kg.wt.}$$



2

The length of the drawer of the cone = 18 cm.

\therefore the circumference of circle

of the cone base = the length of \widehat{AB}

$$= r \times \theta^{\text{rad}} = 18 \times \frac{60^\circ \times \pi}{180^\circ} = 6\pi$$

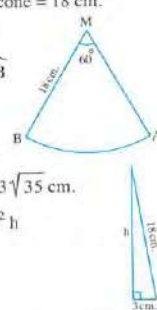
$$\therefore 2\pi r = 6\pi \quad \therefore r = 3 \text{ cm.}$$

$$\therefore h = \sqrt{l^2 - r^2} = \sqrt{(18)^2 - 3^2} = 3\sqrt{35} \text{ cm.}$$

$$\therefore \text{Volume of the cone} = \frac{1}{3} \pi r^2 h$$

$$= \frac{1}{3} \times \pi \times (3)^2 \times 3\sqrt{35}$$

$$= 167.3 \text{ cm}^3$$



3

$$X = 10 \cos 0^\circ + 20 \cos 120^\circ$$

$$+ 30 \cos 240^\circ$$

$$= 10 + 20 \times \frac{-1}{2} + 30 \times \frac{-1}{2} = -15$$

$$Y = 10 \sin 0^\circ + 20 \sin 120^\circ$$

$$+ 30 \sin 240^\circ$$

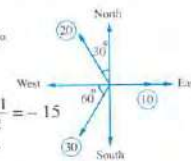
$$= 0 + 20 \times \frac{\sqrt{3}}{2} + 30 \times \frac{-\sqrt{3}}{2} = -5\sqrt{3}$$

$$\therefore \vec{R} = -15\hat{i} - 5\sqrt{3}\hat{j}$$

$$\therefore R = \sqrt{(-15)^2 + (-5\sqrt{3})^2} = 10\sqrt{3} \text{ newton.}$$

$$\therefore \tan \theta = \frac{-5\sqrt{3}}{-15} = \frac{\sqrt{3}}{3} \quad \therefore \theta = 210^\circ$$

\therefore Line of action of the resultant in direction 30° south of west.



Model 10

First Multiple choice questions

- | | | | |
|---------------|---------------|---------------|---------------|
| 1 (d) | 2 (a) | 3 (a) | 4 (a) |
| 5 (b) | 6 (b) | 7 (a) | 8 (c) |
| 9 (d) | 10 (d) | 11 (b) | 12 (a) |
| 13 (a) | 14 (d) | 15 (d) | 16 (d) |
| 17 (a) | 18 (c) | 19 (b) | 20 (c) |
| 21 (b) | | | |

Second Essay questions

1

$$\text{The slant height} = \sqrt{(10\sqrt{3})^2 + (10)^2} = 20 \text{ cm.}$$

(1) Lateral surface area

$$= \frac{1}{2} (\text{perimeter of the base})$$

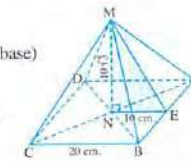
$$\times \text{slant height}$$

$$= \frac{1}{2} \times (4 \times 20) \times 20$$

$$= 800 \text{ cm}^2$$

(2) Volume = $\frac{1}{3}$ area of the base $\times h$

$$= \frac{1}{3} \times (20)^2 \times 10\sqrt{3} = \frac{4000\sqrt{3}}{3} \text{ cm}^3$$



2

$$X = 12 \cos 0^\circ + 9 \cos 90^\circ$$

$$+ 5\sqrt{2} \cos 135^\circ + 7\sqrt{2} \cos 225^\circ + 7 \cos 270^\circ$$

$$= 12 + 0 - 5 - 7 + 0 = 0$$

$$\therefore Y = 12 \sin 0^\circ + 9 \sin 90^\circ$$

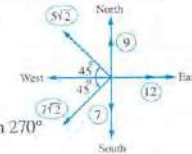
$$+ 5\sqrt{2} \sin 135^\circ$$

$$+ 7\sqrt{2} \sin 225^\circ + 7 \sin 270^\circ$$

$$= 0 + 9 + 5 - 7 - 7 = 0$$

$$\therefore \vec{R} = \vec{0}$$

\therefore The system is in equilibrium.



3

\therefore The set of forces are in equilibrium.

$\therefore \vec{r}$ passes through the point E

$\therefore DE \parallel AC$

$\therefore E$ is the midpoint of \overline{BC}

$\therefore AE \perp BC$

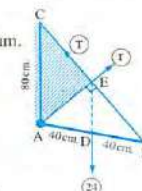
$$\therefore EC = 40\sqrt{3} \quad \therefore AC = 80 \text{ cm.}$$

$\therefore AE = 40 \text{ cm.}$ (Pythagoras theorem)

ΔAEC is the triangle of forces

$$\therefore \frac{r}{40} = \frac{T}{40\sqrt{3}} = \frac{24}{80} \quad \therefore r = 40 \times \frac{24}{80} = 12 \text{ kg.wt.}$$

$$T = 40\sqrt{3} \times \frac{24}{80} = 12\sqrt{3} \text{ kg.wt.}$$



Model

1

Interactive test 1



Answer the following questions :

- 1** The total surface area of a right circular cone which its slant height equal the diameter length of its base is

(a) $4\pi r^2$ (b) $3\pi r^2$ (c) $3\pi r^3$ (d) $4\pi r^3$

- 2** If A, B and C are three points identify a plane, then

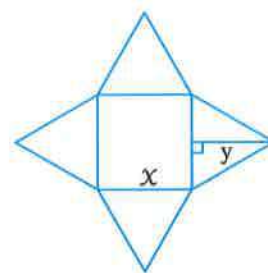
(a) $AB = BC = CA$ (b) $AB + BC = AC$
(c) $AB + BC > AC$ (d) $AB + BC < AC$

- 3** Two forces are equal act at a point and the measure of the angle between them is $\frac{\pi}{3}$ and their resultant is 3 newton, then the magnitude of each is newton.

(a) $\sqrt{3}$ (b) 3 (c) $\frac{3}{2}$ (d) $3\sqrt{3}$

- 4** The opposite figure represents a regular quadrilateral pyramid its height (h), then the relation between x, y and h is

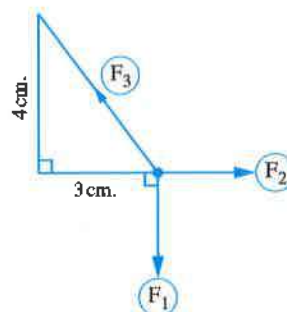
(a) $x^2 + y^2 = h^2$ (b) $x^2 + h^2 = y^2$
(c) $\left(\frac{x}{2}\right)^2 + h^2 = y^2$ (d) $\left(\frac{x}{2}\right)^2 + y^2 = h^2$



- 5** In the opposite figure :

A body is in equilibrium under the action of three forces meeting at a point of magnitudes F_1 , F_2 and F_3 newton, and the sides of the right-angled triangle are parallel to the lines of action of the forces in the same cyclic order, then $F_1 : F_2 : F_3 = \dots\dots\dots$

(a) 3 : 4 : 5 (b) 3 : 5 : 4
(c) 4 : 5 : 3 (d) 4 : 3 : 5



- 6 ABCDHE is a regular hexagon. Forces of magnitudes $2, 4\sqrt{3}, 8, 2\sqrt{3}$ and 4 kg.wt. act at point A in directions $\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD}, \overrightarrow{AH}, \overrightarrow{AE}$ respectively. Find the magnitude and the direction of their resultant.

- 7 Volume of a regular quadrilateral pyramid is 400 cm^3 and its height is 12 cm , then its lateral surface area = cm^2

(a) 240 (b) 260 (c) 300 (d) 360

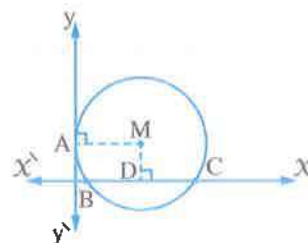
- 8 The area of base of a right circular cone is $36\pi \text{ cm}^2$ and the length of its drawer is 10 cm , find its :

(1) Lateral surface area. (2) Total surface area. (3) Volume.

- 9 In the opposite figure :

If B (2, 0), C (8, 0), then the equation of the circle is

- (a) $(x - 5)^2 + (y - 4)^2 = 25$
 (b) $(x + 5)^2 + (y - 4)^2 = 36$
 (c) $(x - 5)^2 + (y - 4)^2 = 36$
 (d) $(x + 5)^2 + (y - 4)^2 = 25$



- 10 Two forces of magnitude $6, F$ kg.wt. act at a point and measure of the angle between them 135° , if its line of action inclined by an angle 45° , with the line of action of the force F , the magnitude of the resultant = kg.wt.

(a) 6 (b) $6\sqrt{2}$ (c) $6\sqrt{3}$ (d) 10

- 11 A body of weight (W) newton is placed on a smooth plane inclined with the horizontal at an angle of measure 30° and kept in equilibrium by the effect of force of magnitude 36 newton acts in the direction of the line of greatest slope of the plane upwards. then the magnitude of the weight

(a) 36 (b) $72\sqrt{3}$ (c) 72 (d) $36\sqrt{3}$

- 12 If \vec{R} is the resultant of the two forces \vec{F}_1, \vec{F}_2 and \vec{R} is the resultant of the two forces $\vec{F}_1, -\vec{F}_2$, then

- (a) $\vec{R} + \vec{R} = 2\vec{F}_1$ (b) $\vec{R} = \vec{R} + 2\vec{F}_2$
 (c) $R^2 + R^2 = 2(F_1^2 + F_2^2)$ (d) all of previous.

- 13** The equation of the circle which is the image of the circle :

$$x^2 + y^2 - 12x + 6y + 20 = 0 \text{ by translation } (x + 2, y - 2)$$

- (a) $x^2 + y^2 - 10x + 4y + 20 = 0$ (b) $x^2 - 16x + 10y + 20 = 0$
 (c) $(x - 6)^2 + (y + 3)^2 = 20$ (d) $(x - 8)^2 + (y + 5)^2 = 25$

- 14** A force of magnitude $5\sqrt{3}$ newton act in direction 30° east of north, is resolved into two perpendicular components, then the magnitude of its component in direction the east = newton.

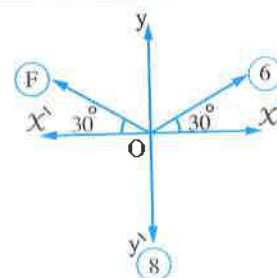
- (a) 5 (b) 7.5 (c) $\frac{5\sqrt{3}}{2}$ (d) 15

- 15** \overline{AB} is a uniform rod of weight 20 kg.wt. the end A attached to a hinge fixed on a vertical wall a horizontal force F acts at B, the body is in equilibrium when it inclined by angle 30° with vertical, find the magnitude of each of the force and reaction of the hinge.

- 16** In the opposite figure :

If the resultant of the forces (in newton) acts along y-axis, then F = newton.

- (a) 8 (b) 6
(c) 14 (d) 2

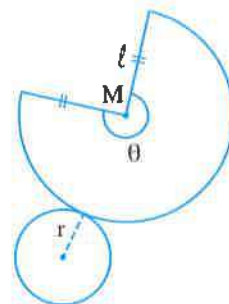


- 17** A cube made of wax, its edge length = 20 cm. it is melted and converted to a right circular cone of height 21 cm. then the radius length of the base of the cone = cm. given that 12% from wax had been lost during melting and reforming. ($\pi \approx \frac{22}{7}$)

- (a) $\frac{20\sqrt{110}}{11}$ (b) $10\sqrt{2}$ (c) 160 (d) $8\sqrt{5}$

- 18** The opposite figure represents net of a cone where the central angle of its circular sector = θ , $180^\circ < \theta < 360^\circ$, then

- (a) $l < 2r$ (b) $l = r$
(c) $l = 2r$ (d) $l > 2r$



- 19** Which of the following system of forces can not be equilibrium ?

- (a) 10 newton, 10 newton, 5 newton. (b) 4 newton, 6 newton, 8 newton.
(c) 11 newton, 7 newton, 8 newton. (d) 8 newton, 4 newton, 14 newton.

20 If the equation $(x - y - 25) \begin{pmatrix} x \\ y \\ -4 \end{pmatrix} = 0$ represents a circle, then the length of its diameter is length unit.

- (a) 10 (b) 20 (c) 100 (d) 200

21 The magnitudes of two forces, meeting at a point, are $5F$, $3F$, then the magnitude of their resultant can not be equal to

- (a) $2F$ (b) $3\sqrt{2}F$ (c) $8F$ (d) $5\sqrt{3}F$

22 In the opposite figure :

The weight of a body is 150 gm. wt.

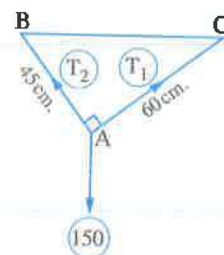
It is tied by two perpendicular

strings their lengths are 60 cm. , 45 cm.

and the other ends are fixed at C and B

on the same horizontal line, then $T_2 - T_1 = \dots\dots\dots \text{ gm.wt.}$

- (a) 120 (b) 90 (c) 60 (d) 30



23 Two lines are skew if

- (a) they are not parallel. (b) they are not intersecting.
(c) they are not coincident. (d) they are not on the same plane.

24 The point which lies on the circle $(x - 2)^2 + y^2 = 13$ is

- (a) $(2, 3)$ (b) $(3, -2)$ (c) $(2, 0)$ (d) $(4, 3)$

Model

2

Interactive test 2



Answer the following questions :

1 Any three points are non-collinear identify

- (a) 1 plane. (b) 2 planes. (c) 3 planes. (d) 4 planes.

2 When the two forces 6 and 8 newton are perpendicular, then the sine of inclination angle of the resultant with the first force equals

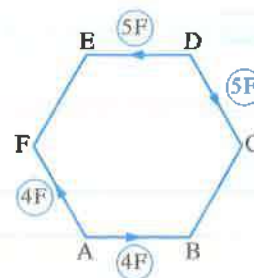
- (a) $\frac{3}{5}$ (b) $\frac{4}{5}$ (c) $\frac{3}{4}$ (d) $\frac{4}{3}$

- 3** The centre of the circle : $x^2 + y^2 - 6x + 8y = 0$ is the point
- (a) (3, -4) (b) (-4, 3) (c) (-3, 4) (d) (-3, -4)
-
- 4** Three forces are equal in magnitude and meeting at a point are in equilibrium, then the measure of the angle between any two of them is
- (a) 60° (b) 120° (c) 90° (d) 150°
-
- 5** The volume of the right cone, if the circumference of its base is 44 cm. and its height is 15 cm. equals cm^3 . ($\pi \approx \frac{22}{7}$)
- (a) 77 (b) 105 (c) 110 (d) 770
-
- 6** Two forces of equal magnitude meeting at a point and the magnitude of their resultant equals 12 kg.wt. if the direction of one of them is reversed then the magnitude of the resultant becomes 6 kg.wt. Find the magnitude of each force.
-
- 7** The forces of magnitudes $2F$, $3F$ and $4F$ newton act on a particle in the directions parallel to the sides of an equilateral triangle in the same cyclic order. then the magnitude of the resultant of these forces = newton.
- (a) $5F$ (b) $2\sqrt{3}F$ (c) $\sqrt{3}F$ (d) F
-
- 8** ABCDEF is a regular hexagon, the force of magnitude 20 newton acts along \overrightarrow{AD} , is resolved into two components in directions \overrightarrow{AC} , \overrightarrow{AF} , then the component in direction \overrightarrow{AF} equals newton.
- (a) 10 (b) $10\sqrt{3}$ (c) 20 (d) $20\sqrt{3}$
-
- 9** The equation of the circle which its centre is (2, -3) and touches the straight line which its equation is : $3x - 4y + 2 = 0$ is
- (a) $(x - 2)^2 + (y + 3)^2 = 2$ (b) $(x + 2)^2 + (y - 3)^2 = 4$
(c) $x^2 + y^2 - 4x + 6y = 12$ (d) $(x - 2)^2 + (y + 3)^2 = 16$
-
- 10** If the length of the base side of a regular quadrilateral pyramid is doubled, then its volume
- (a) will doubled. (b) will be three times.
(c) will be four times. (d) will not change.

11 In the opposite figure :

ABCDEF is a regular hexagon , then the resultant of these forces should be in direction

- (a) \overrightarrow{AD} (b) \overrightarrow{DA}
(c) \overrightarrow{AC} (d) \overrightarrow{EA}



12 A regular quadrilateral pyramid , the side length of its base = 40 cm. , and its lateral height is 25 cm. , find :

- (1) Height of the pyramid. (2) The lateral surface area.
(3) The total surface area. (4) Its volume.

13 If $\vec{F}_1 = 5\hat{i} - 3\hat{j}$, $\vec{F}_2 = -7\hat{i} + 2\hat{j}$, $\vec{F}_3 = 2\hat{i} + \hat{j}$, then $\vec{R} =$

- (a) $7\hat{i} - 2\hat{j}$ (b) $14\hat{i} - 4\hat{j}$ (c) $-14\hat{i} + 4\hat{j}$ (d) $\vec{0}$

14 The ball of a pendulum of weight 600 gm.wt. is displaced until the string makes an angle of measure 30° with the vertical under the action of a force perpendicular to the string. Then the magnitude of the force = dynes.

- (a) $300\sqrt{3}$ (b) 1200 (c) 300 (d) $300\sqrt{2}$

15 Two forces F , F act at a particle and the magnitude of their resultant is F , then the measure of the included angle of the two forces =

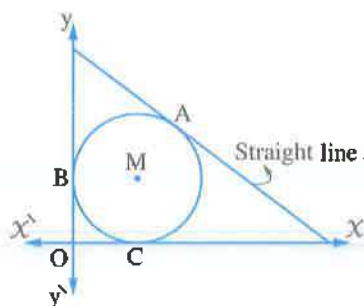
- (a) 60° (b) 45° (c) 120° (d) 135°

16 A circular sector made of paper its radius length is 36 cm. and its central angle is 210° , folded to a right circular cone , find its height.

17 In the opposite figure :

If the equation of the straight line ℓ is $\frac{x}{8} + \frac{y}{6} = 1$, then the equation of the circle is

- (a) $(x - 2)^2 + (y - 2)^2 = 4$
(b) $(x - 2)^2 + (y - 2)^2 = 16$
(c) $(x + 2)^2 + (y + 2)^2 = 4$
(d) $(x + 2)^2 + (y + 2)^2 = 16$

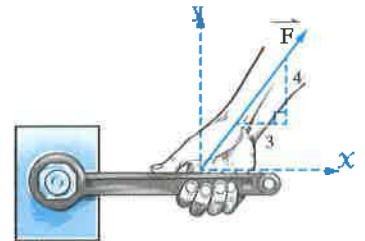


- 18** The ratio between volume of a regular triangular pyramid and volume of greatest cone can put it inside the pyramid equals

(a) $\frac{3\sqrt{3}}{\pi}$ (b) $\frac{3\sqrt{3}}{2\pi}$ (c) $\frac{\sqrt{3}}{\pi}$ (d) $\frac{3\sqrt{3}}{4\pi}$

- 19** In the opposite figure :

If vertical component of the force (\vec{F}) of a person uses a spanner is 60 newton , then the horizontal component of \vec{F} equals newton.



(a) 30 (b) 45
(c) 60 (d) 75

- 20** The magnitude of two forces 4 , 6 N. and the magnitude of the resultant is 10 N. , then the measure of the angle between the two forces equals

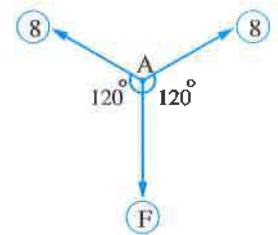
(a) 0° (b) 90° (c) 180° (d) 45°

- 21** The lateral area of a right cone, its base radius length is r , and its drawer ℓ equals

(a) $2\pi r \ell$ (b) $2\pi r^2 \ell$ (c) $\pi r \ell$ (d) $\pi r^2 \ell$

- 22** In the opposite figure :

Particle A is kept in equilibrium under action of the three forces , as shown in the figure , where \vec{F} is in equilibrium with two forces each of magnitude 8 N. and it makes with each an angle of measure 120° , then $F =$ N.



(a) zero (b) 8 (c) 16 (d) $8 \sin 120^\circ$

- 23** The centre of the circle : $x^2 + y^2 - 6x + 8y = 0$ is the point

(a) (3 , -4) (b) (4 , -3) (c) (-3 , 4) (d) (-4 , 3)

- 24** Which of the following statements is not true ?

(a) Any two points in the space have only one plane passing through them.
(b) Any three non-collinear points in the space determine a plane.
(c) The vertices of a triangle determine a plane.
(d) Every two intersecting straight lines are contained in one plane.

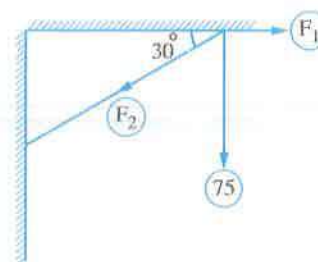


Answer the following questions :

- 1 Two forces of magnitudes 5 , 3 newton and the measure of the angle enclosed between them is 60° , then the magnitude of their resultant \mathbb{R} equals
 (a) 2 (b) 7 (c) 8 (d) 5
- 2 The height of a right circular cone is 12 cm. and the length of its drawer is 15 cm. , then its volume = cm^3
 (a) 324π (b) 715π (c) 32π (d) 180π
- 3 The minimum value of resultant of two forces of magnitudes 5 , 9 newton and meeting at a point equals newton.
 (a) zero (b) 9 (c) 4 (d) 5
- 4 The least number of planes can determine a solid is planes.
 (a) three. (b) four. (c) two. (d) five.
- 5 A weight of magnitude 200 gm.wt. is suspended by two strings of lengths 60 cm. and 80 cm. , from two points on one horizontal line where the distance between them is 100 cm. Find the magnitude of tension in each string.
- 6 The volume of a regular quadrilateral pyramid if the side length of its base = 18 cm. and the lateral height is 15 cm. is equal to cm^3
 (a) 1156 (b) 1254 (c) 1308 (d) 1296
- 7 **In the opposite figure :**

A vertical force of magnitude 75 newton is resolved into two components , one of them is horizontal (F_1) and the other F_2 , then $F_2 =$ newton.

- (a) 75 (b) $75\sqrt{3}$
 (c) 150 (d) $150\sqrt{3}$



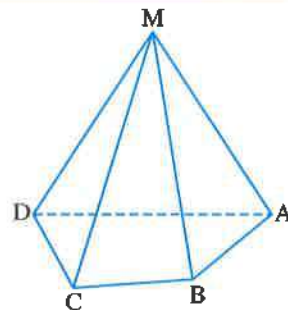
- 8 Two forces of magnitudes 6, 12 newton act at a particle, enclosed between them an angle of measure 120° , then the measure of the angle between the resultant and the first forces =

(a) 120° (b) 60° (c) 90° (d) 30°

- 9 In the opposite figure :

The plane $ABD \cap$ the plane MCD =

(a) \overrightarrow{AM} (b) \overrightarrow{CD}
(c) $\{D\}$ (d) \overrightarrow{MC}



- 10 The forces 8 , $4\sqrt{3}$, $6\sqrt{3}$ and 14 newton act at a point, the measure of the angle between the first force and the second force is 30° , between the second and the third is 120° and between the third and the fourth is 90° taken in the same cyclic order. Find the magnitude and direction of the resultant of these forces.

- 11 If the geometric centre of a regular hexagon is the origin and its area = $3\sqrt{3} \text{ cm}^2$, then the equation of its circumcircle is

(a) $x^2 + y^2 = 2$ (b) $x^2 + y^2 = 4$
(c) $x^2 + y^2 = 6$ (d) $x^2 + y^2 = 8$

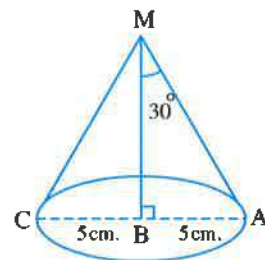
- 12 In the opposite figure :

A right circular cone in which $m(\angle AMB) = 30^\circ$

, the radius length of the base = 5 cm .

, then its total area = cm^2

(a) 50π (b) 75π
(c) 100π (d) 125π



- 13 Two forces of magnitudes 6, 2.5 newton, the magnitude of their resultant = 6.5 newton, then the included angle between the two forces is

(a) acute. (b) obtuse. (c) right. (d) straight.

- 14 A body of weight 100 newton is placed on a smooth plane inclines to the horizontal by an angle 30° , the body kept in equilibrium by a horizontal force. $F \text{ N}$. and the reaction of the plane on the body. is $R \text{ N}$. then $F + R$ = N .

(a) $100\sqrt{3}$ (b) $\frac{100\sqrt{3}}{3}$ (c) $200\sqrt{3}$ (d) $\frac{200\sqrt{3}}{3}$

- 15** In a triangular pyramid of regular faces , if the sum of lengths of its edges = 36 cm. , then the height of the pyramid = cm.

(a) $\sqrt{6}$ (b) $2\sqrt{6}$ (c) 6 (d) 4

- 16** Prove that the two circles :

$$x^2 + y^2 - 2x + 6y + 1 = 0 \quad , \quad 4x^2 + 4y^2 - 8x + 24y + 15 = 0$$

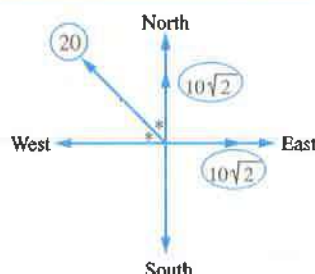
are concentric circles , and find length of radius of each of them.

- 17** In the opposite figure :

The resultant of the forces $10\sqrt{2}$, $10\sqrt{2}$

, 20 newton acts in direction

(a) the eastern north. (b) the north.
(c) the western north. (d) the western south.



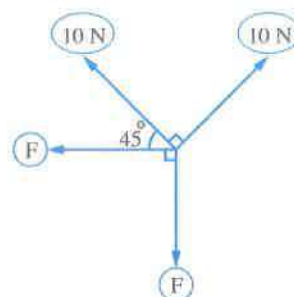
- 18** If the volume of hemisphere of radius length "r" equals the volume of a cone the length of radius of its base = r and its height = h , then

(a) $h = \frac{2}{3} r$ (b) $h = 2 r$ (c) $h = 2 r^2$ (d) $h = 4 r$

- 19** In the opposite figure :

The condition of equilibrium of the given forces is

(a) $F = 10$ newton. (b) $F = 10\sqrt{2}$ newton.
(c) $F = 5\sqrt{2}$ newton. (d) the system will not be equilibrium.



- 20** The circumference of the circle whose equation : $x^2 + y^2 = 8$ is

(a) 8π (b) 64π (c) $2\sqrt{2}\pi$ (d) $4\sqrt{2}\pi$

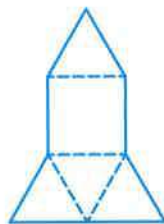
- 21** Two planes coincide if they have in common.

(a) one point (b) two points
(c) three collinear points (d) three non-collinear points

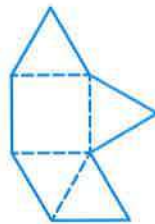
- 22** Force of magnitude $4\sqrt{2}$ N. acts in east direction. It is resolved into two perpendicular directions , then its component in the direction of north of the east equals N.

(a) zero (b) $4\sqrt{2}$ (c) 4 (d) 6

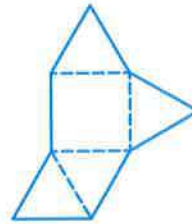
23 Which of the following nets cannot form a pyramid ?



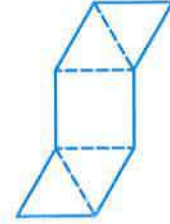
(a)



(b)



(c)



(d)

24 If a body is kept in equilibrium under action of two forces \vec{F}_1 , \vec{F}_2 , then

(a) $\vec{F}_1 = \vec{F}_2$

(b) $F_1 = F_2$

(c) $\vec{F}_1 + \vec{F}_2 \neq \vec{O}$

(d) \vec{F}_1 , \vec{F}_2 are not on the same straight line.

Model

4

Interactive test 4



Answer the following questions :

1 In the opposite figure :

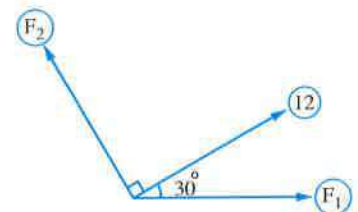
The force of magnitude 12 newton is resolved into two components \vec{F}_1 , \vec{F}_2 make angles of measures 30° , 90° , then $F_2 = \dots\dots\dots$ newton.

(a) 10

(b) $10\sqrt{3}$

(c) $6\sqrt{3}$

(d) $4\sqrt{3}$



2 The height of a regular quadrilateral pyramid is 9 cm. and its volume = 300 cm^3 , then the side length of its base equals cm.

(a) 5

(b) 10

(c) 15

(d) 20

3 Two perpendicular forces of magnitudes 12 newton, 5 newton, act at a point, then the magnitude of their resultant newton.

(a) 5

(b) 12

(c) 13

(d) 17

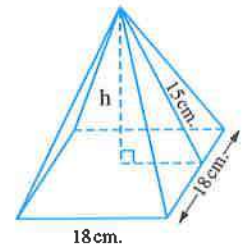
4 ABCD is a rectangle which $AB = 6 \text{ cm}$, $BC = 8 \text{ cm}$, a point $E \in \overline{AD}$ where $AE = 6 \text{ cm}$, the forces of magnitudes F , 5 , K , $6\sqrt{10}$ newton act along \overline{CB} , \overline{CA} , \overline{CD} and \overline{EC} respectively. If the system of forces are in equilibrium, then find value of each of F and K

- 5** All of the following cases form a plane except
- a straight line and a point do not belong to it.
 - two different parallel straight lines.
 - two intersected straight lines.
 - two skew straight lines.
- 6** ABC is right-angle triangle at B where $AB = 3$ cm. , $BC = 4$ cm. , then the volume of the solid which generated by rotation of the triangle complete turn about \overline{BC} is cm^3
- 16π
 - 18π
 - 15π
 - 12π
- 7** A right circle cone , its base on the coordinate plane with equation $x^2 + y^2 = 36$ if the height of the cone = 8 length unit , find :
- Volume of the cone.
 - Total surface area.
- 8** The equation $(x \ y \ 8) \begin{pmatrix} x \\ y \\ -2 \end{pmatrix} = 0$ represents a circle its diameter length = length unit.
- 2
 - 4
 - 6
 - 8
- 9** Two forces of magnitudes 4 , F newton act at a particle , the measure of included angle is 120° , if line of action of the resultant is perpendicular to the first force , then magnitude of the resultant = newton.
- $4\sqrt{2}$
 - $4\sqrt{3}$
 - 4
 - $4\sqrt{5}$
- 10** A body of weight (W) newton is suspended by two light strings inclined to the vertical by angles θ° and 30° the body becomes in equilibrium when the tension of the first string equal 12 newton. and the other is $12\sqrt{3}$ newton , then the weight of the body $W = \dots\dots\dots$ N.
- 60
 - 25
 - 36
 - 24
- 11** If \vec{F}_1 , \vec{F}_2 are two forces , then the measure of the angle enclosed between \vec{F}_1 and the resultant of the two forces $(\vec{F}_1 + \vec{F}_2)$, $(\vec{F}_1 - \vec{F}_2)$ equals
- zero.
 - $\tan^{-1} \left(\frac{F_1}{F_2} \right)$
 - $\tan^{-1} \left(\frac{F_2}{F_1} \right)$
 - $\tan^{-1} \left(\frac{F_1 - F_2}{F_1 + F_2} \right)$

12 In the opposite figure :

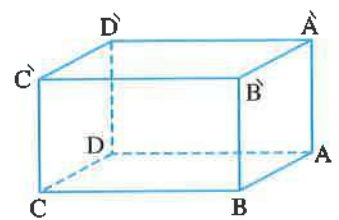
The volume of the regular quadrilateral pyramid in which the side length of its base = 18 cm. and the lateral height = 15 cm. is cm^3 .

- (a) 1296 (b) 1620
(c) 540 (d) 1944


13 Find the equation of the circle which passes through the two points (1, 3), (2, -4) and its centre lies on X-axis.
14 In the opposite figure :

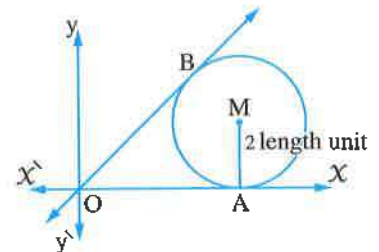
The plane $AA'B'B \cap$ the plane $ACC'A' =$

- (a) $\overleftrightarrow{AA'}$ (b) $\overleftrightarrow{BB'}$
(c) $\overleftrightarrow{CC'}$ (d) \overleftrightarrow{AC}


15 In the opposite figure :

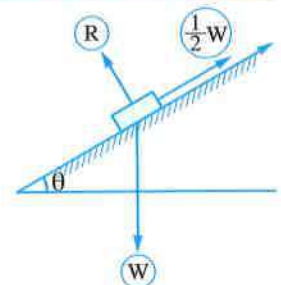
If $OB = 5$ length unit ,
then the equation of the circle M is

- (a) $(x-2)^2 + (y-5)^2 = 25$
(b) $(x-2)^2 + (y-5)^2 = 4$
(c) $(x-5)^2 + (y-2)^2 = 25$
(d) $(x-5)^2 + (y-2)^2 = 4$


16 In the opposite figure :

If the body is in equilibrium under acting
of the shown forces , then $m(\angle \theta) =$

- (a) 30° (b) 60°
(c) 45° (d) 15°

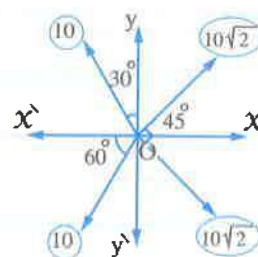

17 The radius length of the base of a right circular cone = 5 cm. and its total surface area = $90\pi \text{ cm}^2$, then its volume = cm^3 .

- (a) 105π (b) 95π (c) 100π (d) 120π

18 In the opposite figure :

The resultant of the system
of forces "R" = newton.

- (a) 20 (b) $10\sqrt{2}$
(c) 10 (d) zero.



19 The equation $\left| \frac{x}{y} - \frac{y}{x} \right| = 36$ represents the equation of a circle with radius
length equals length unit.

- (a) 3 (b) 6 (c) 9 (d) 18

20 Three equal forces, intersecting at one point, are in equilibrium, then the measure of the
angle between any two forces =

- (a) 60° (b) 90° (c) 120° (d) 150°

21 The ratio between the edge length of a uniform triangular pyramid : its height =

- (a) $\sqrt{2} : \sqrt{3}$ (b) $\sqrt{3} : 2$
(c) $\sqrt{6} : 2$ (d) $\sqrt{3} : 3$

22 Two forces of magnitude 8, F newton, the measure of the angle between them $\in]0, \pi[$
If their resultant bisects the angle between them, then F = newton.

- (a) $2\sqrt{2}$ (b) 4 (c) 8 (d) 16

23 The opposite figure represnts a body

remains in equilibrium under action

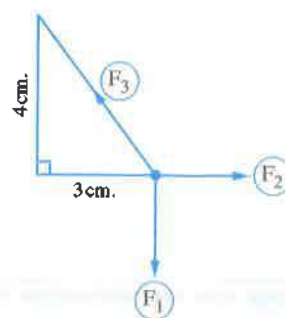
of three forces F_1, F_2, F_3 N.

and the sides of the right angled triangle

are parallel to the lines of action of these forces

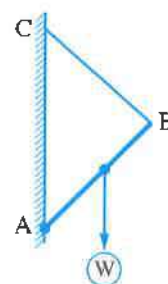
and in one cyclic order then $F_1 : F_2 : F_3 = \dots\dots\dots$

- (a) 3 : 4 : 5 (b) 3 : 5 : 4 (c) 4 : 5 : 3 (d) 4 : 3 : 5



24 In the opposite figure :

\overline{AB} is a uniform rod, its weight W ,
 its end A is connected to a hinge fixed at a vertical wall,
 \overline{BC} is a light string, its end B fixed to the rod
 and the other end C fixed at the vertical wall above A ,
 then the reaction of the hinge



- (a) is in \overline{AB} direction. (b) is perpendicular to \overline{BC}
 (c) is perpendicular to the wall. (d) bisects \overline{BC}

Model**5**Interactive test **5****Answer the following questions :**

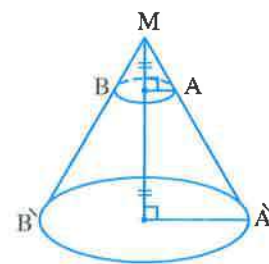
- 1** The two straight lines are skew if they are
 (a) not parallel. (b) not intersecting.
 (c) not coincident. (d) not contained in the same plane.
- 2** The lateral surface area of a right circular cone , radius length of its base = 6 cm. and its height = 8 cm. equals cm^2
 (a) 60π (b) 28π (c) 10π (d) 48π
- 3** Two forces of magnitudes $5F$, $2F$ and their resultant is $7F$ newton , then the measure of the angle between them =
 (a) 180° (b) 60° (c) 20° (d) zero.
- 4** If \vec{F} be equilibrium with two perpendicular forces of magnitudes 8 newton , 15 newton , then $F = \dots\dots\dots$ newton.
 (a) 7 (b) 17 (c) 23 (d) $7\sqrt{2}$
- 5** If the three coplanar forces $\vec{F}_1 = 5\hat{i} + 3\hat{j}$, $\vec{F}_2 = a\hat{i} + 6\hat{j}$, $\vec{F}_3 = -14\hat{i} + b\hat{j}$ act at a point and their resultant $\vec{R} = (10\sqrt{2}, \frac{3}{4}\pi)$, then $a + b = \dots\dots\dots$
 (a) -1 (b) 1 (c) zero. (d) 14

- 6** The total surface area of a right circular cone is $96\pi \text{ cm}^2$, the length of its slant height is 10 cm. Find the radius length of its base and its volume.
- 7** A homogeneous smooth sphere its radius length is 10 cm., its weight = 30 gm.wt. is in equilibrium by a string of length 10 cm. attached to a point of its surface and the other end of the string is fixed at the point in vertical smooth wall, find the tension of the string and the reaction of the wall on the sphere.
- 8** The total surface area of a triangular regular faces pyramid which its edge length = ℓ cm. is equal to cm^2
 (a) $2\sqrt{3}\ell^2$ (b) $\sqrt{3}\ell^2$ (c) $\frac{\sqrt{3}}{3}\ell^2$ (d) $3\sqrt{2}\ell^2$
- 9** The area of any of the lateral faces of a regular quadrilateral pyramid equals to the area of its base, if side length of its base = 6 cm., then its volume = cm^3
 (a) 36 (b) $6\sqrt{3}$ (c) $36\sqrt{15}$ (d) $216\sqrt{15}$
- 10** ABCD is a square of side length = 10 cm., E is the midpoint of \overline{AB} , forces of magnitudes 2, $7\sqrt{5}$, $4\sqrt{2}$ and 4 newton in directions \overrightarrow{CB} , \overrightarrow{CE} , \overrightarrow{CA} and \overrightarrow{CD} respectively, find magnitude and direction of resultant of this forces.
- 11** Two forces of magnitude F , $F\sqrt{3}$ newton, meeting at a point and magnitude of their resultant = R_1 when the measure of included angle = 90° and the resultant became R_2 when the measure of the included angle = 150° , then
 (a) $R_1 = R_2$ (b) $R_1 = 2R_2$ (c) $R_1 = \frac{3}{5}R_2$ (d) $R_1 = \frac{1}{2}R_2$
- 12** The general form of the circle which its diameter \overline{AB} , where A (2, 3), B (-4, 9) is
 (a) $x^2 + y^2 - 4x - 6y + 18 = 0$ (b) $(x+4)^2 + (y-9)^2 = 72$
 (c) $x^2 + y^2 - 2x + 12y + 19 = 0$ (d) $x^2 + y^2 + 2x - 12y + 19 = 0$
- 13** The maximum value of the resultant is 25 newton and minimum value of their resultant is 13 newton of two forces, then their magnitudes are
 (a) 25, 13 (b) 19, 6 (c) 13, 12 (d) 7, 20

14 In the opposite figure :

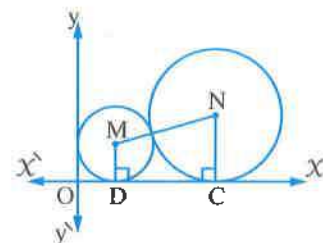
The ratio between the lateral surface area of the cone MAB to the lateral surface area of the cone $M\hat{A}\hat{B}$ equals

- (a) 1 : 2 (b) 1 : 4
(c) 1 : 6 (d) 1 : 8


15 In the opposite figure :

M, N are two circles touching externally their equations are $(x - 2)^2 + (y - 2)^2 = 4$ and $(x - a)^2 + (y - b)^2 = 64$, then $a + b = \dots\dots\dots$

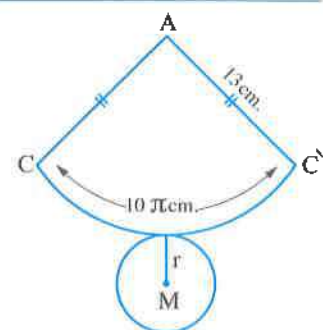
- (a) 8 (b) 10
(c) 18 (d) 28


16 In the opposite figure :

Net of a solid

, its volume = cm^3

- (a) 25π (b) 50π
(c) 75π (d) 100π



17 A force of magnitude 40 newton acts vertically upwards was resolved into two components one of them is horizontal and its magnitude 20 newton, then the magnitude of the other = newton.

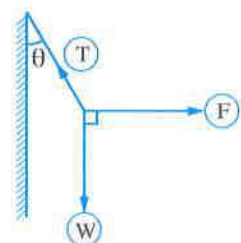
- (a) 20 (b) $20\sqrt{2}$ (c) 10 (d) $20\sqrt{5}$

18 In the opposite figure :

A weight of magnitude (W) newton is suspended in one end of a string and the other end of the string fixed in a point on a vertical wall, the weight is pulled by a horizontal force of magnitude F newton till the string makes an angle θ with vertical.

Which of the following statements is not correct in equilibrium state ?

- (a) $F = W \tan \theta$ (b) $\vec{W} + \vec{F} + \vec{T} = \vec{O}$
(c) $T^2 = F^2 + W^2$ (d) $T = F + W$



- 19 A force \vec{F} is resolved into two components \vec{F}_1 , \vec{F}_2 to make with \vec{F} two angles of measures θ_1 and θ_2 from its both sides respectively, then magnitude of \vec{F}_1 is

(a) $\frac{F \sin \theta_1}{\sin (\theta_1 + \theta_2)}$ (b) $\frac{F \sin \theta_2}{\sin (\theta_1 - \theta_2)}$ (c) $\frac{F \sin \theta_2}{\sin (\theta_1 + \theta_2)}$ (d) $\frac{F \sin (\theta_1 + \theta_2)}{\sin \theta_2}$

- 20 Three coplanar forces intersecting at one point and in equilibrium. If 3 and 7 N. are two forces of them, then the magnitude of the third force could be equals N.

(a) 11 (b) 2 (c) 5 (d) 3

- 21 If a plane intersects a regular quadrilateral pyramid, parallel to its base, then the cross section shape is

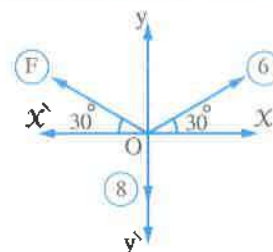
(a) a triangle. (b) a square. (c) a rectangle. (d) a circle.

- 22 The point on the circle $x^2 + (y - 3)^2 = 16$ is

(a) (0, 3) (b) (3, -2) (c) (2, 0) (d) (4, 3)

- 23 If the resultant of the forces shown in the figure acts along the y-axis, then : $F =$ N.

(a) 2 (b) 6
(c) 8 (d) 14



- 24 In the opposite figure :

\overline{AB} is a rod fixed to a hinge at A to a vertical wall.

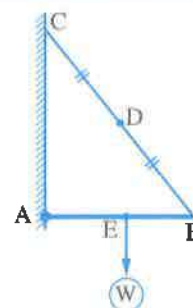
It's kept horizontally by a string fixed to point B and

the other end of the string is fixed to point C

on the wall above A.

Which of the following is the triangle of force ?

(a) $\triangle DBE$ (b) $\triangle DEA$ (c) $\triangle ADE$ (d) $\triangle ACD$



Model

6

Interactive test 6



Answer the following questions :

- 1** The lateral surface area of right cone , which the radius length of its base is r , and length of its drawer l equals

(a) $2\pi lr$ (b) $2\pi lr^2$ (c) πlr (d) πlr^2

- 2** Which two forces from the following pairs , could not have resultant with magnitude = 4 newton ?

(a) 2 newton , 4 newton (b) 3 newton , 3 newton
(c) 2 newton , 6 newton (d) 3 newton , 8 newton

- 3** The point which lies on the circle $(x - 2)^2 + y^2 = 13$ is

(a) (2 , 3) (b) (3 , -2) (c) (2 , 5) (d) (4 , 3)

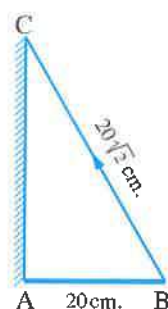
- 4** Number of planes which carry faces of pentagon pyramid is

(a) 5 (b) 6 (c) 10 (d) infinite.

- 5** In the opposite figure :

\overline{AB} is a uniform rod with length 20 cm. and its weight = 30 newton attached by a smooth hinge fixed on a vertical wall in the end A and at the end B suspended by a light string with length $20\sqrt{2}$ cm. its other end fixed at point C on the wall above point A , if the rod is in equilibrium in the horizontal position , then the reaction of the hinge

- (a) act in direction \overline{AB}
(b) its line of action distant 10 cm. from the wall.
(c) bisects \overline{BC}
(d) is of magnitude = 15 newton.



- 6** A body of weight 340 gm.wt. is suspended by two strings with lengths 16 cm. , 30 cm. from two points on same horizontal line , the distance between them 34 cm. , then the magnitude of the tension of each of the two strings respectively = N.

(a) $100\sqrt{3}$, $60\sqrt{3}$ (b) $150\sqrt{2}$, $80\sqrt{2}$ (c) 300 , 160 (d) 300 , 100

- 7 The general form of the equation of circle its centre is $(5, -4)$ and touches X -axis is
- (a) $x^2 + y^2 - 10x + 8y + 25 = 0$ (b) $x^2 + y^2 - 5x + 4y = 0$
 (c) $x^2 + y^2 - 10x + 8y = 25$ (d) $x^2 + y^2 + 10x - 8y + 25 = 0$
- 8 A uniform rod of length 100 cm. , and its weight 150 gm.wt. is suspended from its ends by two strings , the other end of each string fixed on the same point , if the lengths of the two strings are 80 cm. , 60 cm. , then find the magnitude of the tension of each of them.
- 9 If \vec{R} is the resultant of $\vec{F}_1, \vec{F}_2, \vec{R} \perp \vec{F}_1$ and $R = \frac{1}{2} F_2$, then the measure of the angle between the two forces \vec{F}_1, \vec{F}_2 is
- (a) 40° (b) 120° (c) 135° (d) 150°
- 10 A regular quadrilateral pyramid , the side length of its base 18 cm. If its volume is 1296 cm^3 , then find the lateral height and lateral surface area.
- 11 Three coplanar forces of magnitudes 60 , F and K newton meeting at a point and in equilibrium. If the angle between the 1st and the 2nd forces measures 120° and between the 2nd and the 3rd measures 90° , then the value of K = newton.
- (a) $30\sqrt{3}$ (b) $30\sqrt{2}$ (c) 30 (d) 60
- 12 A right cone of volume $27\pi \text{ cm}^3$, circumference of its base $6\pi \text{ cm}$. , then its height. = cm.
- (a) 27 (b) 3 (c) $3\sqrt{3}$ (d) 9
- 13 The ratio between the lateral surface area of the triangular pyramid of regular faces to its total surface area =
- (a) 1 : 3 (b) 1 : 4 (c) 3 : 4 (d) 1 : 2
- 14 ABCDEO is regular hexagon the forces of magnitudes 2 , $4\sqrt{3}$, $4\sqrt{3}$, 4 kg.wt. act at point A in directions $\vec{AB}, \vec{AC}, \vec{AE}, \vec{AO}$ respectively. , then the resultant of this forces acts in direction of
- (a) \vec{AC} (b) \vec{AE} (c) \vec{AD} (d) \vec{AO}

- 15** The length of the tangent segment which drawn from the point $(0, 2r)$ to the circle $x^2 + y^2 = r^2$ is length unit.

(a) r (b) $2r$ (c) $\sqrt{3}r$ (d) $\frac{\sqrt{3}}{2}r$

- 16** ABC is an isosceles triangle where $AB = AC = 10$ cm. , $BC = 12$ cm. , rotates a complete revolution about \overline{BC} , calculate the volume of the solid which generated by rotation.

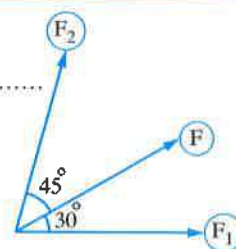
- 17** ABCD $\hat{A}\hat{B}\hat{C}\hat{D}$ is a cube of edge length = 20 cm. a right circular cone is put inside the cube such that the vertex of the cone is the centre of cube base $\hat{A}\hat{B}\hat{C}\hat{D}$, and base of the cone touches the sides of the base ABCD , then the ratio between volume of each the cone and cube is

(a) $\frac{\pi}{12}$ (b) $\frac{\pi}{3}$ (c) $\frac{1}{3}$ (d) $\frac{12}{\pi}$

- 18** In the opposite figure :

The force \vec{F} is the resultant of the two forces \vec{F}_1 , \vec{F}_2 , then $\frac{F_1 + F_2}{F} = \dots\dots\dots$

(a) $\sin 30^\circ + \sin 45^\circ$ (b) $\frac{\sin 75^\circ + \sin 30^\circ}{\sin 75^\circ}$
 (c) $\frac{\sin 45^\circ + \sin 30^\circ}{\sin 75^\circ}$ (d) $\frac{\sin 75^\circ}{\sin 30^\circ} + \frac{\sin 75^\circ}{\sin 45^\circ}$

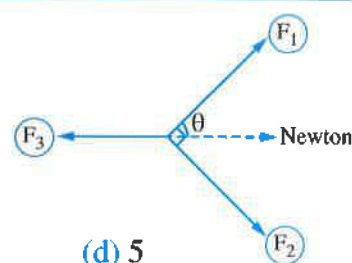


- 19** Two forces meeting at a point of magnitudes F_1 , F_2 where $0 \leq F_1 \leq 13$, $8 \leq F_2 \leq 17$, the measure of the included angle is 180° and magnitude of their resultant R , then

(a) $3 \leq R \leq 4$ (b) $0 \leq R \leq 4$ (c) $0 \leq R \leq 17$ (d) $5 \leq R \leq 17$

- 20** The opposite figure represents three forces \vec{F}_1 , \vec{F}_2 and \vec{F}_3 of magnitudes 4 , 3 and 2 newton respectively , if $\sin \theta = \frac{3}{5}$, then magnitude of their resultant equals newton.

(a) 1 (b) 2 (c) 3 (d) 5



- 21** The vertical straight lines in the space are

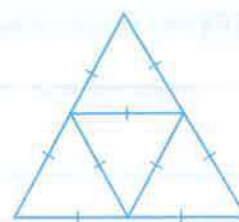
(a) parallel. (b) skew. (c) lie in one plane. (d) intersecting.

- 22** If \vec{F} is in equilibrium with two perpendicular forces of magnitude 8 N. , 15 N. , then $F = \dots\dots\dots$ N.

(a) 7 (b) 17 (c) 23 (d) $17\sqrt{2}$

23 Which of the following can be unfolded to form the opposite net ?

- (a) quadrilateral pyramid.
- (b) regular quadrilateral pyramid.
- (c) regular triangular pyramid.
- (d) otherwise.



24 The magnitude of two forces 3 and 5 N. and thier resultant is 2 N. , then the measure of the angle between the resultant and the second force =

- (a) 180°
- (b) 90°
- (c) 0°
- (d) 30°

Model

7

Interactive test **7**



Answer the following questions :

1 If the resultant of two forces acting at a point reaches the maximum value , then the measure of the angle between their line of actions equals

- (a) 180°
- (b) 120°
- (c) 0°
- (d) 60°

2 A regular quadrilateral pyramid , the side length of its base 10 cm. , and its lateral height 13 cm. , then its lateral area = cm^2

- (a) 260
- (b) 360
- (c) 130
- (d) 520

3 The centre of the circle $x^2 + y^2 - 6x + 8y = 0$ is the point

- (a) (3 , -4)
- (b) (4 , -3)
- (c) (-3 , 4)
- (d) (-4 , 3)

4 If the forces \vec{F}_1 , \vec{F}_2 , \vec{F}_3 are three forces measured by newton are in equilibrium and meeting at a point and $\vec{F}_1 = 2\hat{i} - 3\hat{j}$, $\vec{F}_2 = 3\hat{i} + 5\hat{j}$, then $\vec{F}_3 = \dots\dots\dots$ newton.

- (a) $5\hat{i} + 2\hat{j}$
- (b) $-5\hat{i} - 2\hat{j}$
- (c) $\sqrt{29}$
- (d) $\sqrt{34}$

5 A body of weight W newton is placed on smooth inclined plane , where the angle of the inclination of the plane with the horizontal is 30° , the body kept in equilibrium by a force of magnitude 36 newton and acts in the direction of the line of greatest slope upward , then the magnitude of the weight = N.

- (a) $36\sqrt{3}$
- (b) $36\sqrt{2}$
- (c) 72
- (d) $72\sqrt{3}$

- 6 The general form of the equation of the circle which its centre is $(-2, 5)$ and passes through $(3, 2)$ is

(a) $x^2 + y^2 - 4x + 10y - 5 = 0$

(b) $x^2 + y^2 + 4x - 10y - 5 = 0$

(c) $x^2 + y^2 + 2x - 5y - 5 = 0$

(d) $x^2 + y^2 + 4x - 10y - 25 = 0$

- 7 If the straight line $L \parallel$ the plane X , $A \in X$, then $L \cap X = \dots\dots\dots$

(a) \emptyset

(b) L

(c) $\{A\}$

(d) X

- 8 The lateral surface area of a regular quadrilateral pyramid 240 cm^2 , and its slant height is 12 cm . , find :

(1) Height of the pyramid.

(2) Volume of the pyramid.

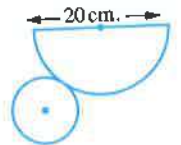
- 9 If we folded the opposite net to become a cone , then the radius length of its base =

(a) 10 cm .

(b) 8 cm .

(c) 5 cm .

(d) 2.5 cm .



- 10 A metal sphere of weight 400 kg.wt . acts in its centre , placed between two smooth planes , one of them is vertical and the other inclined 60° with vertical , then find the reaction of each plane.

- 11 The volume of right cone , where the length of its drawer 15 cm . and the total surface area $= 216 \pi \text{ cm}^2$ equals cm^3

(a) 205π

(b) 320π

(c) 380π

(d) 324π

- 12 If R is the resultant of the two forces F_1, F_2 where $F_2 > F_1$, then which of the following conditions is enough to make $R \perp F_1$?

(a) $R^2 = F_1^2 + F_2^2$

(b) $R^2 = F_2^2 - F_1^2$

(c) $\vec{F}_1 \perp \vec{F}_2$

(d) all of previous.

- 13 ABCD is a square of side length 12 cm . $H \in \overline{BC}$ where $BH = 5 \text{ cm}$. forces of magnitudes $2, 13, 4\sqrt{2}, 9 \text{ gm.wt}$. act in directions of $\overrightarrow{AB}, \overrightarrow{AH}, \overrightarrow{CA}$ and \overrightarrow{AD} respectively. Find the resultant of these forces.

- 14 If $x^2 + y^2 + 2(\cos \theta)x - 2(\sin \theta)y - 8 = 0$ represents the equation of a circle , then $r = \dots\dots\dots$ length unit.

(a) $\sqrt{2}$

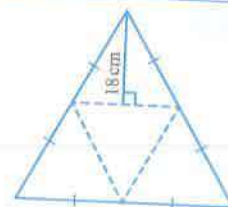
(b) $2\sqrt{2}$

(c) 3

(d) 8

- 15 Four coplanar forces of magnitudes $F_1, 6\sqrt{2}, 6\sqrt{2}, F_2$ gm.wt. acting at a point in direction of east, the eastern north, western north and south respectively. If the resultant of this forces equal 7 gm.wt. and acts in direction of east, then $(F_1, F_2) = \dots\dots\dots$
- (a) (7, 0) (b) (7, 12) (c) $(7, 12\sqrt{2})$ (d) $(6\sqrt{2}, 6\sqrt{2})$

- 16 When we fold the opposite net, then the total surface area of the produced solid is cm^2
- (a) $108\sqrt{3}$ (b) $324\sqrt{3}$
(c) 758 (d) $432\sqrt{3}$



- 17 ABCDE is a regular pentagon, a force of magnitude 20 newton acts along \overrightarrow{AC} , then was resolved in two directions \overrightarrow{AB} and \overrightarrow{AE} , then the magnitude of the component in direction \overrightarrow{AB} equals newton.
- (a) 10 (b) 20 (c) $20\sqrt{3}$ (d) 12.4

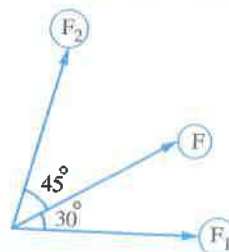
- 18 The radius length of the base of a right circular cone is 15 cm., and its height = 20 cm., then its lateral area = cm^2
- (a) 600π (b) 375π (c) 1875π (d) 5625π

- 19 If the force of magnitude F is in equilibrium with the two forces 5, 3 N. and their included angle is 60° , then $F = \dots\dots\dots$ N.
- (a) $\sqrt{19}$ (b) $\sqrt{34}$ (c) 7 (d) 15

20 In the opposite figure :

The force \vec{F} is the resultant of the two forces \vec{F}_1, \vec{F}_2 , then $\frac{F_1 + F_2}{F} = \dots\dots\dots$

- (a) $\sin 30^\circ + \sin 45^\circ$ (b) $\frac{\sin 75^\circ + \sin 30^\circ}{\sin 75^\circ}$
(c) $\frac{\sin 45^\circ + \sin 30^\circ}{\sin 75^\circ}$ (d) $\frac{\sin 75^\circ}{\sin 30^\circ} + \frac{\sin 75^\circ}{\sin 45^\circ}$



- 21 The centre of the circle $2x^2 + 2y^2 - 6x + 8y = 0$ is the point
- (a) (3, -4) (b) (-4, 3) (c) $(\frac{3}{2}, -2)$ (d) (-3, -4)

- 22** Two perpendicular forces of magnitude 6 , 8 N. , then the sine of angle between the resultant and first force =

(a) $\frac{3}{5}$ (b) $\frac{4}{5}$ (c) $\frac{3}{4}$ (d) $\frac{4}{3}$

- 23** The least number of planes could form a solid is

(a) 1 (b) 2 (c) 3 (d) 4

- 24** If $\vec{F}_1 = 2\hat{i} - 2\hat{j}$, $\vec{F}_2 = 4\hat{i} - 8\hat{j}$ and their resultant $\vec{R} = 2a\hat{i} - 3b\hat{j}$, then $a + b =$

(a) 3 (b) $3\frac{1}{3}$ (c) $6\frac{1}{3}$ (d) 12

Model

8

Interactive test 8



Answer the following questions :

- 1** Two forces of magnitudes 8 , F gm.wt. and the measure of the included angle $\in]0, \pi[$, the line of action of their resultant bisects the included angle , then F = gm.wt.

(a) $2\sqrt{2}$ (b) 4 (c) 8 (d) 16

- 2** The volume of the regular quadrilateral pyramid , where the perimeter of its base = 36 cm. and its height 10 cm. is cm^3

(a) 810 (b) 180 (c) 360 (d) 270

- 3** The circumference of the circle which its equation is $x^2 + y^2 = 8$ is

(a) 8π (b) 64π (c) $2\sqrt{2}\pi$ (d) $4\sqrt{2}\pi$

- 4** If three forces meeting at a point and acting up on a particle are in equilibrium , then the magnitude of each force is proportional to the of the included angle between the two other forces.

(a) cosine (b) sine (c) tangent (d) cotangent

- 5** Two forces are equal in magnitude and each of them equal F newton if the magnitude of the resultant is F newton , then the measure of the included angle =

(a) 0° (b) 30° (c) 60° (d) 120°

- 6 The volume of regular hexagon pyramid is $8\sqrt{3} \text{ cm}^3$ and its height is 4 cm. , find the perimeter of its base.
- 7 Force of magnitude $10\sqrt{2} \text{ gm.wt.}$ acts in direction the eastern south , it was resolved into two perpendicular components , then the component in the south direction = gm.wt.
 (a) $10\sqrt{3}$ (b) $10\sqrt{2}$ (c) 10 (d) 5
- 8 The general form of the equation of the circle where its centre is (2 , - 1) and radius length is 3 cm. is
 (a) $x^2 + y^2 - 4x + 2y - 4 = 0$ (b) $x^2 + y^2 - 2x + y - 4 = 0$
 (c) $x^2 + y^2 + 4x - 2y - 4 = 0$ (d) $x^2 + y^2 - 4x + 2y - 16 = 0$
- 9 A body of weight 24 newton is suspended at one end of a string of length 130 cm. , the other end is fixed at a point of a vertical wall. A horizontal force acts on the body to become in equilibrium. , then the magnitude of horizontal force = newton when the body is at a distance = 50 cm. from the wall.
 (a) 26 (b) 22 (c) 13 (d) 10
- 10 In the opposite figure :
 The central angle of the sector which if it is folded becomes this cone is
 (a) acute. (b) obtuse. (c) straight. (d) reflex.
- 11 The forces of magnitudes F , 80 , K , 50 , $80\sqrt{3}$ newton act at a point in the directions of east , 30° east of north , north , west and south respectively.
 Find the values of F and K if the resultant is 40 newton in magnitude in the direction of 60° north of east.
- 12 Number of the planes which passes through two given points is
 (a) zero. (b) 1 (c) 2 (d) infinite.
- 13 A right circular cone , length of its drawer 17 cm. , and its height 15 cm. , then the radius length of its base = cm.
 (a) 8 (b) 13 (c) 7 (d) 12

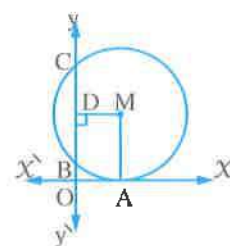


14 In the opposite figure :

Circle M touches X -axis at A , $OB = 2$ length units

, $BC = 6$ length units , then equation of the circle M is

- (a) $(x + 4)^2 + (y + 5)^2 = 16$
 (b) $(x - 4)^2 + (y - 5)^2 = 25$
 (c) $(x - 4)^2 + (y - 5)^2 = 16$
 (d) $(x + 4)^2 + (y + 5)^2 = 25$

**15** A body of weight 6 kg.wt. is placed on a smooth plane inclines to the horizontal by an angle of measure 30° and kept in equilibrium by a horizontal force. , then the magnitude of the reaction of the plane on the body = kg.wt.

- (a) $2\sqrt{3}$ (b) $4\sqrt{3}$ (c) $12\sqrt{3}$ (d) $8\sqrt{3}$

16 Find K which makes the two circle $C_1 : (x + 2)^2 + (y + 11)^2 = K$
 $C_2 : (x - 3)^2 + (y - 1)^2 = 16$ are touching each other.**17** If the resultant of two perpendicular forces , inclined to the greatest one by angle of measure θ , then which of the following values is suitable value of θ ?

- (a) 90° (b) 70° (c) 45° (d) 10°

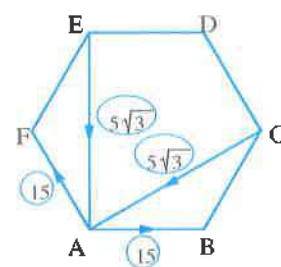
18 In the opposite figure :

ABCDEF is a regular hexagon , forces of magnitudes

$15, 5\sqrt{3}, 5\sqrt{3}$ and 15 act along $\overrightarrow{AB}, \overrightarrow{CA}, \overrightarrow{EA}$ and \overrightarrow{AF}

, then the magnitude of the resultant $R = \dots\dots\dots$ newton.

- (a) 5 (b) 10
 (c) 25 (d) zero.

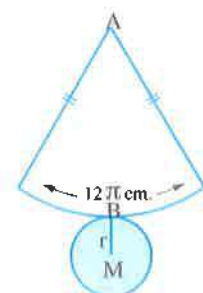
**19** ABCDEF is a regular hexagon. A force of magnitude 20 newton acts in direction of \overrightarrow{AD} , then the components of the force in direction of $\overrightarrow{AC}, \overrightarrow{AF}$ respectively are

- (a) $10\sqrt{3}, 10$ (b) $5\sqrt{3}, 10$ (c) $10, 10\sqrt{3}$ (d) $20\sqrt{3}, 20$

20 The opposite net describes a solid its volume = $96\pi \text{ cm}^3$

, then its total area = cm^2

- (a) 16π (b) 32π
 (c) 48π (d) 96π

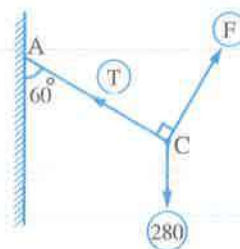


21 Which of the following statements is true ?

- (a) The lateral faces of the right pyramid are congruent.
- (b) The regular pyramid is a right pyramid.
- (c) The heights of the lateral faces of the right pyramid are equal.
- (d) The least number of planes that can determine a solid = 3 planes.

22 In the opposite figure :

A lamp of weight 280 gm.wt. is attached to the end of a string. It is in equilibrium under the effect of a force perpendicular to the string when it is inclined to the vertical by an angle of measure 60° , then $\frac{F}{T} = \dots\dots\dots$



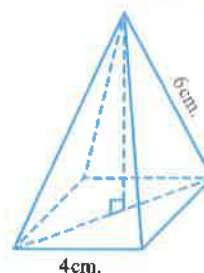
- (a) 2
- (b) $\frac{1}{2}$
- (c) $\frac{1}{\sqrt{3}}$
- (d) $\sqrt{3}$

23 The resultant of two forces F , $2 F$ is perpendicular to one of them , then $R = \dots\dots\dots$

- (a) $\sqrt{5} F$
- (b) $\sqrt{3} F$
- (c) $3 F$
- (d) F

24 The opposite figure represents a regular quadrilateral pyramid of height = cm.

- (a) $7\sqrt{2}$
- (b) $2\sqrt{7}$
- (c) $4\sqrt{2}$
- (d) $2\sqrt{5}$



Model

9

Interactive test

9



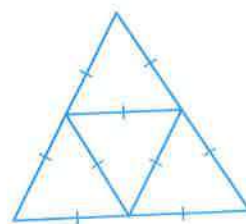
Answer the following questions :

1 Two perpendicular forces of magnitudes $2 F - 5$, $F + 2$ newton act at a particle , the magnitude of their resultant is $3\sqrt{5}$ newton , then $F = \dots\dots\dots$ newton

- (a) 2
- (b) 3
- (c) 4
- (d) 5

2 Which solid, its net is the opposite figure ?

- (a) Quadrilateral pyramid.
 (b) Regular quadrilateral pyramid.
 (c) Triangular pyramid with regular faces.
 (d) Otherwise.



3 Volume of right circular cone is 100 cm^3 , then its volume when its height is doubled becomes cm^3

- (a) 100 (b) 200 (c) 400 (d) 800

4 A body of weight 18 kg.wt. is placed on a smooth plane inclines to the horizontal at angle of measure 30° , the body kept in equilibrium by a force F inclines to line of greatest slope upward by an angle of measure 30° , then the magnitude of this force = kg.wt.

- (a) 12 (b) 9 (c) $3\sqrt{3}$ (d) $6\sqrt{3}$

5 Force of magnitude $4\sqrt{2}$ acts in east direction it was resolved into two perpendicular component, then the magintude of the component in direction of eastern north equals newton.

- (a) 4 (b) $4\sqrt{2}$ (c) 8 (d) $8\sqrt{2}$

6 A regular quadrilateral pyramid. The perimeter of its base = 40 cm. and its height 12 cm. , then its lateral surface area = cm^2

- (a) 200 (b) 240 (c) 260 (d) 320

7 The equation of the circle which the straight line : $x + y = 2$ touches it, and its centre is $(3, 5)$ is

- (a) $(x - 3)^2 + (y - 5)^2 = 3\sqrt{2}$ (b) $(x + 3)^2 + (y + 5)^2 = 18$
 (c) $(x - 3)^2 + (y - 5)^2 = 12$ (d) $(x - 3)^2 + (y - 5)^2 = 18$

8 A weight of 16 newton is suspended at the end of a light string and the other end is fixed at a point of a vertical wall. A force of magnitude $F \text{ newton}$ acts on the weight in a perpendicular direction of the string till it becomes in equilibrium when the string is inclined to the wall with an angle of measure 30° , then the magnitude of the tension in the string. = newton.

- (a) 8 (b) $8\sqrt{2}$ (c) $8\sqrt{3}$ (d) 12

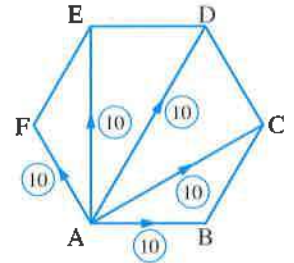
- 9** A uniform rod \overline{AB} of length 6 metres and weight 8 kg.wt. is attached to a hinge fixed in a vertical wall at its end A. The rod is kept horizontally by attaching it at a point C on the rod (where $AC = 4$ metres) by a string which its other end is fixed at the point D on the wall above A exactly and at a distance 4 metres from it. Calculate the magnitude of the tension in the string and the reaction of the hinge.
- 10** The equation of the circle which touches the x -axis at the point $(-2, 0)$ and intercepts from the positive part of y -axis a chord of length $4\sqrt{3}$ length unit is
- (a) $(x + 2)^2 = 48$ (b) $(x + 2)^2 + (y - 4)^2 = 48$
 (c) $(x - 2)^2 + (y + 4)^2 = 24$ (d) $(x + 2)^2 + (y - 4)^2 = 16$
- 11** Two forces are equal in magnitude and the magnitude of their resultant is 24 newton and the measure of the angle between the resultant and one of the two forces is 30° , then the magnitude of each of the two forces = newton.
- (a) 8 (b) $8\sqrt{3}$ (c) $8\sqrt{2}$ (d) 12
- 12** A circular sector, the radius length of its circle is 18 cm. and the measure of its central angle = 60° , it is folded and their radii are connected to form greatest lateral area of a right circular cone. Find the volume of this cone.
- 13** The ratio between length of the edge of triangular pyramid of regular faces to its height =
- (a) $\sqrt{2} : \sqrt{3}$ (b) $\sqrt{3} : 2$ (c) $\sqrt{6} : 2$ (d) $\sqrt{3} : 3$
- 14** Three forces of magnitudes 10, 20, 30 newton act at a particle, the first in direction of east and the second in direction of 30° west of north and third in direction of 60° south of west. Find the magnitude and direction of the resultant of these forces.
- 15** Right circular cone, area of its base = $25\pi \text{ cm}^2$, length of its slant height = 13 cm., then its lateral area = cm^2 .
- (a) 50π (b) 65π (c) 90π (d) 100π
- 16** Two forces of magnitudes $F, 2F$ newton act at a particle, and the line of action of its resultant is perpendicular to one of the two forces, then the measure of the included angle between the two forces =
- (a) 60° (b) 90° (c) 120° (d) 135°

17 The point which lies on the circle : $(x-2)^2 + y^2 = 13$ is

- (a) (2 , 3) (b) (3 , -2) (c) (2 , 5) (d) (4 , 3)

18 Five forces equal in magnitude each equals 10 newton act on one of vertices of a regular hexagon in direction of the other vertices as shown in the opposite figure , then the resultant of this forces is newton.

- (a) 50 (b) 20
(c) $30\sqrt{3}$ (d) $(20 + 10\sqrt{3})$

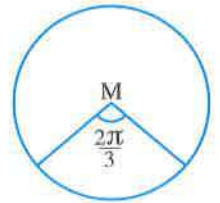


19 In the opposite figure :

A circle is divided into two circular sectors such that they form two right cone nets

, then : $\frac{\text{the lateral area of the smallest cone}}{\text{the lateral area of the greatest cone}} = \dots\dots\dots$

- (a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{8}$ (d) $\frac{1}{16}$



20 Two forces of magnitudes 4 and 6 newton. The measure of the angle between them is 90° , then the tangent of the angle between the resultant and the first force equal

- (a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) $2\sqrt{13}$ (d) $\frac{\sqrt{6}}{2}$

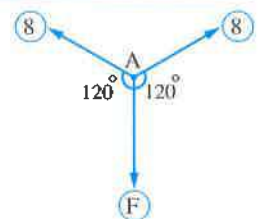
21 Two forces equal in magnitude, the measure of the angle between them is 90° and the magnitude of their resultant is 8 N. , then the magnitude of each one = N.

- (a) $2\sqrt{2}$ (b) 4 (c) $4\sqrt{2}$ (d) 8

22 The centre of the circle : $x^2 + y^2 - 6x + 8y = 0$ is the point

- (a) (3 , -4) (b) (4 , -3) (c) (-3 , 4) (d) (-4 , 3)

23 A is a particle balanced under the effect of the three forces as shown in the opposite figure where \vec{F} is balanced with two forces the magnitude of each is 8 newton and the angle between them is of measure 120° , then F = newton.



- (a) zero (b) 8 (c) 16 (d) $8 \sin 120^\circ$

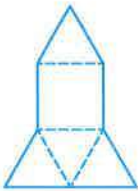
24 Two non parallel planes intersect at

- (a) a point. (b) a straight line. (c) a plane. (d) a ray.

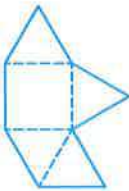


Answer the following questions :

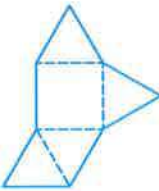
- 1 The point which lies on the circle : $x^2 + (y - 5)^2 = 20$ is
 (a) (2 , 3) (b) (3 , -2) (c) (2 , 5) (d) (4 , 3)
- 2 Two forces of magnitudes 3 , 4 newton their resultant is 7 newton , then the measure of the angle between them is
 (a) 0° (b) 60° (c) 180° (d) 90°
- 3 If \vec{F}_1 , \vec{F}_2 and \vec{F}_3 are three forces meeting at a point and they are in equilibrium , then the magnitude of the resultant of \vec{F}_1 and $\vec{F}_2 = \dots\dots\dots$
 (a) F_1 (b) $F_1 + F_2$ (c) F_3 (d) zero
- 4 Two forces of magnitudes 8 , F newton act at a particle , if the measure of the included angle is 120° , and their resultant $F\sqrt{3}$ newton , then $F = \dots\dots\dots$ newton.
 (a) 4 (b) $4\sqrt{2}$ (c) $4\sqrt{3}$ (d) 8
- 5 The length of the drawer of a right circular cone is 17 cm. and its height = 15 cm. , then its total surface area = cm^2
 (a) 200π (b) 136π (c) 320π (d) 400π
- 6 The length of the base side of a regular quadrilateral pyramid is 20 cm. and its height is $10\sqrt{3}$ cm. , then find :
 (1) The lateral surface area. (2) The volume of the pyramid.
- 7 If O is the origin of perpendicular Cartesian coordinate plane and $\vec{F} = (8 \text{ kg.wt.} , 135^\circ)$ is a force acts at the point O , then the component of \vec{F} in direction of y-axis equals
 (a) $-4\sqrt{2}$ (b) $4\sqrt{2}$ (c) $4\sqrt{3}$ (d) 4
- 8 ABCDEF is a regular hexagon , forces of magnitudes $6\sqrt{3}$, 5 , $6\sqrt{3}$ newton in directions \vec{AC} , \vec{AD} , \vec{AE} respectively. then the magnitude and direction of the resultant of these forces is
 (a) 18 N. in \vec{AD} direction. (b) 23 N. in \vec{AD} direction.
 (c) 20 N. in \vec{AE} direction. (d) 23 N. in \vec{AC} direction.

- 9** A body of weight 32 newton is suspended at the end of a string with length 10 cm. and the other end of the string is fixed at a point on a vertical wall and the body is pulled by horizontal force to make the body in equilibrium when it distant 6 cm. from the wall. , then the magnitude of this force = newton.
- (a) 24 (b) 40 (c) 36 (d) 28
-
- 10** A body of weight 18 newton is placed on a smooth plane inclines to the horizontal by angle of measure 30° and kept in equilibrium by a horizontal force of magnitude F newton. , then the magnitude of the reaction of the plane on the body = newton.
- (a) $6\sqrt{3}$ (b) $8\sqrt{3}$ (c) $12\sqrt{3}$ (d) $10\sqrt{3}$
-
- 11** The equation of the circle which its centre $(-4, 3)$ and passes through the origin point is
- (a) $(x + 4)^2 + (y - 3)^2 = 5$ (b) $(x - 4)^2 + (y + 3)^2 = 25$
 (c) $(x + 4)^2 + (y - 3)^2 = 625$ (d) $(x + 4)^2 + (y - 3)^2 = 25$
-
- 12** A cylindrical shaped vessel contains water , a metallic body in the form of a right cone , its height is 12 cm. and the length of its base radius is 2 cm. is completely immersed in it raising the surface of the water in the vessel with 1 cm. Find the length of base diameter of the vessel.
-
- 13** Which of the following nets does not make a regular quadrilateral pyramid when it is folded ?
- 

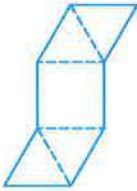
(a)



(b)



(c)

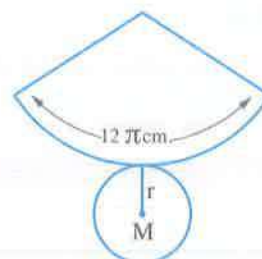


(d)
-
- 14** Five coplanar forces meeting at a point their magnitudes are 12 , 9 , $5\sqrt{2}$, $7\sqrt{2}$ and 7 kg.wt. act due east , north , western north , western south and south respectively , prove that the system is in equilibrium.

- 15 The opposite figure describes a solid its volume = $96 \pi \text{ cm}^3$

, its total surface area = cm^2

- (a) 96π (b) 48π
(c) 32π (d) 16π



- 16 In the opposite figure :

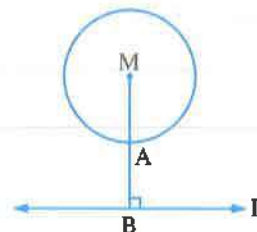
If the equation of the circle is : $x^2 + y^2 - 6x + 4y - 12 = 0$

, $\overline{MB} \perp L$ where $L : 3x - 4y + 23 = 0$,

\overline{MB} intersects the circle at A,

then length of \overline{AB} = length units.

- (a) 3 (b) 5 (c) 8 (d) 12



- 17 Two forces of magnitudes F , $F\sqrt{3}$ newton act at a particle, the magnitude of their resultant $R = F$ newton, and θ_1 is the measure of the angle between 1st force and the resultant and θ_2 is the angle between the 2nd force and the resultant, then

- (a) $\theta_1 = \theta_2$ (b) $\theta_1 = \frac{1}{2} \theta_2$ (c) $\theta_1 = 3 \theta_2$ (d) $\theta_1 = 4 \theta_2$

- 18 Which of the following statements is not true ?

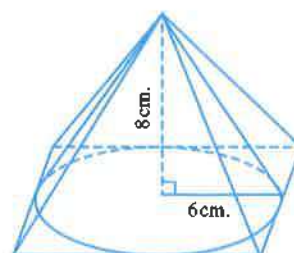
- (a) Any two different parallel straight line identify a plane ?
(b) Any two intersecting different straight lines have a common point.
(c) The two skew lines aren't contained in one plane.
(d) Any three non collinear points, there is at least one plane passes through them.

- 19 In the opposite figure :

A right regular pyramid and right circular cone are common in the vertex and base such that base of the cone touches the sides of the base of the pyramid internally,

then the ratio between the lateral area of the right circular cone and the lateral area of the pyramid =

- (a) $\frac{4}{\pi}$ (b) $\frac{5}{6}$ (c) $\frac{7}{8}$ (d) $\frac{\pi}{4}$



- 20 Force of magnitude $5\sqrt{3}$ N. acts in direction 30° east of the north. It is resolved into two perpendicular components, then the component in direction of the east equals N.

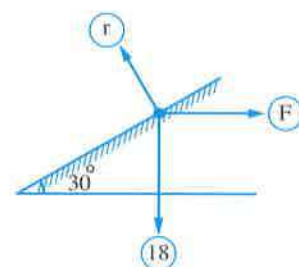
(a) $\frac{5\sqrt{3}}{2}$ (b) $\frac{15}{2}$ (c) $\frac{15\sqrt{3}}{2}$ (d) $15\sqrt{3}$

- 21 If $\vec{F}_1, \vec{F}_2, \vec{F}_3$ are three forces intersect at one point and in equilibrium, then the magnitude of the resultant of \vec{F}_1 and $\vec{F}_2 = \dots\dots\dots$

(a) F_1 (b) $F_1 + F_2$ (c) F_3 (d) zero

- 22 In the opposite figure :

A body of weight 18 N. placed on a smooth inclined plane, inclined to the horizontal at an angle of measure 30° and kept in equilibrium by a horizontal force F N. , then $F + r = \dots\dots\dots$ N.



(a) $6\sqrt{3}$ (b) $12\sqrt{3}$ (c) $18\sqrt{3}$ (d) $24\sqrt{3}$

- 23 The diameter length of the circle : $4x^2 + 4y^2 + 16x - 8y - 16 = 0$ equals length units.

(a) 3 (b) 6 (c) 12 (d) 24

- 24 The ratio between the edge length of regular triangular pyramid and its high =

(a) $\sqrt{2} : \sqrt{3}$ (b) $\sqrt{3} : 2$ (c) $\sqrt{6} : 2$ (d) $\sqrt{3} : 3$

Second

Multiple choice examinations

Model

1

Answer the following questions : (Calculators are allowed)

Choose the correct answer from the given answers :

- 1** Two forces of magnitudes $3F$ and $2F$ newton act at a point and their resultant is $5F$ newton, then the measure of the angle between them =
- (a) 0° (b) 60° (c) 20° (d) 180°
-
- 2** A force of magnitude $5\sqrt{3}$ newton acts in the direction of 30° East of North, is resolved into two perpendicular components, then the magnitude of its component in the East direction = newton.
- (a) 5 (b) $7\frac{1}{2}$ (c) $\frac{5\sqrt{3}}{2}$ (d) 15
-
- 3** A uniform smooth sphere of weight 1.5 gm.wt. and radius length 25 cm. is suspended at a point on its surface by a light string of length 25 cm. and the other end of the string is fixed at the point in vertical smooth wall. If the sphere is in equilibrium, then the tension in the string = gm.wt.
- (a) $\sqrt{3}$ (b) 6 (c) $2\sqrt{3}$ (d) 3
-
- 4** If the three coplanar forces $\vec{F}_1 = 5\hat{i} + 3\hat{j}$, $\vec{F}_2 = a\hat{i} + 6\hat{j}$, $\vec{F}_3 = -14\hat{i} + b\hat{j}$ act at a point and their resultant $\vec{R} = (10\sqrt{2}, \frac{3}{4}\pi)$, then $a + b =$
- (a) -1 (b) 1 (c) zero (d) 14
-
- 5** Two forces of magnitudes 5 , 3 newton act at a point and the measure of the angle between them is 60° , then the magnitude of their resultant R equals
- (a) 2 (b) 7 (c) 8 (d) 5
-
- 6** Three forces are equal in magnitude and meeting at a point are in equilibrium, then the measure of the angle between any two forces of them is
- (a) 60° (b) 120° (c) 90° (d) 150°

- 7** Two forces of magnitudes 4, F newton act at a particle, the measure of the angle between them is 120° . If line action of the resultant is perpendicular to the first force, then magnitude of the resultant = newton.
 (a) $4\sqrt{2}$ (b) $4\sqrt{3}$ (c) 4 (d) $4\sqrt{5}$
- 8** ABCDEF is regular hexagon, then forces of magnitudes 4, $8\sqrt{3}$, $4\sqrt{3}$, 8 newton act at a point A in directions \overrightarrow{AB} , \overrightarrow{AC} , \overrightarrow{AE} and \overrightarrow{AF} respectively, then the magnitude of their resultant = newton.
 (a) 12 (b) $12\sqrt{3}$ (c) 576 (d) 24
- 9** The minimum value of resultant of two forces of magnitude 5, 9 newton and meeting at a point equals newton.
 (a) zero (b) 9 (c) 4 (d) 5
- 10** A body of weight 18 newton is placed on a smooth plane inclines to the horizontal by an angle of measure 30° , then body kept in equilibrium by a horizontal force of magnitude F newton, then F = newton.
 (a) $6\sqrt{3}$ (b) $12\sqrt{3}$ (c) 6 (d) 18
- 11** If $\vec{F}_1 = 5\vec{i} - 3\vec{j}$, $\vec{F}_2 = -7\vec{i} + 2\vec{j}$, $\vec{F}_3 = 2\vec{i} + \vec{j}$, then $\vec{R} =$
 (a) $7\vec{i} - 2\vec{j}$ (b) $4\vec{i} - 4\vec{j}$ (c) $-14\vec{i} + 4\vec{j}$ (d) \vec{O}
- 12** All of the following cases determine a plane except
 (a) a straight line and a point not belong to it.
 (b) two different parallel straight lines.
 (c) two intersected straight lines. (d) two skew straight lines.
- 13** The centre of the circle : $x^2 + y^2 - 6x + 8y = 0$ is the point
 (a) (3, -4) (b) (-4, 3) (c) (-3, 4) (d) (-3, -4)
- 14** If the equation $(x^2 + y^2 - 25)\left(\frac{x}{y} - 4\right) = 0$ represents a circle, then the length of its diameter = length unit..
 (a) 10 (b) 20 (c) 100 (d) 200

- 15** The volume of the right cone , the circumference of its base is 44 cm. and its height is 15 cm. = cm^3 ($\pi = \frac{22}{7}$)
 (a) 77 (b) 105 (c) 110 (d) 770
- 16** The volume of the regular quadrilateral pyramid , where the perimeter of its base = 36 cm. and its height 10 cm. equals cm^3
 (a) 810 (b) 180 (c) 360 (d) 270
- 17** The two straight lines are skew if they are
 (a) not parallel. (b) not intersecting.
 (c) not coincident. (d) not contained in the same plane.
- 18** If the length of the base side of a regular quadrilateral pyramid is doubled , then its volume =
 (a) will doubled. (b) will be three times. (c) will be four times. (d) will not change.
- 19** The length of the tangent segment which drawn of the circle : $x^2 + y^2 = r^2$ from the point $(0, 2r)$ is
 (a) r (b) $2r$ (c) $\sqrt{3}r$ (d) $\frac{\sqrt{3}}{2}r$
- 20** The lateral surface area of a right circular cone , radius length of its base = 6 cm. , and its height = 8 cm. equals cm^2
 (a) 60π (b) 28π (c) 10π (d) 48π
- 21** The two circles : $x^2 + y^2 - 2x + 6y + 1 = 0$, $4x^2 + 4y^2 - 8x + 24y - 60 = 0$ are
 (a) touching externally. (b) touching internally.
 (c) concentric. (d) distant.

Model

2

Answer the following questions :

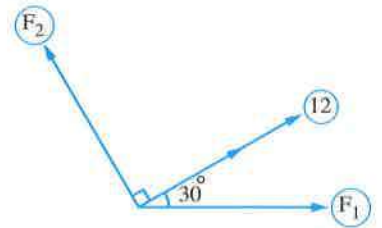
Choose the correct answer :

- 1** The centre of the circle $x^2 + y^2 - 6x + 8y = 0$ is the point
 (a) $(3, -4)$ (b) $(4, -3)$ (c) $(-3, 4)$ (d) $(-4, 3)$

2 In the opposite figure :

The force of magnitude 12 newton is resolved into two components \vec{F}_1, \vec{F}_2 make angles of measures $30^\circ, 90^\circ$, then $F_2 = \dots\dots\dots$ newton.

- (a) 10 (b) $10\sqrt{3}$
(c) $6\sqrt{3}$ (d) $4\sqrt{3}$

**3 If the straight line $L \parallel$ the plane $X, A \in X$, then $L \cap X = \dots\dots\dots$**

- (a) \emptyset (b) L (c) $\{A\}$ (d) X

4 If the three coplanar forces $\vec{F}_1 = 5\hat{i} + 3\hat{j}$, $\vec{F}_2 = a\hat{i} + 6\hat{j}$, $\vec{F}_3 = -14\hat{i} + b\hat{j}$ act at a point and their resultant $\vec{R} = (10\sqrt{2}, \frac{3}{4}\pi)$, then $a + b = \dots\dots\dots$

- (a) -1 (b) 1 (c) zero (d) 14

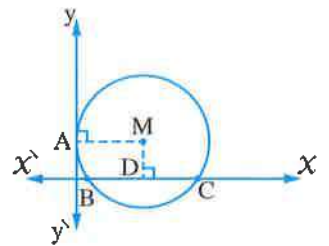
5 Two forces are equal in magnitude and each of them equal F newton if the magnitude of the resultant is F newton, then the measure of the included angle = $\dots\dots\dots$

- (a) 0° (b) 30° (c) 60° (d) 120°

6 In the opposite figure :

If $B(2, 0), C(8, 0)$, then the equation of the circle is $\dots\dots\dots$

- (a) $(x - 5)^2 + (y - 4)^2 = 25$
(b) $(x + 5)^2 + (y - 4)^2 = 36$
(c) $(x - 5)^2 + (y - 4)^2 = 36$
(d) $(x + 5)^2 + (y - 4)^2 = 25$

**7 Two forces of magnitudes 3, 4 newton their resultant is 7 newton, then the measure of the angle between them is $\dots\dots\dots$**

- (a) zero (b) 60° (c) 180° (d) 90°

8 Number of the planes which passes through two given points is $\dots\dots\dots$

- (a) zero (b) 1 (c) 2 (d) infinite.

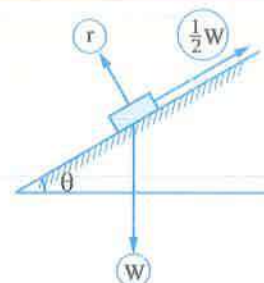
- 9 Two forces of magnitudes 4, F newton act at a particle, the measure of included angle is 120° , if line action of the resultant is perpendicular to the first force, then magnitude of the resultant = newton.

(a) $4\sqrt{2}$ (b) $4\sqrt{3}$ (c) 4 (d) $4\sqrt{5}$

- 10 In the opposite figure :

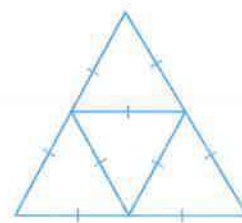
If the body is in equilibrium under acting of the shown forces, then $m(\angle \theta) = \dots\dots\dots$

(a) 30° (b) 60°
(c) 45° (d) 15°



- 11 Which solid, its net is the opposite figure ?

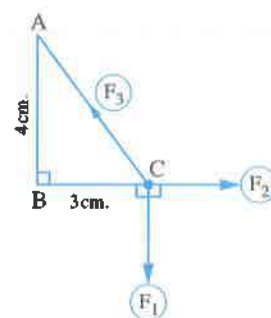
(a) Quadrilateral pyramid.
(b) Regular quadrilateral pyramid.
(c) Triangular pyramid with regular faces.
(d) Otherwise.



- 12 In the opposite figure :

A body is in equilibrium under the action of three forces meeting at a point of magnitudes F_1 , F_2 and F_3 newton, and the sides of the right-angled triangle are parallel to the lines of action of the forces in the same cyclic order, then $F_1 : F_2 : F_3 = \dots\dots\dots$

(a) 3 : 4 : 5 (b) 3 : 5 : 4 (c) 4 : 5 : 3 (d) 4 : 3 : 5



- 13 If the resultant of two forces acting at a point reaches the maximum value, then the measure of the angle between their line of actions equals

(a) 180° (b) 120° (c) 0° (d) 60°

- 14 The radius length of the base of a right circular cone = 5 cm. and its total surface area = $90\pi \text{ cm}^2$, then its volume = cm^3 .

(a) 105π (b) 95π (c) 100π (d) 120π

15 Which of the following system of forces can not be equilibrium ?

- (a) 10 newton , 10 newton , 5 newton. (b) 4 newton , 6 newton , 8 newton.
 (c) 11 newton , 7 newton , 8 newton. (d) 8 newton , 4 newton , 14 newton.

16 A regular quadrilateral pyramid. The perimeter of its base = 40 cm. and its height 12 cm. , then its lateral surface area = cm^2

- (a) 200 (b) 240 (c) 260 (d) 320

17 The volume of the regular quadrilateral pyramid , where the perimeter of its base = 36 cm. and its height 10 cm. is cm^3

- (a) 810 (b) 180 (c) 360 (d) 270

18 Two perpendicular forces of magnitudes $2F - 5$, $F + 2$ newton act at a particle , the magnitude of their resultant is $3\sqrt{5}$ newton , then $F =$

- (a) 2 (b) 3 (c) 4 (d) 5

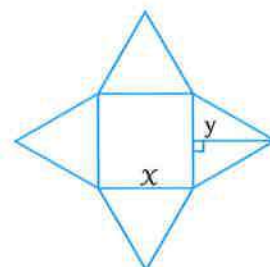
19 In the opposite figure :

Represents a regular quadrilateral pyramid its

height (h) , then the relation between

x , y and h is =

- (a) $x^2 + y^2 = h^2$ (b) $x^2 + h^2 = y^2$
 (c) $\left(\frac{x}{2}\right)^2 + h^2 = y^2$ (d) $\left(\frac{x}{2}\right)^2 + y^2 = h^2$



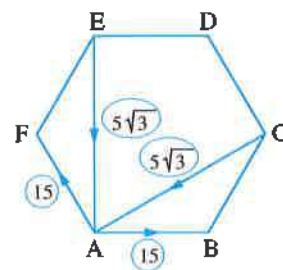
20 In the opposite figure :

ABCDEF is a regular hexagon , forces of magnitudes

15 , $5\sqrt{3}$, $5\sqrt{3}$ and 15 act along \overrightarrow{AB} , \overrightarrow{CA} , \overrightarrow{EA} and \overrightarrow{AF}

, then the magnitude of the resultant $R =$ newton.

- (a) 5 (b) 10
 (c) 25 (d) zero



21 The general form of the equation of the circle which its centre is $(-2, 5)$ and passes through $(3, 2)$ is

- (a) $x^2 + y^2 - 4x + 10y - 5 = 0$ (b) $x^2 + y^2 + 4x - 10y - 5 = 0$
 (c) $x^2 + y^2 + 2x - 5y - 5 = 0$ (d) $x^2 + y^2 + 4x - 10y - 25 = 0$

Model

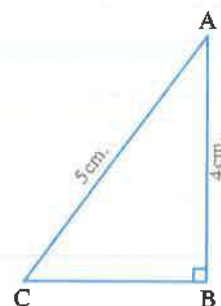
3

Answer the following questions : (Calculators are allowed)

Choose the correct answer :

- 1** If the triangle ABC is rotated about \overleftrightarrow{AB} with a full turn , then the volume of the formed solid = cm^2

(a) 6π (b) 8π
(c) 10π (d) 12π



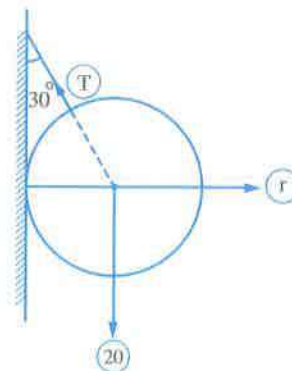
- 2** The volume of regular quadrilateral pyramid is 200 cm^3 and the length its height is 6 cm. , then its lateral surface area = cm^2

(a) $30\sqrt{61}$ (b) $20\sqrt{61}$ (c) $10\sqrt{61}$ (d) $40\sqrt{61}$

- 3** In the opposite figure :

A smooth sphere of weight 20 newton rests against a smooth vertical wall. It is suspended at a point on its surface by means of a string and the other end is fixed to the wall at a point lies directly above the point of tangency of the sphere and the wall , if the string makes with the vertical an angle of measure 30° , then in case of equilibrium $T : r = \dots\dots\dots$

(a) $2 : 1$ (b) $1 : 2$
(c) $\sqrt{3} : 1$ (d) $2 : \sqrt{3}$



- 4** A force of magnitude $10\sqrt{2}$ newton acts in the direction of East. it is resolved into two perpendicular components , one in the direction of eastern north , then the components of the force in the perpendicular direction is newton.

(a) 10 (b) 20 (c) $10\sqrt{3}$ (d) $10\sqrt{2}$

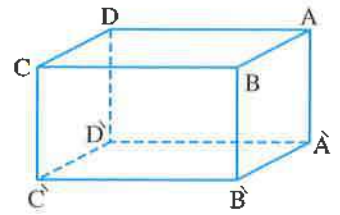
- 5** Two forces $\vec{F}_1 = 3\hat{i} + b\hat{j}$ and $\vec{F}_2 = a\hat{i} + 2\hat{j}$ act at a particle and they are in equilibrium , then $a + b = \dots\dots\dots$

(a) 6 (b) -5 (c) 5 (d) zero

6 In the opposite figure :

All the following expressions are true except

- (a) the plane $\overleftrightarrow{ABB'A'} \cap \text{then plane } ABCD = \overleftrightarrow{AB}$
 (b) The two straight lines \overleftrightarrow{BC} and $\overleftrightarrow{DD'}$ are two skew straight lines.
 (c) $\overleftrightarrow{BB'} \cap \overleftrightarrow{AD} = \emptyset$ (d) $m(\angle ADC) > 90^\circ$

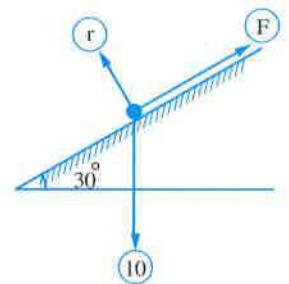
**7** If the equation of the circle is : $x^2 + y^2 - 4x - 6y + 4 = 0$, then each of the following is correct except

- (a) the centre of the circle is (2 , 3)
 (b) the circumference of the circle = 6π length unit.
 (c) the equation of the circle by translation of magnitude two units in the positive direction of x -axis is : $(x - 4)^2 + (y - 3)^2 = 9$
 (d) the equation of the diameter of the circle : $x + y = 7$

8 In the opposite figure :

The body is in equilibrium on the inclined plane , then all the following expressions are true except

- (a) the measure of the angle between the reaction of the plane (r) and the weight of the body = 150°
 (b) $F = 5\sqrt{3}$ newton. (c) $r = \sqrt{3} F$
 (d) The component of the weight in the direction of the plane downwards = 5 newton.

**9** If the magnitude of the resultant of two forces acting at a point reached to its maximum value , then the measure of the included angle between the two forces =

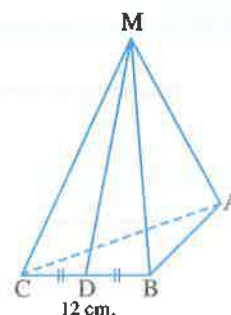
- (a) 90° (b) 180° (c) 0° (d) 30°

10 Two forces of magnitudes 3 and 5 newton and enclose between them an angle of measure 60° , then the magnitude of their resultant = newton.

- (a) 7 (b) 14 (c) 8 (d) 2

11 In the opposite figure :

A triangular pyramid with regular faces , the length of its edge is 12 cm. , D is the midpoint of \overline{BC} , then all the following statements are true except



- (a) $MD = 8\sqrt{3}$ cm.
- (b) the height of the pyramid $= 4\sqrt{6}$ cm.
- (c) the total area of the pyramid $= 144\sqrt{3}$ cm²
- (d) the plane $AMD \cap$ the plane $MBC = \emptyset$

12 If R is the resultant of two forces , $R \in [4, 12]$, then one of the following statements is not true

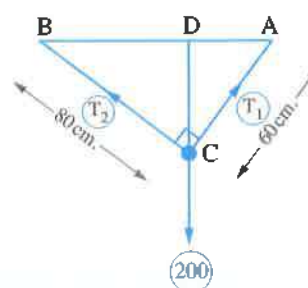
- (a) the resultant of two forces $= 4\sqrt{3}$ force unit when the angle between the two forces $= 120^\circ$
- (b) the resultant of two forces $= 4\sqrt{5}$ force unit when the two forces are perpendicular.
- (c) the resultant of two forces $= 12$ force unit when the angle between the two forces $= 180^\circ$
- (d) the resultant of two forces $= 4$ force unit when the angle between the two forces $= 180^\circ$

13 The least number of planes that can determine a solid is

- (a) 2 planes.
- (b) 3 planes.
- (c) 4 planes.
- (d) 5 planes.

14 In the opposite figure :

A body of weight 200 newton is in equilibrium by suspending it by two perpendicular strings , $AC = 60$ cm. , $CB = 80$ cm. , then all following statements is false except :

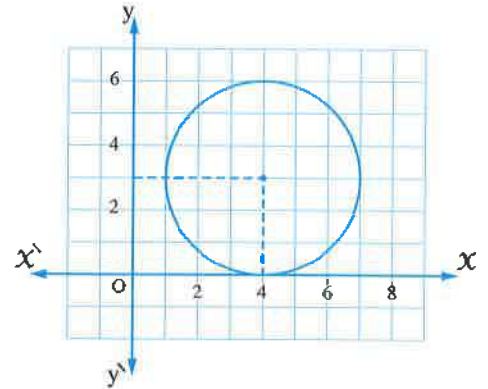


- (a) $AB = 200$ cm.
- (b) $T_1 = 120$ newton , $T_2 = 160$ newton
- (c) $T_1 = 160$ newton , $T_2 = 120$ newton.
- (d) The body can not be in equilibrium under the effect of these forces.

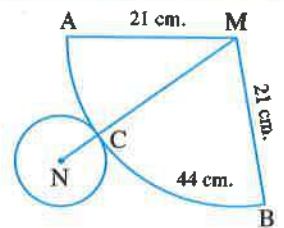
15 In the opposite figure :

The equation of the circle

- (a) $(x - 3)^2 + (y - 4)^2 = 9$
 (b) $(x - 4)^2 + (y - 3)^2 = 9$
 (c) $(x + 3)^2 + (y + 4)^2 = 9$
 (d) $(x + 4)^2 + (y + 3)^2 = 9$

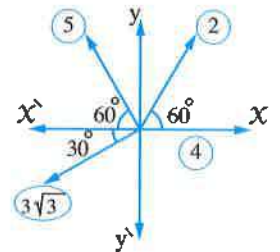
**16 The opposite figure shows a net of a right cone**, then its height = cm. ($\pi = \frac{22}{7}$)

- (a) 7
 (b) $14\sqrt{2}$
 (c) 14
 (d) $7\sqrt{3}$

**17 In the opposite figure :**

The true one from the following is

- (a) the sum of the algebraic components of the forces in direction $\overrightarrow{OX} = 2\hat{i}$
 (b) the sum of the algebraic components of the forces in direction $\overrightarrow{OY} = -2\sqrt{3}\hat{j}$
 (c) $\vec{R} = -2\hat{i} + 2\sqrt{3}\hat{j}$
 (d) the set of forces in equilibrium with the force $(4, 120^\circ)$

**18 The two points $(\frac{3}{5}, -\frac{4}{5})$ and $(-\frac{3}{5}, \frac{4}{5})$ are the ends of a diameter in a circle , then the false statement of the following statements is**

- (a) the equation of the circle is $x^2 + y^2 = 1$
 (b) the point $(1, 1)$ lies on the circle.
 (c) the straight line $x = 1$ touch the circle.
 (d) the point $(\frac{5}{13}, -\frac{12}{13})$ lies on the circle.

19 The equation of the circle M is $x^2 + y^2 = 4$ and the equation of the circle N is $(x - 3)^2 + y^2 = 9$ and , then

- (a) the two circles M and N are touching externally.
- (b) the two circles M and N are touching internally.
- (c) the two circles M and N are two distant circles.
- (d) the two circles M and N are two intersecting circles.

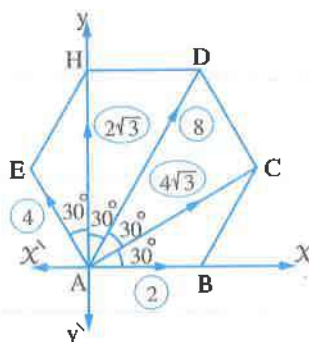
20 Two forces F and F , such that $R = F$, then the measure of the angle between the two forces =

- (a) 30°
- (b) 60°
- (c) 120°
- (d) 150°

21 In the opposite figure :

The false statement is

- (a) the sum of algebraic components of the forces in direction $\overrightarrow{OX} = 10$
- (b) the sum of algebraic components of the forces in direction $\overrightarrow{OY} = 10\sqrt{3}$
- (c) the resultant of the set of forces $= 20\sqrt{3}$
- (d) the resultant acts in the direction of \overrightarrow{AD}



Model

4

Answer the following questions :

Choose the correct answer :

1 The magnitudes of two forces acting on a particle are 5 , 8 newton , then the smallest value of their resultant = newton.

- (a) 2
- (b) 3
- (c) 7
- (d) 13

2 Two forces of magnitudes $3\sqrt{2}$ and 6 newton and the measure of the angle between them is 135° , then the measure of the angle between their resultant and the second forces is

- (a) 30°
- (b) 45°
- (c) 60°
- (d) 90°

- 3 Two equal forces, the magnitude of each is 6 newton, the magnitude of their resultant is 6 newton, then the angle between them is

(a) 30° (b) 60° (c) 120° (d) 150°

- 4 Two forces of equal of magnitudes of, enclosing between them an angle of measure $\frac{\pi}{2}$ if their resultant is 8 newton, then the value of each force is newton.

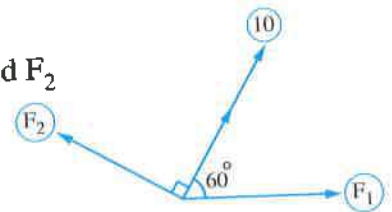
(a) $2\sqrt{2}$ (b) 4 (c) $4\sqrt{2}$ (d) 8

- 5 In the opposite figure :

If the force 10 newton is resolved into two components F_1 and F_2 inclined to forces by 60° and 90° respectively

, then $F_2 =$ newton.

(a) $5\sqrt{3}$ (b) 10 (c) $10\sqrt{3}$ (d) 20



- 6 A force of magnitude 6 newton acts in direction of North. It resolved into two perpendicular components, so its component in the direction of East = newton.

(a) zero (b) 3 (c) $3\sqrt{2}$ (d) 6

- 7 In the opposite figure :

The force \vec{F} is the resultant of the two forces \vec{F}_1 and \vec{F}_2

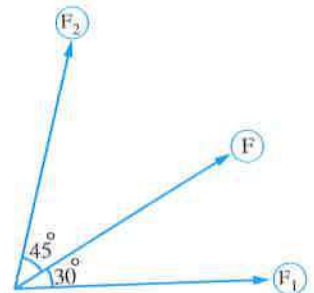
, then $\frac{F_1 + F_2}{F} =$

(a) $\sin 30^\circ + \sin 45^\circ$

(b) $\frac{\sin 75^\circ + \sin 30^\circ}{\sin 75^\circ}$

(c) $\frac{\sin 45^\circ + \sin 30^\circ}{\sin 75^\circ}$

(d) $\frac{\sin 75^\circ}{\sin 30^\circ} + \frac{\sin 75^\circ}{\sin 45^\circ}$



- 8 If $\vec{F}_1 = 5\vec{i}$, $\vec{F}_2 = 7\vec{i} - 5\vec{j}$, then $\|\vec{R}\| =$ force unit.

(a) 5 (b) $\sqrt{73}$ (c) 12 (d) 13

- 9 If three forces meeting at a point and acting up on a particle are in equilibrium, then the magnitude of each force is proportional to the of the included angle between the other two forces.

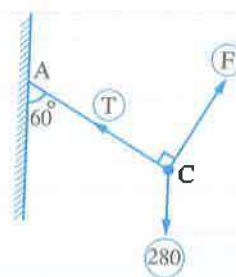
(a) tangent (b) sin (c) cos (d) cotangent.

- 10** A lamp of weight 280 gm.wt. is attached to the end of a string.

It is in equilibrium under the effect of a force perpendicular to the string when it is inclined to the vertical by an angle of

measure 60° , then $\frac{F}{T} = \dots\dots\dots$

- (a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\sqrt{3}$ (d) 2



- 11** If a body of magnitude (F) is in equilibrium with two forces of magnitudes 5 and 3 newton and the measure of the angle between them is 60° , then $F = \dots\dots\dots$ newton.

- (a) $\sqrt{19}$ (b) $\sqrt{34}$ (c) 7 (d) 15

- 12** The radius length of the base of a right circular cone = 5 cm. and its total surface area = $90\pi \text{ cm}^2$, then its volume is $\dots\dots\dots \text{cm}^3$.

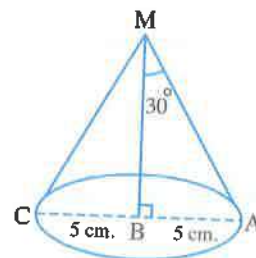
- (a) 95π (b) 100π (c) 105π (d) 120π

- 13** In the opposite figure :

A right circular cone in which $m(\angle AMB) = 30^\circ$
the radius length of the base = 5 cm.

, then its total area = $\dots\dots\dots \text{cm}^2$

- (a) 50π (b) 75π
(c) 100π (d) 125π



- 14** The volume of a regular quadrilateral pyramid, where the perimeter of its base is 36 cm. and its height is 10 cm. is $\dots\dots\dots \text{cm}^3$.

- (a) 180 (b) 270 (c) 360 (d) 810

- 15** A regular quadrilateral pyramid, the perimeter of its base is 40 cm. and its height is 12 cm. , then its total surface area = $\dots\dots\dots \text{cm}^2$.

- (a) 200 (b) 240 (c) 260 (d) 360

- 16** The circumference of the circle which its equation is $x^2 + y^2 = 8$ is $\dots\dots\dots$

- (a) 8π (b) 64π (c) $2\sqrt{2}\pi$ (d) $4\sqrt{2}\pi$

- 17** If the straight line $L \parallel$ the plane X , $A \in X$, then $L \cap X = \dots\dots\dots$
 (a) \emptyset (b) L (c) $\{A\}$ (d) X
- 18** The general form of the equation of a circle which its center is $(-2, 5)$ and passes through the point $(3, 2)$ is $\dots\dots\dots$
 (a) $x^2 + y^2 - 4x + 10y - 5 = 0$ (b) $x^2 + y^2 + 4x - 10y - 5 = 0$
 (c) $x^2 + y^2 + 2x - 5y - 5 = 0$ (d) $x^2 + y^2 + 4x - 10y - 25 = 0$
- 19** The equation of a circle which its center is $(-4, 3)$ and passes through the origin point $\dots\dots\dots$
 (a) $(x + 4)^2 + (y - 3)^2 = 5$ (b) $(x + 4)^2 + (y - 3)^2 = 625$
 (c) $(x + 4)^2 + (y - 3)^2 = 25$ (d) $(x - 4)^2 + (y + 3)^2 = 25$
- 20** If we cut a regular quadrilateral pyramid by a plane parallel to its base, then the resulting section is $\dots\dots\dots$
 (a) triangle. (b) square. (c) rectangle. (d) circle.
- 21** The two lines be skew if they are $\dots\dots\dots$
 (a) not parallel. (b) not intersecting.
 (c) not coincident. (d) not contained in the same plane.

Model**5****Answer the following questions :****Choose the correct answer from those given :**

- 1** If the resultant of two forces acting at a point reaches the maximum value, then the measure of the angle between their line of actions equals $\dots\dots\dots$
 (a) 180° (b) 120° (c) $zero^\circ$ (d) 60°
- 2** The height of a regular quadrilateral pyramid is 4 cm. and its volume 12 cm^3 , then the side length of its base = $\dots\dots\dots$
 (a) 1 (b) 2 (c) 3 (d) 4

3 In the opposite figure :

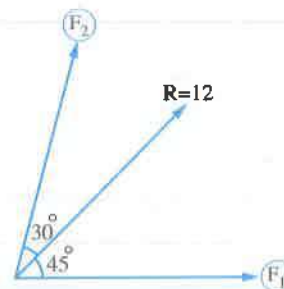
$F_1 = \dots\dots\dots$

(a) $12 \cos 75^\circ$

(b) $12 \cos 45^\circ$

(c) $6 \sec 45^\circ$

(d) $6 \csc 75^\circ$



4 In the opposite figure :

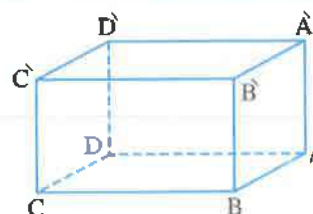
plane $A \hat{A} B \cap \text{plane } A \hat{C} C = \dots\dots\dots$

(a) \overrightarrow{AA}

(b) \overrightarrow{BB}

(c) \overrightarrow{CC}

(d) \overrightarrow{AC}



5 Two forces of magnitude 4 , F newton act at a point and measure of the angle between them 120° and their resultant is perpendicular on 1^{st} , then value of F = newton.

(a) 4

(b) 8

(c) 6

(d) 2

6 Right circular cone , length of its base radius is 6 cm. and its height 8 cm. , then its lateral area = cm^2

(a) 60π

(b) 28π

(c) 10π

(d) 48π

7 The circumference of the circle which its equation is : $(x - 3)^2 + (y + 2)^2 = 25$ equal length unit.

(a) 2π

(b) 3π

(c) 10π

(d) 25π

8 If the three coplanar forces $\vec{F}_1 = 5 \hat{i} + 3 \hat{j}$, $\vec{F}_2 = a \hat{i} + 6 \hat{j}$, $\vec{F}_3 = -14 \hat{i} + b \hat{j}$ act point and their resultant $\vec{R} = (10\sqrt{2} , \frac{3}{4} \pi)$, then $a + b = \dots\dots\dots$

(a) - 1

(b) 1

(c) zero

(d) 14

9 In the triangular pyramid of regular faces , if the sum of lengths of its edges is 18 cm. , the its total area = cm^2

(a) $\frac{27\sqrt{2}}{4}$

(b) $\frac{27\sqrt{3}}{4}$

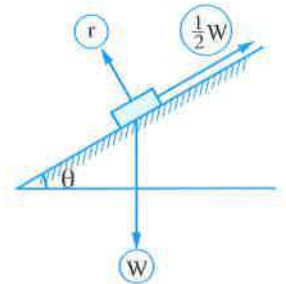
(c) $\frac{27\sqrt{3}}{2}$

(d) $9\sqrt{3}$

10 In the opposite figure :

If the body is in equilibrium under acting of the shown forces , then $m (\angle \theta) = \dots\dots\dots^\circ$

- (a) 30 (b) 60
(c) 45 (d) 15

**11** The volume of the right cone is $27 \pi \text{ cm}^3$, and the circumference of its base is $6 \pi \text{ cm}$, then its height is $\dots\dots\dots \text{ cm}$.

- (a) 27 (b) 18 (c) 9 (d) 6

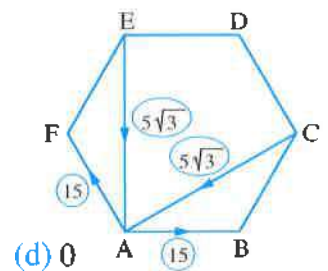
12 If the length of the base side of a regular quadrilateral pyramid is doubled , then its volume $\dots\dots\dots$

- (a) will doubled. (b) will not change.
(c) will be three times. (d) will be four times.

13 In the opposite figure :

ABCDEF is a regular hexagon , forces of magnitudes 15 , $5\sqrt{3}$, $5\sqrt{3}$ and 15 newton act along \overrightarrow{AB} , \overrightarrow{CA} , \overrightarrow{EA} and \overrightarrow{AF} , then the magnitude of the resultant $R = \dots\dots\dots$ newton.

- (a) 5 (b) 10 (c) 25

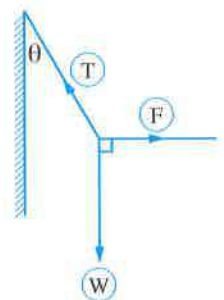
**14** If three forces meeting at a point and acting up on a particle are in equilibrium , then the magnitude of each force proportional to the $\dots\dots\dots$ of the included angle between the two others forces.

- (a) cosine (b) sine (c) tangent (d) cotangent

15 In the opposite figure :

A weight of magnitude W newton is suspended in one end of string and the other end of the string fixed in a point on a vertical wall , the weight is pulled by a horizontal force of magnitude F newton till the string become makes an angle θ with vertical which of the following statements is not correct in equilibrium state ?

- (a) $F = W \tan \theta$ (b) $\vec{W} + \vec{F} + \vec{T} = \vec{O}$ (c) $T^2 = F^2 + W^2$ (d) $T = F + W$



16 The volume of regular hexagon pyramid is $8\sqrt{3} \text{ cm}^3$. and its height is 4 cm.

, then perimeter of its base =

- (a) 2 (b) 12 (c) 6 (d) $6\sqrt{3}$

17 A force of magnitude $5\sqrt{3}$ newton act in direction 30° east of north is resolved into two perpendicular components in direction , then value of component in the east direction = newton.

- (a) 5 (b) 7.5 (c) $\frac{5\sqrt{3}}{2}$ (d) 15

18 The equation of the circle which its center $(-4, 4)$ and touches X-axis and y-axis is

- (a) $x^2 + y^2 + 8x - 8y + 16 = 0$ (b) $x^2 + y^2 = 16$
(c) $x^2 + y^2 - 8x + 8y + 16 = 0$ (d) $x^2 + y^2 = 8$

19 Two forces are equal act at a point and the measure of the angle between them is 90° and their resultant is 8 newton , then the magnitude of each is newton.

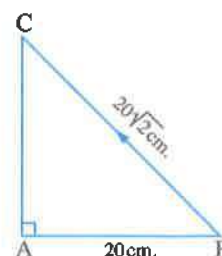
- (a) $2\sqrt{2}$ (b) 4 (c) $4\sqrt{2}$ (d) 8

20 Volume of regular triangular pyramid is 12 cm^3 and area of its base is 4 cm^2 , then its height =

- (a) 3 (b) 6 (c) 9 (d) 2

21 In the opposite figure :

\overline{AB} is a uniform rod with length 20 cm. and its weight = 30 newton attached by a smooth hinge fixed on a vertical wall in the end A and at the end B suspended by a light string with length $20\sqrt{2} \text{ cm}$. its other end fixed at point C on the wall above point A if the rod in equilibrium in the horizontal position , then the reaction of the hinge



- (a) action direction \overrightarrow{AB}
(b) its line of action distant 10 cm. from the wall.
(c) bisects \overline{BC} (d) is magnitude = 15 newton.

Guide answers of final models

Model 1

- 1 (b) 2 (c) 3 (a) 4 (c) 5 (d)

6

Let \vec{AB} in the direction of \vec{OX}

$$\therefore X = 2 \cos 0^\circ + 4\sqrt{3} \cos 30^\circ$$

$$+ 8 \cos 60^\circ$$

$$+ 2\sqrt{3} \cos 90^\circ$$

$$+ 4 \cos 120^\circ$$

$$= 2 \times 1 + 4\sqrt{3} \times \frac{\sqrt{3}}{2}$$

$$+ 8 \times \frac{1}{2} + 2\sqrt{3} \times 0 + 4 \times \frac{-1}{2} = 10$$

$$\therefore Y = 2 \sin 0^\circ + 4\sqrt{3} \sin 30^\circ + 8 \sin 60^\circ$$

$$+ 2\sqrt{3} \sin 90^\circ + 4 \sin 120^\circ$$

$$= 2 \times 0 + 4\sqrt{3} \times \frac{1}{2} + 8 \times \frac{\sqrt{3}}{2} + 2\sqrt{3} \times 1 + 4 \times \frac{\sqrt{3}}{2}$$

$$= 10\sqrt{3}$$

$$\therefore \vec{R} = 10\hat{i} + 10\sqrt{3}\hat{j}$$

$$\therefore R = \sqrt{(10)^2 + (10\sqrt{3})^2} = 20 \text{ kg.wt.}$$

$$\therefore \tan \theta = \frac{10\sqrt{3}}{10} = \sqrt{3} \quad \therefore \theta = 60^\circ$$

\therefore The magnitude of \vec{R} = 20 kg.wt. and makes an angle of measure 60° with \vec{OX}

7 (b)

8

(1) The area of the base = πr^2

$$\therefore 36\pi = \pi r^2$$

$$\therefore r = 6 \text{ cm.}$$

\therefore The lateral area = $\pi r L = \pi \times 6 \times 10 = 60\pi \text{ cm}^2$

(2) The total area = $\pi r (L + r) = \pi \times 6 (10 + 6)$

$$= 96\pi \text{ cm}^2$$

$$\therefore h = \sqrt{(10)^2 - (6)^2} = 8 \text{ cm.}$$

(3) Volume = $\frac{1}{3} \pi r^2 h = \frac{1}{3} \pi \times 6^2 \times 8 = 96\pi \text{ cm}^3$

9 (a)

10 (a)

11 (c)

12 (d)

13 (d)

14 (c)

15

Suppose that :

$$AB = 4l$$

$$\therefore CB = 2l$$

$$\therefore CE = l$$

$$\therefore AC = 2\sqrt{3}l$$

$$\text{In } \triangle ACE : AE = \sqrt{13}l$$

$$\therefore \triangle ACE \text{ is the } \Delta \text{ of forces}$$

$$\frac{20}{2\sqrt{3}l} = \frac{F}{\sqrt{13}l}$$

$$\therefore F = \frac{10\sqrt{3}}{3} \text{ kg.wt.}$$

$$R = \frac{10\sqrt{39}}{3} \text{ kg.wt.}$$

16 (b)

17 (d)

18 (a)

19 (d)

20 (b)

21 (d)

22 (d)

23 (d)

24 (d)

Model 2

1 (a) 2 (b) 3 (a) 4 (b) 5 (d)

6

$$R_1 = 2F \cos \frac{\alpha}{2} = 12$$

$$\therefore F \cos \frac{\alpha}{2} = 6$$

$$\therefore R_2 = 2F \cos \left(\frac{180^\circ - \alpha}{2} \right) = 6$$

$$\therefore F \sin \frac{\alpha}{2} = 3$$

squaring the two equations and adding them

$$\therefore F^2 \cos^2 \frac{\alpha}{2} + F^2 \sin^2 \frac{\alpha}{2} = 6^2 + 3^2$$

$$\therefore F^2 \left(\cos^2 \frac{\alpha}{2} + \sin^2 \frac{\alpha}{2} \right) = 45$$

$$\therefore F = \sqrt{45} = 3\sqrt{5} \text{ kg.wt.}$$

7 (c)

8 (a)

9 (d)

10 (c)

11 (b)

12

(1) In $\triangle MNE$:

$$MN = \sqrt{(25)^2 - (20)^2} = 15 \text{ cm.}$$

$$\therefore \text{height} = 15 \text{ cm.}$$

(2) The lateral area

$$= \frac{1}{2} \times \text{base perimeter}$$

$$\times \text{slant height}$$

$$= \frac{1}{2} \times (4 \times 40) \times 25$$

$$= 2000 \text{ cm}^2$$

(3) The total area = $2000 + (40)^2 = 3600 \text{ cm}^2$

(4) The volume = $\frac{1}{3} \times (40)^2 \times 15 = 8000 \text{ cm}^3$

13 (d)

14 (c)

15 (c)

16

Length of the arc \widehat{AB}

$$= MB \cdot \theta' = 36 \times \frac{210}{180} \pi = 42\pi \text{ cm.}$$

Circumference of the base

$$\text{of the cone} = 42\pi$$

$$\therefore 2\pi r = 42\pi$$

$$\therefore r = 21$$

$$\therefore h = \sqrt{36^2 - 21^2} = 3\sqrt{95} \text{ cm.}$$

$$\approx 29.2 \text{ cm.}$$

$$\therefore h = \sqrt{36^2 - 21^2} = 3\sqrt{95} \text{ cm.}$$

$$\approx 29.2 \text{ cm.}$$

17 (a)

18 (a)

19 (b)

20 (a)

21 (c)

22 (b)

23 (a)

24 (a)

Model 3

1 (c)

2 (a)

3 (c)

4 (b)

5

$$\therefore (60)^2 + (80)^2 = (100)^2$$

$\therefore \triangle ACB$ is right-angled at C

From Lami's rule

$$\therefore \frac{200}{\sin 90^\circ} = \frac{T_1}{\sin \theta_1} = \frac{T_2}{\sin \theta_2}$$

$$\therefore \sin \theta_1 = \frac{BC}{AB} = \frac{80}{100} = \frac{4}{5}$$

$$\therefore \sin \theta_2 = \frac{AC}{AB} = \frac{60}{100} = \frac{3}{5}$$

$$\therefore \frac{200}{1} = \frac{T_1}{\frac{4}{5}} = \frac{T_2}{\frac{3}{5}}$$

$$\therefore T_1 = 200 \times \frac{4}{5} = 160 \text{ gm.wt.}$$

$$\therefore T_2 = 200 \times \frac{3}{5} = 120 \text{ gm.wt.}$$

6 (d)

7 (c)

8 (c)

9 (b)

10

Suppose \vec{OX} is the direction

of the first force

$$X = 8 \cos 0^\circ + 4\sqrt{3} \cos 30^\circ$$

$$+ 6\sqrt{3} \cos 150^\circ + 14 \cos 240^\circ$$

$$= 8 \times 1 + 4\sqrt{3} \times \frac{\sqrt{3}}{2} + 6\sqrt{3} \times \left(-\frac{\sqrt{3}}{2} \right) + 14 \times \left(-\frac{1}{2} \right)$$

$$= 8 + 6 - 9 - 7 = -2$$

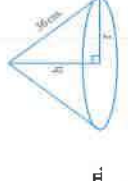
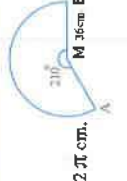
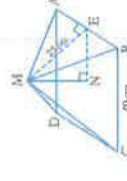
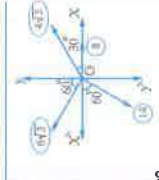
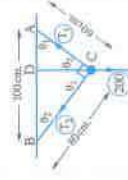
$$\therefore Y = 8 \sin 0^\circ + 4\sqrt{3} \sin 30^\circ$$

$$+ 6\sqrt{3} \sin 150^\circ + 14 \sin 240^\circ$$

$$= 8 \times 0 + 4\sqrt{3} \times \frac{1}{2} + 6\sqrt{3} \times \frac{1}{2} + 14 \times \left(-\frac{\sqrt{3}}{2} \right)$$

$$= -2\sqrt{3}$$

$$\therefore \vec{R} = -2\hat{i} - 2\sqrt{3}\hat{j}$$



9 (d)

10

∴ The volume of the pyramid = 1296

$$\therefore \frac{1}{3} \times (18)^2 \times h = 1296$$

$$\therefore h = 12 \text{ cm.}$$

$$\therefore \text{Slant height} = \sqrt{h^2 + \left(\frac{1}{2} \text{ side}\right)^2}$$

$$= \sqrt{12^2 + \left(\frac{18}{2}\right)^2} = 15 \text{ cm.}$$

$$\therefore \text{Lateral surface area} = \frac{1}{2} \text{ perimeter of the base} \times \text{S.h.}$$

$$= \frac{1}{2} \times (4 \times 18) \times 15 = 540 \text{ cm}^2$$

11 (a)

12 (d)

13 (c)

14 (c)

15 (c)

16

The solid which generated by rotation around BC is two cones have common base and are congruent.

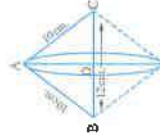
$$\therefore AD = \sqrt{10^2 - 6^2} = 8 \text{ cm.}$$

∴ The volume of the result solid

$$= 2 \left(\frac{1}{3} \pi r^2 h \right)$$

$$= 2 \times \frac{1}{3} \times \pi \times 8^2 \times 6$$

$$= 256 \pi \text{ cm}^2$$



17 (a)

18 (c)

19 (c)

20 (c)

21 (a)

22 (b)

23 (c)

24 (c)

Model 7

1 (c)

2 (a)

3 (a)

4 (b)

5 (c)

6 (b)

7 (a)

8

∴ Lateral surface area = 240

$$\therefore \frac{1}{2} (\text{perimeter of the base}) \times \text{slant height} = 240$$

If l is side length of the base.

$$\therefore \frac{1}{2} \times 4 \times l \times 12 = 240$$

$$\therefore l = 10 \text{ cm.}$$

(1) ∴ Height of pyramid

$$= \sqrt{12^2 - 5^2} = \sqrt{119} \text{ cm.}$$

(2) Volume of pyramid = $\frac{1}{3} \times \text{area of base} \times h$

$$= \frac{1}{3} \times (10)^2 \times \sqrt{119} = \frac{100}{3} \sqrt{119} \approx 363.6 \text{ cm}^3$$



9 (c)

10

∴ The two planes are smooth

∴ r_1 and r_2 are

perpendicular to the two

planes and passes through

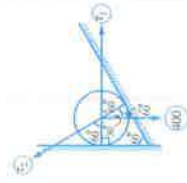
the centre of the sphere.

Applying lami's rule

$$\therefore \frac{r_1}{\sin 150^\circ} = \frac{r_2}{\sin 90^\circ} = \frac{400}{\sin 120^\circ}$$

$$\therefore r_1 = \frac{400 \times \sin 150^\circ}{\sin 120^\circ} = \frac{400 \sqrt{3}}{3} \text{ kg wt.}$$

$$\therefore r_2 = \frac{400 \times \sin 90^\circ}{\sin 120^\circ} = \frac{800 \sqrt{3}}{3} \text{ kg wt.}$$



11 (d)

12 (b)

13

∴ ΔAHB is right-angled at B

$$\therefore AH = \sqrt{(5)^2 + (12)^2} = 13 \text{ cm.}$$

$$\therefore \sin (\angle BAH) = \frac{5}{13}$$

$$\therefore \cos (\angle BAH) = \frac{12}{13}$$

∴ AC is a diagonal of

the square ABCD

$$\therefore \alpha = 45^\circ$$



$$\therefore X = 2 \cos 0^\circ + 13 \cos (\angle BAH) + 9 \cos 90^\circ$$

$$+ 4\sqrt{2} \cos 225^\circ$$

$$= 2 \times 1 + 13 \times \frac{12}{13} + 9 \times 0 + 4\sqrt{2} \times \frac{-1}{\sqrt{2}} = 10$$

$$\therefore Y = 2 \sin 0^\circ + 13 \sin (\angle BAH) + 9 \sin 90^\circ$$

$$+ 4\sqrt{2} \sin 225^\circ$$

$$= 2 \times 0 + 13 \times \frac{5}{13} + 9 \times 1 + 4\sqrt{2} \times \frac{-1}{\sqrt{2}} = 10$$

$$\therefore \vec{R} = 10\hat{i} + 10\hat{j}$$

$$\therefore R = \sqrt{(10)^2 + (10)^2} = 10\sqrt{2} \text{ gm.wt.}$$

$$\therefore \tan \theta = \frac{10}{10} = 1 \quad \therefore X > 0, Y > 0 \quad \therefore \theta = 45^\circ$$

∴ \vec{R} acts due to \vec{AC}

14 (c)

15 (b)

16 (d)

17 (b)

19 (c)

20 (c)

23 (d)

18 (b)

21 (c)

22 (b)

Model 8

1 (c)

2 (d)

3 (d)

4 (b)

5 (d)

∴ Volume of the pyramid = $\frac{1}{3} \times \text{base area} \times \text{height}$

$$\therefore 8\sqrt{3} = \frac{1}{3} \times \text{base area} \times 4$$

$$\therefore \text{Base area} = 6\sqrt{3} \text{ cm}^2$$

$$\therefore \frac{6}{4} \times X^2 \times \cot \frac{\pi}{6} = 6\sqrt{3}$$

$$\therefore X^2 = 4$$

$$\therefore X = 2$$

∴ Side length of the hexagon = 2 cm.

∴ Base perimeter = $6 \times 2 = 12 \text{ cm.}$

7 (c)

8 (a)

9 (d)

10 (d)

11

$$\therefore X = F \cos 0^\circ + 80 \cos 60^\circ$$

$$+ K \cos 90^\circ + 50 \cos 180^\circ$$

$$+ 80\sqrt{3} \cos 270^\circ$$

$$= F \times 1 + 80 \times \frac{1}{2} + K \times 0$$

$$+ 50 \times -1 + 80\sqrt{3} \times 0$$

$$= F - 10$$

$$\therefore Y = F \sin 0^\circ + 80 \sin 60^\circ + K \sin 90^\circ + 50 \sin 180^\circ$$

$$+ 80\sqrt{3} \sin 270^\circ$$

$$= F \times 0 + 80 \times \frac{\sqrt{3}}{2} + K \times 1 + 50 \times 0$$

$$+ 80\sqrt{3} \times -1 = K - 40\sqrt{3}$$

$$\therefore \vec{R} = (F - 10)\hat{i} + (K - 40\sqrt{3})\hat{j}$$

$$\therefore R = 40 \text{ newton due to } 60^\circ \text{ North of East}$$

$$\therefore \vec{R} = 40 \cos 60^\circ \hat{i} + 40 \sin 60^\circ \hat{j}$$

$$= 20\hat{i} + 20\sqrt{3}\hat{j}$$

From (1) and (2) : $\therefore F - 10 = 20$

$$\therefore F = 30 \text{ newton}$$

$$\therefore K - 40\sqrt{3} = 20\sqrt{3}$$

$$\therefore K = 60\sqrt{3} \text{ newton}$$



12 (d)

13 (a)

14 (b)

15 (b)

1E

From circle C_1 :

$$r_1 = \sqrt{k} \rightarrow P_1 (-2, -1) \text{ is its centre}$$

From circle C_2 :

$$r_2 = \sqrt{16} = 4 \rightarrow P_2 (3, 1) \text{ is its centre}$$

The distance between the two centres

$$P_1 P_2 = \sqrt{(3+2)^2 + (1+1)^2} = 13 \text{ length unit.}$$

In case the two circles are touching externally

$$\therefore P_1 P_2 = r_1 + r_2 \quad \therefore 13 = \sqrt{k} + 4$$

$$\therefore \sqrt{k} = 13 - 4 = 9 \quad \therefore k = 9^2 = 81$$

In case the two circles are touching internally

$$\therefore P_1 P_2 = |r_1 - r_2| \quad \therefore 13 = |\sqrt{k} - 4|$$

$$\therefore 13 = \sqrt{k} - 4 \quad \text{i.e. } \sqrt{k} = 13 + 4 = 17$$

then $k = 289$

$$\text{or } -13 = \sqrt{k} - 4 \quad \text{i.e. } \sqrt{k} = -13 + 4 = -9 \text{ refused}$$

$$\therefore k = 81 \quad \text{or } 289$$

17 (d)

18 (d)

19 (a)

20 (d)

21 (b)

22 (d)

23 (b)

24 (b)

Model 9

1 (c)

2 (c)

3 (b)

4 (d)

5 (a)

6 (c)

7 (d)

8 (c)

9

$$\therefore AC = AD = 4 \text{ metres}$$

$$\therefore m(\angle ACD) = 45^\circ$$

$$\therefore CD = 4\sqrt{2} \text{ metres}$$

From $\triangle MCN$:

$$MN = 1 \text{ metre, } m(\angle NCM) = 45^\circ$$

$$\therefore m(\angle NMC) = 90^\circ$$

$$\therefore NC = \sqrt{2} \text{ metres}$$

$$\therefore DN = 4\sqrt{2} - \sqrt{2} = 3\sqrt{2} \text{ metres}$$

From $\triangle AMN$: $AN = \sqrt{10}$ metres (Pythagoras)

\therefore Since $\triangle AND$ is the triangle of forces

$$\therefore \frac{T}{\sqrt{10}} = \frac{T}{3\sqrt{2}} = \frac{8}{4}$$

$$\therefore T = 2\sqrt{10} \text{ kg wt.}, \therefore T = 6\sqrt{2} \text{ kg wt.}$$

10 (d)

11 (b)

12

The length of the drawer of the cone = 18 cm.

\therefore the circumference of circle

of the cone base = the length of \widehat{AB}

$$= r \times \theta^{\text{rad}} = 18 \times \frac{60^\circ \times \pi}{180^\circ} = 6\pi$$

$$\therefore 2\pi r = 6\pi \quad \therefore r = 3 \text{ cm.}$$

$$\therefore h = \sqrt{r^2 - r'^2} = \sqrt{(18)^2 - 3^2} = 3\sqrt{35}$$

$$\therefore \text{Volume of the cone} = \frac{1}{3} \pi r^2 h$$

$$= \frac{1}{3} \times \pi \times (3)^2 \times 3\sqrt{35}$$

$$= 167.3 \text{ cm}^3$$

13 (c)

14

$$X = 10 \cos 0^\circ + 20 \cos 120^\circ$$

$$+ 30 \cos 240^\circ$$

$$= 10 + 20 \times \frac{-1}{2} + 30 \times \frac{-1}{2} = -15$$

$$Y = 10 \sin 0^\circ + 20 \sin 120^\circ$$

$$+ 30 \sin 240^\circ$$

$$= 0 + 20 \times \frac{\sqrt{3}}{2} + 30 \times \frac{-\sqrt{3}}{2} = -5\sqrt{3}$$

$$\therefore \vec{R} = -15\hat{i} - 5\sqrt{3}\hat{j}$$

$$\therefore R = \sqrt{(-15)^2 + (-5\sqrt{3})^2} = 10\sqrt{3} \text{ newton.}$$

$$\tan \theta = \frac{-5\sqrt{3}}{-15} = \frac{\sqrt{3}}{3}$$

$$\therefore \theta = 210^\circ$$

\therefore Line of action of the resultant in direction 30° south of west.

15 (b)

16 (c)

17 (d)

18 (d)

19 (a)

20 (b)

21 (c)

22 (a)

23 (b)

24 (b)

Model 10

1 (d)

2 (a)

3 (c)

4 (a)

5 (a)

6

The slant height = $\sqrt{(10\sqrt{3})^2 + (10)^2} = 20 \text{ cm.}$

(1) Lateral surface area

$$= \frac{1}{2} (\text{perimeter of the base})$$

$$\times \text{slant height}$$

$$= \frac{1}{2} \times (4 \times 20) \times 20$$

$$= 800 \text{ cm}^2$$

$$(2) \text{ Volume} = \frac{1}{3} \text{ area of the base} \times h$$

$$= \frac{1}{3} \times (20)^2 \times 10\sqrt{3} = \frac{4000\sqrt{3}}{3} \text{ cm}^3$$

7 (b)

8 (b)

9 (a)

10 (c)

11 (d)

12

$$\therefore \text{Volume of the cone} = \frac{1}{3} \times \pi (2)^2 \times 12 = 16\pi \text{ cm}^3$$

$$\therefore \text{Volume of the dislodges water in the cylinder}$$

$$= 16\pi \text{ cm}^3$$

$$\therefore 16\pi = \pi r^2 \times 1 \quad \therefore r^2 = 16$$

$$\therefore r = 4 \text{ cm.}$$

The length of the diameter of the base of the vessel = $4 \times 2 = 8 \text{ cm.}$

13 (b)

14

$$X = 12 \cos 0^\circ + 9 \cos 90^\circ$$

$$+ 5\sqrt{2} \cos 135^\circ + 7\sqrt{2} \cos 225^\circ + 7 \cos 270^\circ$$

$$= 12 + 0 - 5 - 7 + 0 = 0$$

$$\therefore Y = 12 \sin 0^\circ + 9 \sin 90^\circ$$

$$+ 5\sqrt{2} \sin 135^\circ$$

$$+ 7\sqrt{2} \sin 225^\circ + 7 \sin 270^\circ$$

$$= 0 + 9 + 5 - 7 - 7 = 0$$

$$\therefore \vec{R} = \vec{0}$$

\therefore The system is in equilibrium.

15 (a)

16 (a)

17 (d)

18 (d)

19 (d)

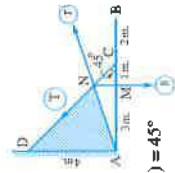
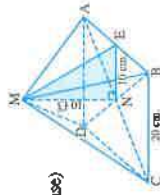
20 (a)

21 (c)

22 (c)

23 (b)

24 (c)



Guide answers of multiple choice examinations

Model 1

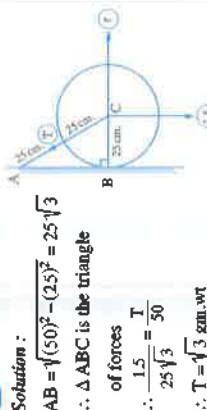
1 (a)



2 (c)

Solution :
Component = $5\sqrt{3} \sin 30^\circ$
 $= \frac{5\sqrt{3}}{2}$

3 (a)



4 (c)

Solution :
 $\vec{R} = (10\sqrt{2} \cos \frac{3\pi}{4}, 10\sqrt{2} \sin \frac{3\pi}{4}) = (-10, 10)$
 $\vec{R} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = (-9 + a, 9 + b)$
 $\therefore -9 + a = -10 \quad \therefore a = -1$
 $9 + b = 10 \quad \therefore b = 1 \quad \therefore a + b = \text{zero}$

5 (b)

Solution :
 $R = \sqrt{5^2 + 3^2 + 2 \times 5 \times 3 \cos 60^\circ} = 7 \text{ newton.}$

6 (b)

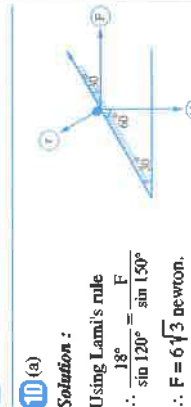
7 (b)

Solution :
 \vec{R} is perpendicular to the force of magnitude 4 N
 $\therefore 4 + F \cos 120^\circ = 0 \quad \therefore F = 8 \text{ newton}$
 $\therefore R = \sqrt{4^2 + 8^2 + 2 \times 4 \times 8 \times \cos 120^\circ}$
 $= 4\sqrt{3} \text{ newton.}$

8 (d)

Solution :
Let \vec{AB} in direction of \vec{OX}
 $\therefore X = 4 \cos 0^\circ + 8\sqrt{3} \cos 30^\circ$
 $+ 4\sqrt{3} \cos 90^\circ$
 $+ 8 \cos 120^\circ = 12$
 $Y = 4 \sin 0^\circ + 8\sqrt{3} \sin 30^\circ$
 $+ 4\sqrt{3} \sin 90^\circ$
 $+ 8 \sin 120^\circ = 12\sqrt{3}$
 $\therefore \vec{R} = (12, 12\sqrt{3})$
 $\therefore |\vec{R}| = \sqrt{(12)^2 + (12\sqrt{3})^2} = 24 \text{ newton.}$

9 (c)



10 (a)

Solution :
Using Lami's rule
 $\therefore \frac{18^\circ}{\sin 120^\circ} = \frac{F}{\sin 150^\circ}$
 $\therefore F = 6\sqrt{3} \text{ newton.}$

11 (d)

Solution :
 $\vec{R} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = (0, 0) = \vec{O}$

12 (d)

Solution :
 $(x \ y \ 25) \begin{pmatrix} x \\ y \end{pmatrix} = (x^2 + y^2 - 100) = 0$
 $\therefore x^2 + y^2 = 100$
 $\therefore r = 10 \rightarrow \text{diameter length} = 20 \text{ length unit.}$

15 (d)

Solution :
side length of the base = $36 \div 4 = 9$
 $\therefore \text{base area} = 9 \times 9 = 81 \text{ cm}^2$
 $\therefore v = \frac{1}{3} \times 81 \times 10 = 270 \text{ cm}^3$

17 (d)

Solution :
 $v = \frac{1}{3} \ell^2 h, \therefore h = \frac{1}{3} (2 \ell^2 h) = 4 \left(\frac{1}{3} \ell^2 h \right) = 4 v$

18 (c)



20 (a)

Solution :
drawn = $\sqrt{8^2 + 6^2} = 10 \text{ cm.}$
 $\therefore \text{lateral surface area} = \pi \times 6 \times 10 = 60 \pi$

21 (c)

Solution :
with respect to the first circle
 $M_1 = (1, -3)$
with respect to the second circle
 $\therefore 4x^2 + 4y^2 - 8x + 24y - 60 = 0$
 $\therefore x^2 + y^2 - 2x + 6y - 15 = 0$
 $M_2 = (1, -3)$
 $\therefore \text{The two circles are concentric}$

Model 2

1 (a)

Solution :
 $F_2 = \frac{12 \sin 30^\circ}{\sin (30^\circ + 90^\circ)} = 4\sqrt{3} \text{ newton.}$

2 (d)

Solution :
 $\sin 90^\circ = \frac{\sin (180^\circ - \theta)}{\sin \theta}$
 $\therefore \sin \theta = \frac{1}{2} \quad \therefore \theta = 30^\circ$

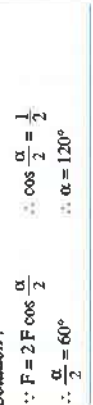
3 (a)

Solution :
 $\vec{R} = (10\sqrt{2} \cos \frac{3\pi}{4}, 10\sqrt{2} \sin \frac{3\pi}{4})$
 $= (-10, 10)$
 $\therefore \vec{R} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = (a - 9, 9 + b)$
 $\therefore a - 9 = -10 \quad \therefore a = -1$
 $9 + b = 10 \quad \therefore b = 1$
 $\therefore a + b = -1 + 1 = 0$

4 (c)

Solution :
 $F = 2F \cos \frac{\alpha}{2}$
 $\therefore \frac{\alpha}{2} = 60^\circ \quad \therefore \alpha = 120^\circ$

5 (d)



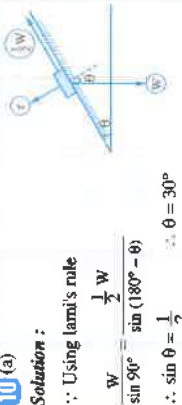
6 (a)

Solution :
 $\therefore D$ is the midpoint of \vec{BC}
 $D = (5, 0) \quad \therefore r = 5, M = (5, y)$
 $MB = \sqrt{(5-2)^2 + (y-0)^2} = 5$
 $9 + y^2 = 25 \quad \therefore y^2 = 16$
 $\therefore y = 4$ (negative solution is refused)
 $M = (5, 4)$
 $\therefore \text{equation of the circle} : (x-5)^2 + (y-4)^2 = 25$

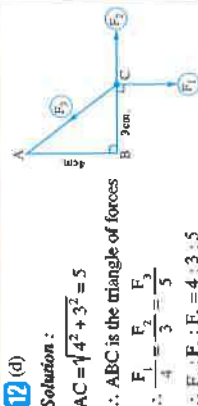
7 (a)

Solution :
 $\therefore \vec{R}$ is perpendicular to the force of magnitude 4 N.
 $\therefore 4 + F \cos 120^\circ = 0$
 $\therefore F = 8 \text{ newton}$
 $\therefore R = \sqrt{4^2 + 8^2 + 2 \times 4 \times 8 \times \cos 120^\circ} = 4\sqrt{3} \text{ N.}$

10 (a)



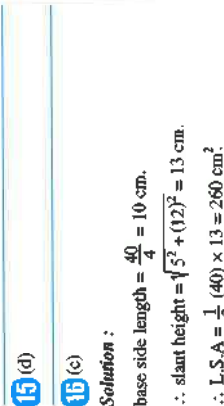
11 (c)



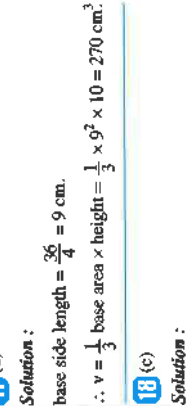
12 (d)



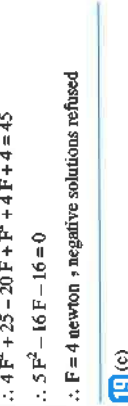
13 (c)



14 (c)



15 (c)



20 (d)

Solution :
 $X = 15 \cos 0^\circ + 15 \cos 120^\circ$
 $+ 5\sqrt{3} \cos 210^\circ$
 $+ 5\sqrt{3} \cos 270^\circ = 0$
 $Y = 15 \sin 0^\circ + 15 \sin 120^\circ$
 $+ 5\sqrt{3} \sin 210^\circ$
 $+ 5\sqrt{3} \sin 270^\circ = 0$
 $\therefore \vec{R} = (0, 0) \quad \therefore |\vec{R}| = 0$

21 (b)

Solution :
 $r = \sqrt{(-2-3)^2 + (5-2)^2} = \sqrt{34}$
 \therefore equation of the circle :
 $(x+2)^2 + (y-5)^2 = 34$
 $\therefore x^2 + y^2 + 4x - 10y - 5 = 0$

Model 3

1 (d)

Solution :
 The resulted solid is a right cone of radius length of the base
 $r = \sqrt{5^2 - 4^2} = 3$ cm. \therefore height = 4 cm.
 $\therefore v = \frac{1}{3} \pi (3^2) \times 4 = 12 \pi$ cm³

2 (b)

Solution :
 Let side length of the base = l
 $\therefore 200 = \frac{1}{3} l^2 \times 6 \quad \therefore l = 10$ cm.
 \therefore Slant height = $\sqrt{5^2 + 6^2} = \sqrt{61}$ cm.
 \therefore L.S.A. = $\frac{1}{2} \times 40 \times \sqrt{61} = 20\sqrt{61}$ cm²

3 (a)

Solution :
 Using lami's rule
 $\frac{20^\circ}{\sin 120^\circ} = \frac{T}{\sin 150^\circ} = \frac{T}{\sin 90^\circ}$
 $\therefore \frac{T}{T} = \sin 90^\circ = 2 : 1$

4 (a)

Solution :
 $\vec{R} = \vec{F}_1 + \vec{F}_2 = (3+a, 2+b) = (0, 0)$
 $\therefore a = -3, b = -2 \quad \therefore a+b = -5$

5 (d)

7 (d) 8 (b) 9 (c)

10 (a)

Solution :
 $R = \sqrt{3^2 + 5^2} = 2 \times 3 \times 5 \cos 60^\circ = 7$ N

11 (d)

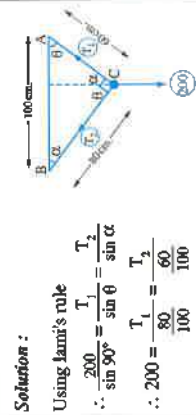
Solution :
 $\therefore R \in [4, 12] \quad \therefore F_1 - F_2 = 4$ (1)
 $F_1 + F_2 = 12$ (2)
 adding (1), (2) :
 $2F_1 = 16 \quad \therefore F_1 = 8, F_2 = 4$
 \therefore The resultant = 12 when $\alpha = 0$
 \therefore (c) is not true

12 (c)

Solution :
 $\therefore R \in [4, 12] \quad \therefore F_1 - F_2 = 4$ (1)
 $F_1 + F_2 = 12$ (2)
 adding (1), (2) :
 $2F_1 = 16 \quad \therefore F_1 = 8, F_2 = 4$
 \therefore The resultant = 12 when $\alpha = 0$
 \therefore (c) is not true

13 (c)

14 (c)



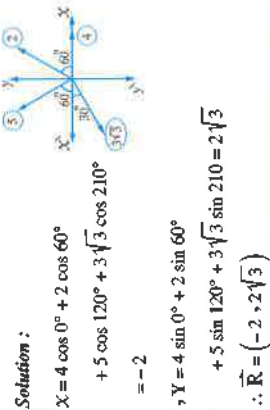
Solution :
 Using lami's rule
 $\frac{200}{\sin 90^\circ} = \frac{T_1}{\sin 60^\circ} = \frac{T_2}{\sin 30^\circ}$
 $\therefore 200 = \frac{T_1}{\frac{\sqrt{3}}{2}} = \frac{T_2}{\frac{1}{2}}$
 $\therefore T_1 = 160$ N, $T_2 = 120$ N

15 (b)

16 (b)

Solution :
 $\therefore 2\pi r = 44$ cm.
 $\therefore r = 7$ cm.
 $\therefore h = \sqrt{(21)^2 - 7^2} = 14\sqrt{2}$ cm.

17 (c)



18 (b)

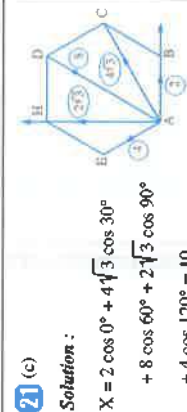
Solution :
 centre of the circle = $(0, 0)$
 radius length = $\sqrt{\left(\frac{2}{5} - 0\right)^2 + \left(\frac{-4}{5} - 0\right)^2} = 1$
 \therefore Equation of the circle : $x^2 + y^2 = 1$
 $\therefore 1^2 + 1^2 = 2 \neq 1$
 $\therefore (1, 1)$ does not lie on the circle

19 (d)

Solution :
 $M = (0, 0), r_1 = 2, N = (3, 0), r_2 = 3$
 $\therefore MN = 3, r_1 + r_2 = 5 > |r_1 - r_2| = 1$
 \therefore The two circles are intersecting.

20 (c)

Solution :
 $\therefore F = 2F \cos \frac{\alpha}{2} \quad \therefore \cos \frac{\alpha}{2} = \frac{1}{2}$
 $\therefore \frac{\alpha}{2} = 60^\circ \quad \therefore \alpha = 120^\circ$



21 (c)

Solution :

$$\begin{aligned} X &= 2 \cos 0^\circ + 4\sqrt{3} \cos 30^\circ \\ &\quad + 8 \cos 60^\circ + 2\sqrt{3} \cos 90^\circ \\ &\quad + 4 \cos 120^\circ = 10 \\ Y &= 2 \sin 0^\circ + 4\sqrt{3} \sin 30^\circ + 8 \sin 60^\circ \\ &\quad + 2\sqrt{3} \sin 90^\circ + 4 \sin 120^\circ = 10\sqrt{3} \\ \therefore \vec{R} &= (10, 10\sqrt{3}) \\ \therefore \vec{R} &= \sqrt{(10)^2 + (10\sqrt{3})^2} = 20 \\ \tan \theta &= \frac{10\sqrt{3}}{10} = \sqrt{3} \quad \therefore \theta = 60^\circ \\ \therefore \vec{R} &\text{ acts in direction of AD} \end{aligned}$$

Model 4

1 (b)

2 (b)

Solution :

$$\tan \theta = \frac{3\sqrt{2} \sin 135^\circ}{6 + 3\sqrt{2} \cos 135^\circ} = 1 \quad \therefore \theta = 45^\circ$$

3 (c)

Solution :

$$\begin{aligned} \therefore 6 &= 2 \times 6 \cos \frac{\alpha}{2} \quad \therefore \cos \frac{\alpha}{2} = \frac{1}{2} \\ \therefore \frac{\alpha}{2} &= 60^\circ \quad \therefore \alpha = 120^\circ \end{aligned}$$

4 (c)

$$\therefore 8 = 2F \cos \frac{\pi}{2} \quad \therefore F = 4\sqrt{2} \text{ N}$$

5 (c)

Solution :

$$F_2 = \frac{10 \sin 60^\circ}{\sin (90^\circ + 60^\circ)} = 10\sqrt{3} \text{ N}$$

6 (a)

7 (c)

Solution :

$$\begin{aligned} F_1 &= \frac{F \sin 45^\circ}{\sin 75^\circ}, F_2 = \frac{F \sin 30^\circ}{\sin 75^\circ} \\ \therefore F_1 + F_2 &= \frac{\sin 45^\circ + \sin 30^\circ}{\sin 75^\circ} \\ \therefore \frac{F_1 + F_2}{F} &= \frac{\sin 45^\circ + \sin 30^\circ}{\sin 75^\circ} \end{aligned}$$

8 (d)

Solution :

$$\begin{aligned} \vec{R} &= \vec{F}_1 + \vec{F}_2 = (12, -5) \\ \therefore \|\vec{R}\| &= \sqrt{(12)^2 + (-5)^2} = 13 \end{aligned}$$

9 (b)

10 (c)

Solution :

Using lamit's rule

$$\begin{aligned} \therefore \frac{T}{\sin 150^\circ} &= \frac{F}{\sin 120^\circ} = \frac{280^\circ}{\sin 90^\circ} \\ \therefore T &= 140 \text{ gm.wt} \\ F &= 140\sqrt{3} \text{ gm.wt} \\ \therefore \frac{F}{T} &= \sqrt{3} \end{aligned}$$

11 (c)

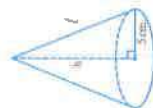
Solution :

$$F = \sqrt{5^2 + 3^2} + 2 \times 3 \times 5 \cos 60^\circ = 7 \text{ newton}$$

12 (b)

Solution :

$$\begin{aligned} \therefore \pi r \ell + \pi r^2 &= 90 \pi \\ \therefore 5 \ell + 25 &= 90 \\ \therefore \ell &= 13 \\ \therefore h &= \sqrt{(13)^2 - 5^2} = 12 \text{ cm} \\ \therefore v &= \frac{1}{3} \times 5^2 \pi \times 12 = 100 \pi \text{ cm}^3 \end{aligned}$$



13 (b)

Solution :

$$\begin{aligned} \ell &= 10 \text{ cm} \\ \therefore \text{T.S.A.} &= \pi \times 5 \times 10 + \pi \times 5^2 = 75 \pi \text{ cm}^2 \end{aligned}$$

14 (b)

Solution :

$$\begin{aligned} \text{side length of the base} &= \frac{26}{4} = 9 \\ \therefore v &= \frac{1}{3} \times 9^2 \times 10 = 270 \text{ cm}^3 \end{aligned}$$

15 (d)

Solution :

$$\begin{aligned} \text{side length of the base} &= \frac{40}{4} = 10 \text{ cm} \\ \therefore \text{Slant height} &= \sqrt{5^2 + (10)^2} = 13 \text{ cm} \\ \therefore \text{T.S.A.} &= \frac{1}{2} (40) \times 13 + (10)^2 = 360 \text{ cm}^2 \end{aligned}$$

16 (d)

Solution :

$$\begin{aligned} r^2 &= 8 \quad \therefore r = 2\sqrt{2} \\ \therefore \text{Circumference} &= 4\sqrt{2} \pi \end{aligned}$$

17 (a)

Solution :

$$\begin{aligned} r &= \sqrt{(3+2)^2 + (2-5)^2} = \sqrt{34} \text{ cm} \\ \therefore \text{The equation of the circle is } &(x+2)^2 + (y-5)^2 = 34 \\ \therefore x^2 + 4x + 4 + y^2 - 10y + 25 &= 34 \\ \therefore x^2 + y^2 + 4x - 10y - 5 &= 0 \end{aligned}$$

19 (c)

Solution :

$$\begin{aligned} r &= \sqrt{(-4-0)^2 + (3-0)^2} = 5 \\ \therefore \text{Equation of the circle is : } &(x+4)^2 + (y-3)^2 = 25 \end{aligned}$$

20 (b) 21 (d)

Model 5

1 (c)

2 (c)

Solution :

$$\begin{aligned} \therefore 12 &= \frac{1}{3} \text{ base area} \times 4 \quad \therefore \text{base area} = 9 \\ \therefore \text{Side length} &= 3 \text{ cm} \end{aligned}$$

3 (d)

Solution :

$$F_1 = \frac{12 \sin 30^\circ}{\sin 75^\circ} = 6 \csc 75^\circ$$

4 (a)

5 (b)

Solution :

$$\therefore 4 + F \cos 120^\circ = 0 \quad \therefore F = 8 \text{ N}$$

6 (a)

Solution :

$$\begin{aligned} \ell &= \sqrt{6^2 + 8^2} = 10 \text{ cm} \\ \therefore \text{L.S.A.} &= \pi \times 6 \times 10 \\ &= 60 \pi \text{ cm}^2 \end{aligned}$$



7 (c)

Solution :

$$r = 5 \quad \therefore \text{circumference} = 10 \pi$$

8 (c)

Solution :

$$\begin{aligned} \vec{R} &= (10\sqrt{2} \cos \frac{3\pi}{4}, 10\sqrt{2} \sin \frac{3\pi}{4}) \\ &= (-10, 10) \\ \therefore \vec{F}_1 + \vec{F}_2 + \vec{F}_3 &= (-9+a, 9+b) \\ \therefore -9+a &= -10 \quad \therefore a = -1 \\ 9+b &= 10 \quad \therefore b = 1 \quad \therefore a+b = 0 \end{aligned}$$

9 (d)

Solution :

$$\begin{aligned} \text{edge length} &= \frac{18}{6} = 3 \text{ cm} \\ \therefore \text{Total area} &= 4 \times \left(\frac{1}{2} \times 3 \times 3 \sin 60^\circ \right) = 9\sqrt{3} \end{aligned}$$

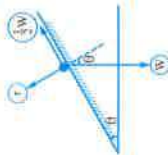
10 (a)

Solution :

Using lam's rule

$$\frac{1}{2} \frac{W}{\sin(180^\circ - \theta)} = \frac{W}{\sin 90^\circ}$$

$$\therefore \sin \theta = \frac{1}{2} \quad \therefore \theta = 30^\circ$$



11 (c)

Solution :

radius of the base = 3 cm.

$$\therefore 27\pi = \frac{1}{3}\pi(3^2)(h) \quad \therefore h = 9 \text{ cm.}$$

12 (d)

13 (d)

Solution :

Let \vec{AB} in direction of \vec{OX}

$$\therefore X = 15 \cos 0 + 15 \cos 120^\circ$$

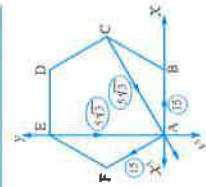
$$+ 5\sqrt{3} \cos 210^\circ$$

$$+ 5\sqrt{3} \cos 270^\circ = 0$$

$$Y = 15 \sin 0 + 15 \sin 120^\circ$$

$$+ 5\sqrt{3} \sin 210^\circ + 5\sqrt{3} \sin 270^\circ = 0$$

$$\therefore \vec{R} = (0, 0) \quad \therefore |\vec{R}| = 0$$



14 (b)

15 (d)

16 (b)

Solution :

$$8\sqrt{3} = \frac{1}{3} \text{ base area} \times 4$$

$$\therefore \text{base area} = 6\sqrt{3}$$

$$\therefore \frac{6}{4} x^2 \cot \frac{180}{6} = 6\sqrt{3} \quad \therefore x^2 = 4$$

$$\therefore \text{side length of hexagon} = 2 \text{ cm.}$$

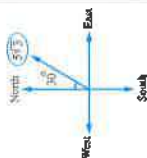
$$\therefore \text{perimeter of base} = 12 \text{ cm.}$$

17 (c)

Solution :

Component in east direction

$$= 5\sqrt{3} \sin 30^\circ = \frac{5\sqrt{3}}{2}$$



18 (c)

Solution :

$$M = (-4, 4), r = 4$$

$$\therefore \text{The equation is } (x+4)^2 + (y-4)^2 = 16$$

$$\therefore x^2 + 8x + 16 + y^2 - 8y + 16 = 16$$

$$\therefore x^2 + y^2 + 8x - 8y + 16 = 0$$

19 (c)

Solution :

$$\therefore 8 = 2F \cos \frac{90^\circ}{2}$$

$$\therefore F = 4\sqrt{2}$$

20 (c)

Solution :

$$12 = \frac{1}{3} \times 4 \times h \quad \therefore h = 9 \text{ cm.}$$

21 (c)

Solution :

The reaction bisects \overline{BC}

$\therefore \triangle ACD$ is the triangle

of forces

$$\therefore \frac{30}{20} = \frac{x}{10\sqrt{2}} = \frac{T}{10\sqrt{2}}$$

$$\therefore T = 15\sqrt{2}$$

